Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/7.5.1-u-a+b-arcsech-c-x-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [190]. This is test number [200].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (190)	% 0.00 (0)
Mathematica	% 97.37 (185)	% 2.63 (5)
Maple	% 79.47 (151)	% 20.53 (39)
Maxima	% 45.26 (86)	% 54.74 (104)
Fricas	% 63.16 (120)	% 36.84 (70)
Sympy	% 25.26 (48)	% 74.74 (142)
Giac	% 23.68 (45)	% 76.32 (145)
Mupad	% 27.37 (52)	% 72.63 (138)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

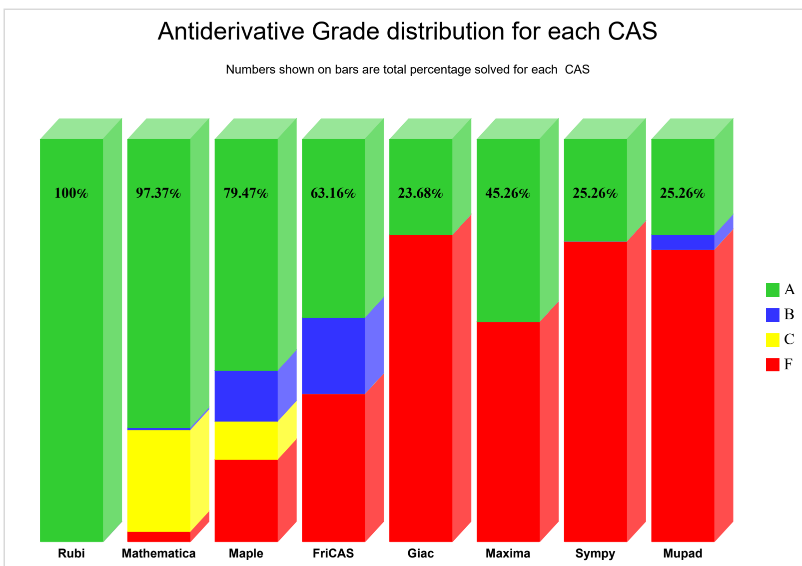
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

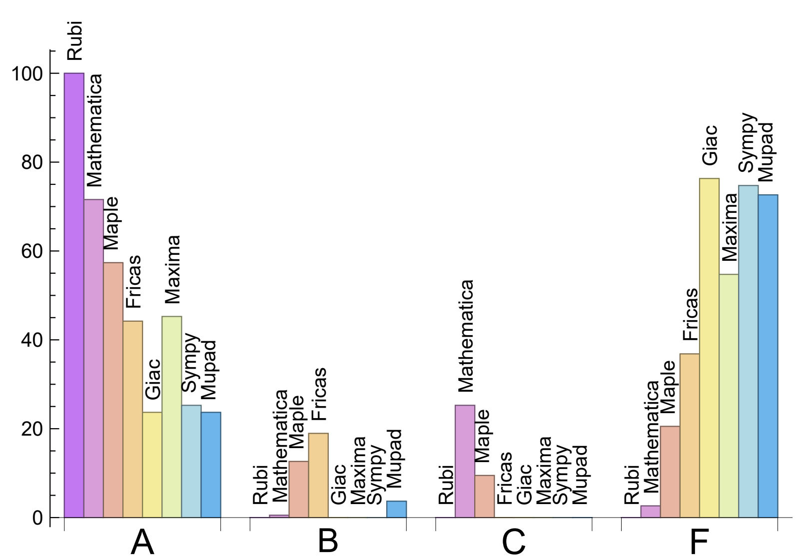
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	71.58	0.53	25.26	2.63
Maple	57.37	12.63	9.47	20.53
Maxima	45.26	0.00	0.00	54.74
Fricas	44.21	18.95	0.00	36.84
Sympy	25.26	0.00	0.00	74.74
Giac	23.68	0.00	0.00	76.32
Mupad	23.68	3.68	0.00	72.63

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	5	100.00 %	0.00 %	0.00 %
Maple	39	71.79 %	0.00 %	28.21 %
Maxima	104	82.69 %	7.69 %	9.62 %
Fricas	70	97.14 %	2.86 %	0.00 %
Sympy	142	80.28 %	19.72 %	0.00 %
Giac	145	98.62 %	0.00 %	1.38 %
Mupad	138	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

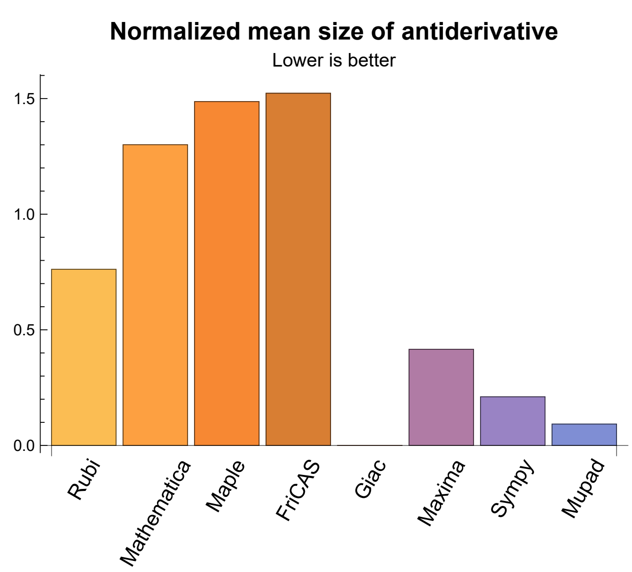
1.3 Performance

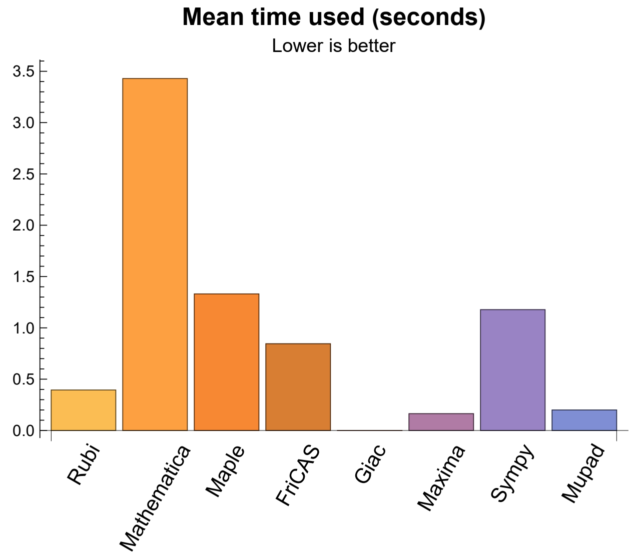
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.39	195.62	0.76	138.50	1.00
Mathematica	3.43	359.69	1.30	135.00	0.98
Maple	1.33	419.17	1.49	151.00	1.07
Maxima	0.16	59.27	0.42	0.00	0.00
Fricas	0.85	296.65	1.52	119.00	0.88
Sympy	1.18	30.50	0.21	0.00	0.00
Giac	0.00	0.00	0.00	0.00	0.00
Mupad	0.20	7.63	0.09	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {26, 111, 115, 116, 123}

Mathematica {1, 3, 5, 6, 10, 11, 12, 13, 14, 15, 26, 34, 36, 37, 42, 43, 44, 45, 46, 68, 78, 81, 82, 83, 84, 85, 86, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 127, 128, 129}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

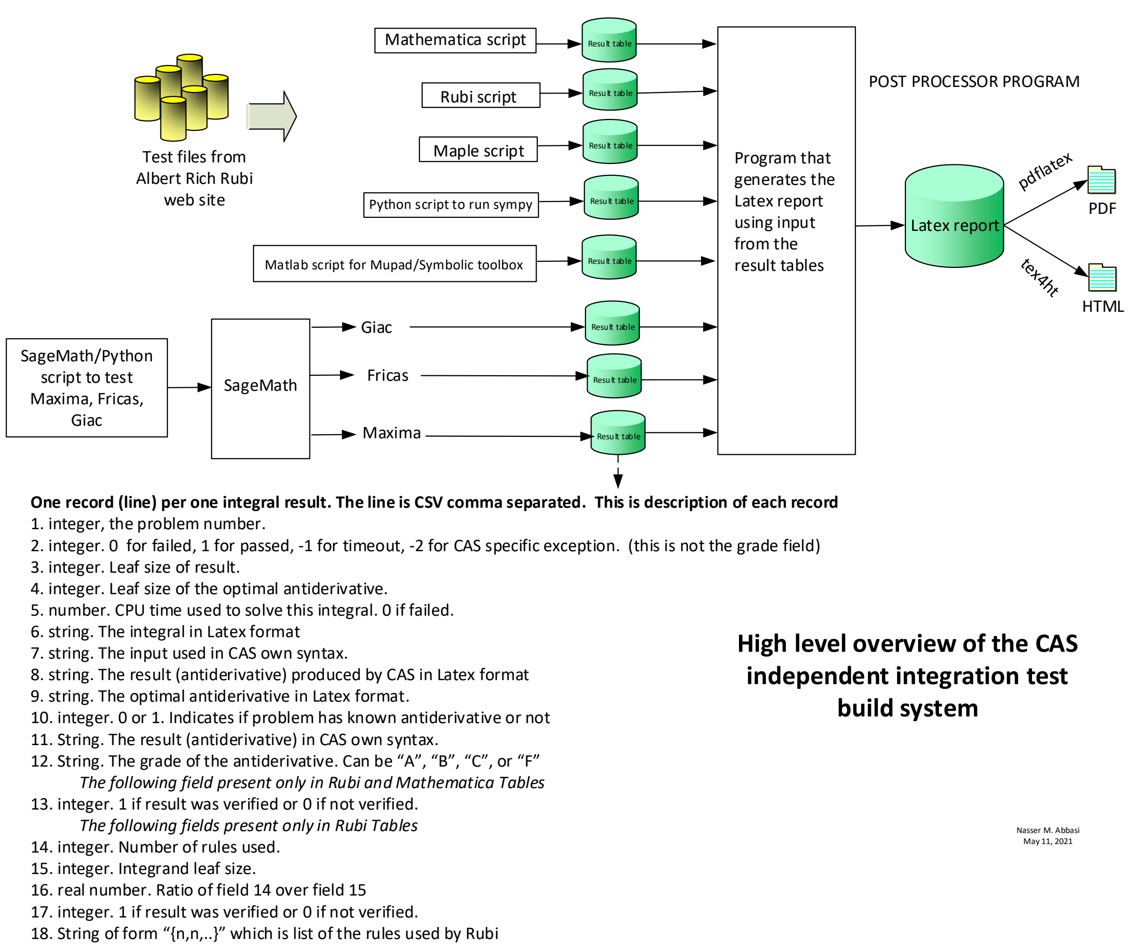
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 79, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 168, 169, 170, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }

B grade: { 45 }

C grade: { 19, 21, 23, 74, 75, 78, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 100, 101, 102, 103, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 138, 139, 148, 149, 157, 158, 166, 167, 175, 176 }

F grade: { 118, 126, 177, 178, 179 }

2.1.3 Maple

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 37, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 69, 70, 72, 73, 74, 75, 76, 77, 79, 82, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 182, 183, 184, 185, 189 }

B grade: { 4, 33, 35, 38, 42, 44, 46, 47, 48, 49, 50, 61, 62, 66, 67, 68, 80, 81, 84, 85, 86, 117, 124, 125 }

C grade: { 78, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129 }

F grade: { 10, 12, 14, 43, 45, 71, 130, 131, 132, 138, 139, 140, 141, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 181, 186, 187, 188, 190 }

2.1.4 Maxima

A grade: { 4, 7, 16, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 38, 47, 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 26, 33, 34, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 78, 79, 80, 81, 82, 83, 84, 85, 86, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

2.1.5 FriCAS

A grade: { 2, 8, 9, 16, 17, 18, 19, 20, 22, 27, 28, 29, 30, 31, 32, 39, 40, 41, 48, 49, 50, 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 88, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 130, 131, 133, 134, 135, 136, 137, 140, 142, 143, 144, 145, 146, 147, 150, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190 }

B grade: { 4, 7, 21, 23, 24, 25, 33, 35, 38, 47, 74, 75, 76, 77, 79, 80, 89, 90, 91, 101, 102, 103, 117, 124, 125, 132, 141, 151, 152, 159, 160, 161, 168, 169, 170, 188 }

C grade: { }

F grade: { 1, 3, 5, 6, 10, 11, 12, 13, 14, 15, 26, 34, 36, 37, 42, 43, 44, 45, 46, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 78, 81, 82, 83, 84, 85, 86, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 138, 139, 148, 149, 157, 158, 166, 167, 175, 176, 177, 178, 179 }

2.1.6 Sympy

A grade: { 4, 20, 22, 24, 35, 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 95, 96, 97, 106, 107, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 164, 165, 180, 183, 184, 185, 189, 190 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 186, 187, 188 }

2.1.7 Giac

A grade: { 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

2.1.8 Mupad

A grade: { 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190 }

B grade: { 24, 25, 27, 28, 76, 77, 91 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	182	289	0	0	0	0	-1
normalized size	1	1.00	1.11	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.464	0.946	0.000	0.583	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	77	151	0	125	0	0	-1
normalized size	1	1.00	0.74	1.45	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.118	0.798	0.000	0.608	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	138	240	0	0	0	0	-1
normalized size	1	1.00	1.18	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.254	0.715	0.000	0.689	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	101	40	106	42	0	-1
normalized size	1	1.00	1.00	1.91	0.75	2.00	0.79	0.00	-0.02
time (sec)	N/A	0.057	0.065	0.612	0.329	0.625	0.993	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	90	190	0	0	0	0	-1
normalized size	1	1.00	1.43	3.02	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.200	0.336	0.000	0.527	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	136	0	0	0	0	-1
normalized size	1	1.00	0.98	2.12	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	0.036	0.141	0.000	0.678	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	61	35	97	0	0	-1
normalized size	1	1.00	0.86	1.24	0.71	1.98	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.100	0.119	0.328	0.474	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	54	77	0	106	0	0	-1
normalized size	1	1.00	0.60	0.86	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.044	0.123	0.000	0.562	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	73	112	0	116	0	0	-1
normalized size	1	1.00	0.72	1.10	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.109	0.421	0.000	0.953	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	281	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.639	1.342	0.000	0.636	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	188	246	0	0	0	0	-1
normalized size	1	1.00	1.02	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.636	0.857	0.000	0.709	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	199	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.502	0.916	0.000	0.672	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	152	0	0	0	0	-1
normalized size	1	1.00	0.99	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.397	0.645	0.000	0.696	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	128	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.130	0.398	0.000	0.568	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	84	181	0	0	0	0	-1
normalized size	1	1.00	0.95	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.053	0.148	0.000	0.627	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	98	55	155	0	0	-1
normalized size	1	1.00	0.90	1.18	0.66	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.088	0.127	0.322	0.710	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	147	126	0	174	0	0	-1
normalized size	1	1.00	1.07	0.92	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.153	0.129	0.000	0.629	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	120	192	0	186	0	0	-1
normalized size	1	1.00	0.67	1.07	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.128	0.427	0.000	0.678	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	143	138	135	183	0	0	-1
normalized size	1	1.00	1.01	0.97	0.95	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.222	0.087	0.412	0.587	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	97	81	78	100	94	0	-1
normalized size	1	1.00	0.89	0.74	0.72	0.92	0.86	0.00	-0.01
time (sec)	N/A	0.047	0.110	0.065	0.311	0.510	5.567	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	123	118	106	174	0	0	-1
normalized size	1	1.00	1.12	1.07	0.96	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.136	0.064	0.410	0.553	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	72	57	90	68	0	-1
normalized size	1	1.00	1.00	0.94	0.74	1.17	0.88	0.00	-0.01
time (sec)	N/A	0.031	0.079	0.060	0.305	0.470	1.973	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	103	96	73	162	0	0	-1
normalized size	1	1.00	1.32	1.23	0.94	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.090	0.062	0.404	0.640	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	63	36	73	46	0	50
normalized size	1	1.00	1.27	1.40	0.80	1.62	1.02	0.00	1.11
time (sec)	N/A	0.014	0.054	0.061	0.302	0.583	0.526	0.000	1.391

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	60	42	31	119	0	0	44
normalized size	1	1.00	1.50	1.05	0.78	2.98	0.00	0.00	1.10
time (sec)	N/A	0.015	0.107	0.042	0.301	0.543	0.000	0.000	1.344

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	100	0	0	0	0	-1
normalized size	1	1.00	0.84	1.79	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	0.048	0.155	0.000	0.543	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	58	32	66	0	0	46
normalized size	1	1.00	1.05	1.45	0.80	1.65	0.00	0.00	1.15
time (sec)	N/A	0.020	0.057	0.061	0.307	0.486	0.000	0.000	1.476

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	117	112	105	77	0	0	61
normalized size	1	1.00	1.24	1.19	1.12	0.82	0.00	0.00	0.65
time (sec)	N/A	0.040	0.077	0.072	0.307	0.431	0.000	0.000	1.464

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	77	56	79	0	0	-1
normalized size	1	1.00	0.96	1.00	0.73	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.069	0.065	0.303	0.579	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	137	135	147	90	0	0	-1
normalized size	1	1.00	1.09	1.07	1.17	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.113	0.066	0.306	0.599	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	85	73	89	0	0	-1
normalized size	1	1.00	0.86	0.78	0.67	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.093	0.066	0.305	0.634	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	157	155	185	100	0	0	-1
normalized size	1	1.00	0.99	0.98	1.17	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.158	0.074	0.318	0.591	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	212	264	0	244	0	0	-1
normalized size	1	1.00	1.71	2.13	0.00	1.97	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.349	0.721	0.000	0.535	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	224	372	0	0	0	0	-1
normalized size	1	1.00	1.60	2.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	1.275	0.642	0.000	0.496	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	112	168	84	205	99	0	-1
normalized size	1	1.00	1.72	2.58	1.29	3.15	1.52	0.00	-0.02
time (sec)	N/A	0.075	0.253	0.579	0.320	0.702	1.167	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	126	250	0	0	0	0	-1
normalized size	1	1.00	1.62	3.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.266	0.297	0.000	0.699	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	116	250	0	0	0	0	-1
normalized size	1	1.00	1.40	3.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.174	0.128	0.000	0.620	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	87	124	78	143	0	0	-1
normalized size	1	1.00	1.43	2.03	1.28	2.34	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.243	0.139	0.318	0.587	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	183	192	0	165	0	0	-1
normalized size	1	1.00	1.55	1.63	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.179	0.141	0.000	0.580	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	134	192	0	181	0	0	-1
normalized size	1	1.00	1.10	1.57	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.290	0.440	0.000	0.524	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	268	264	0	204	0	0	-1
normalized size	1	1.00	1.77	1.75	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.282	0.473	0.000	0.616	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	337	546	0	0	0	0	-1
normalized size	1	1.00	1.51	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	2.146	0.955	0.000	0.615	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	440	0	0	0	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	1.113	1.017	0.000	0.539	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	219	343	0	0	0	0	-1
normalized size	1	1.00	1.74	2.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.954	0.715	0.000	0.606	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	282	0	0	0	0	0	-1
normalized size	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.442	0.417	0.000	0.521	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	182	454	0	0	0	0	-1
normalized size	1	1.00	1.60	3.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.249	0.151	0.000	0.475	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	165	227	144	228	0	0	-1
normalized size	1	1.00	1.62	2.23	1.41	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.359	0.171	0.329	0.542	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	245	321	0	271	0	0	-1
normalized size	1	1.00	1.50	1.97	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.478	0.174	0.000	0.480	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	256	387	0	305	0	0	-1
normalized size	1	1.00	1.20	1.82	0.00	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.420	0.506	0.000	0.719	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	332	485	0	351	0	0	-1
normalized size	1	1.00	1.37	2.00	0.00	1.45	0.00	0.00	-0.00
time (sec)	N/A	0.196	0.787	0.481	0.000	0.605	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.017	2.375	1.069	0.000	0.573	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	0.034	0.486	0.000	0.441	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.027	0.325	0.257	0.000	0.559	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	54	0	0	0	0	-1
normalized size	1	1.00	0.93	1.17	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.081	0.132	0.000	0.542	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	60	0	0	0	0	-1
normalized size	1	1.00	0.89	0.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.142	0.085	0.242	0.000	0.526	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	110	0	0	0	0	-1
normalized size	1	1.00	0.78	0.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.166	0.402	0.000	0.507	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.016	17.757	1.076	0.000	0.510	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	9.714	0.474	0.000	0.609	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	5.431	0.263	0.000	0.678	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	164	0	0	0	0	-1
normalized size	1	1.00	0.95	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.325	0.194	0.000	0.500	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	186	0	0	0	0	-1
normalized size	1	1.00	1.08	2.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.362	0.260	0.000	0.640	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	250	420	0	0	0	0	-1
normalized size	1	1.00	1.32	2.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.295	0.543	0.435	0.000	0.842	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.016	6.282	1.030	0.000	1.046	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	3.919	0.678	0.000	0.614	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	2.691	0.237	0.000	0.646	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	103	244	0	0	0	0	-1
normalized size	1	1.00	0.90	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.307	0.171	0.000	0.547	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	122	277	0	0	0	0	-1
normalized size	1	1.00	1.09	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.431	0.287	0.000	0.959	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	204	628	0	0	0	0	-1
normalized size	1	1.00	0.85	2.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	0.628	0.469	0.000	0.613	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	5.607	2.891	0.000	0.697	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	3.613	2.683	0.000	0.721	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	97	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.149	2.598	0.000	1.232	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	0.415	1.846	0.000	0.645	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	0.824	1.718	0.000	0.558	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	190	283	221	358	0	0	-1
normalized size	1	1.00	0.72	1.07	0.84	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.425	0.069	0.424	0.762	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	147	215	152	280	0	0	-1
normalized size	1	1.00	0.73	1.07	0.76	1.39	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.246	0.066	0.421	0.695	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	122	125	70	177	0	0	99
normalized size	1	1.00	0.86	0.88	0.49	1.25	0.00	0.00	0.70
time (sec)	N/A	0.118	0.377	0.063	0.319	0.679	0.000	0.000	1.527

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	60	42	31	119	0	0	44
normalized size	1	1.00	1.50	1.05	0.78	2.98	0.00	0.00	1.10
time (sec)	N/A	0.015	0.077	0.040	0.313	0.682	0.000	0.000	1.392

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	393	514	0	0	0	0	-1
normalized size	1	1.00	1.72	2.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.933	0.580	0.679	0.000	0.690	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	222	231	0	578	0	0	-1
normalized size	1	1.00	1.51	1.57	0.00	3.93	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.243	0.122	0.000	1.081	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	342	1090	0	1212	0	0	-1
normalized size	1	1.00	1.12	3.56	0.00	3.96	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.695	0.087	0.000	0.878	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	2653	830	0	0	0	0	-1
normalized size	1	1.00	7.73	2.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.621	10.779	0.228	0.000	173.840	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	2938	415	0	0	0	0	-1
normalized size	1	1.00	10.53	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	13.559	0.100	0.000	9.079	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1707	288	0	0	0	0	-1
normalized size	1	1.00	9.13	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	11.748	0.100	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	1675	253	0	0	0	0	-1
normalized size	1	1.00	15.95	2.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	11.339	0.094	0.000	7.574	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	4527	902	0	0	0	0	-1
normalized size	1	1.00	16.28	3.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	13.741	0.117	0.000	5.007	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	8675	1632	0	0	0	0	-1
normalized size	1	1.00	14.24	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	14.965	0.129	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	1.955	2.095	0.000	1.190	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	162	224	244	259	0	0	-1
normalized size	1	1.00	0.71	0.98	1.07	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.127	0.344	0.067	0.419	1.007	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	144	182	182	238	0	0	-1
normalized size	1	1.00	0.83	1.05	1.05	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.224	0.065	0.420	1.020	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	169	135	107	209	0	0	-1
normalized size	1	1.00	1.51	1.21	0.96	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.370	0.065	0.415	0.981	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	107	114	66	182	0	0	98
normalized size	1	1.00	1.11	1.19	0.69	1.90	0.00	0.00	1.02
time (sec)	N/A	0.066	0.263	0.067	0.316	0.531	0.000	0.000	1.809

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	76	123	91	106	0	0	-1
normalized size	1	1.00	0.60	0.98	0.72	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.119	0.068	0.315	0.612	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	101	142	132	128	0	0	-1
normalized size	1	1.00	0.55	0.78	0.72	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.176	0.072	0.316	0.630	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	117	160	165	149	0	0	-1
normalized size	1	1.00	0.49	0.67	0.69	0.63	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.222	0.076	0.318	0.580	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	126	150	177	168	228	0	-1
normalized size	1	1.00	0.54	0.65	0.76	0.72	0.98	0.00	-0.00
time (sec)	N/A	0.164	0.218	0.069	0.323	0.663	15.657	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	106	132	138	147	177	0	-1
normalized size	1	1.00	0.59	0.73	0.77	0.82	0.98	0.00	-0.01
time (sec)	N/A	0.135	0.188	0.063	0.316	0.498	5.809	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	85	113	96	125	126	0	-1
normalized size	1	1.00	0.52	0.69	0.59	0.76	0.77	0.00	-0.01
time (sec)	N/A	0.193	0.135	0.066	0.318	0.689	2.186	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	98	166	0	0	0	0	-1
normalized size	1	1.00	0.33	0.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.875	0.314	1.493	0.000	0.521	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	149	170	0	0	0	0	-1
normalized size	1	1.00	0.48	0.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	0.614	0.786	0.000	0.704	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	207	300	328	341	0	0	-1
normalized size	1	1.00	0.75	1.09	1.19	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.499	0.069	0.422	1.823	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	174	228	224	305	0	0	-1
normalized size	1	1.00	0.85	1.12	1.10	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.127	0.345	0.069	0.427	0.809	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	158	197	152	287	0	0	-1
normalized size	1	1.00	0.89	1.11	0.86	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.315	0.075	0.414	1.840	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	149	205	134	267	0	0	-1
normalized size	1	1.00	0.85	1.16	0.76	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.308	0.073	0.311	0.701	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	134	193	175	167	0	0	-1
normalized size	1	1.00	0.63	0.91	0.82	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.290	0.076	0.326	1.075	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	160	225	232	199	0	0	-1
normalized size	1	1.00	0.57	0.80	0.83	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.398	0.084	0.324	0.547	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	168	212	245	227	332	0	-1
normalized size	1	1.00	0.60	0.76	0.88	0.82	1.19	0.00	-0.00
time (sec)	N/A	0.237	0.310	0.071	0.329	1.084	16.463	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	139	180	185	192	252	0	-1
normalized size	1	1.00	0.60	0.78	0.80	0.83	1.10	0.00	-0.00
time (sec)	N/A	0.252	0.324	0.069	0.327	0.999	6.165	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	176	286	0	0	0	0	-1
normalized size	1	1.00	0.48	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.098	0.444	1.818	0.000	1.904	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	212	252	0	0	0	0	-1
normalized size	1	1.00	0.57	0.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.047	0.839	1.602	0.000	0.509	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	921	411	0	0	0	0	-1
normalized size	1	1.00	1.77	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.273	1.755	6.534	0.000	1.580	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	459	441	860	513	0	0	0	0	-1
normalized size	1	0.96	1.87	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.229	0.406	0.893	0.000	0.717	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	849	302	0	0	0	0	-1
normalized size	1	1.00	1.81	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.966	0.391	2.693	0.000	0.667	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	386	3157	0	0	0	0	-1
normalized size	1	1.00	0.93	7.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.989	0.935	0.892	0.000	0.641	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	933	372	0	0	0	0	-1
normalized size	1	1.00	1.78	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.261	1.862	6.503	0.000	0.715	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	631	611	1278	870	0	0	0	0	-1
normalized size	1	0.97	2.03	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.548	5.618	1.628	0.000	0.789	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	580	562	1208	661	0	0	0	0	-1
normalized size	1	0.97	2.08	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.463	1.290	1.107	0.000	0.855	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	345	840	0	602	0	0	-1
normalized size	1	1.00	2.35	5.71	0.00	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.245	1.053	0.101	0.000	0.663	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	0	3326	0	0	0	0	-1
normalized size	1	1.00	0.00	6.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.363	42.089	2.174	0.000	1.180	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	840	840	1270	2016	0	0	0	0	-1
normalized size	1	1.00	1.51	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.088	1.772	18.231	0.000	0.465	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	786	786	1226	1880	0	0	0	0	-1
normalized size	1	1.00	1.56	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.574	1.747	5.842	0.000	0.659	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	786	786	1216	1870	0	0	0	0	-1
normalized size	1	1.00	1.55	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.836	1.790	6.059	0.000	1.304	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	844	844	1305	1952	0	0	0	0	-1
normalized size	1	1.00	1.55	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.953	1.393	12.207	0.000	1.069	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	778	760	2000	1779	0	0	0	0	-1
normalized size	1	0.98	2.57	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.709	7.740	1.580	0.000	0.653	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	486	3331	0	1346	0	0	-1
normalized size	1	1.00	2.81	19.25	0.00	7.78	0.00	0.00	-0.01
time (sec)	N/A	0.189	1.593	0.112	0.000	0.900	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	486	3289	0	1232	0	0	-1
normalized size	1	1.00	2.24	15.16	0.00	5.68	0.00	0.00	-0.00
time (sec)	N/A	0.293	1.099	0.103	0.000	0.715	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	741	741	0	5713	0	0	0	0	-1
normalized size	1	1.00	0.00	7.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.542	63.224	2.779	0.000	0.574	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1272	1272	2022	3455	0	0	0	0	-1
normalized size	1	1.00	1.59	2.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.263	6.217	9.406	0.000	0.731	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1276	1276	2030	2537	0	0	0	0	-1
normalized size	1	1.00	1.59	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.040	6.143	9.280	0.000	1.123	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1272	1272	2015	3446	0	0	0	0	-1
normalized size	1	1.00	1.58	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.910	6.073	10.821	0.000	0.507	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	409	0	0	1995	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	4.46	0.00	0.00	-0.00
time (sec)	N/A	1.395	3.102	3.720	0.000	3.531	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	365	0	0	1669	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	5.07	0.00	0.00	-0.00
time (sec)	N/A	0.427	1.495	3.394	0.000	1.993	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	307	0	0	1382	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	6.25	0.00	0.00	-0.00
time (sec)	N/A	0.357	1.278	2.553	0.000	2.023	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.099	5.525	1.622	0.000	0.564	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.104	6.341	0.870	0.000	0.612	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.098	13.897	2.839	0.000	0.638	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	3.171	1.624	0.000	0.687	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.087	1.927	1.326	0.000	0.625	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	576	0	0	0	0	0	-1
normalized size	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	4.215	180.000	0.000	0.646	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	446	641	0	0	0	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	6.379	180.000	0.000	0.753	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	382	0	0	1989	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	4.76	0.00	0.00	-0.00
time (sec)	N/A	0.530	2.986	3.371	0.000	3.980	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	342	0	0	1667	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	5.61	0.00	0.00	-0.00
time (sec)	N/A	0.432	1.544	2.426	0.000	1.568	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.120	6.375	1.500	0.000	0.662	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.125	6.222	0.710	0.000	0.795	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.122	13.649	2.730	0.000	0.600	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	4.475	1.438	0.000	0.592	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.104	6.870	1.137	0.000	0.562	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.128	16.579	180.000	0.000	0.528	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	620	0	0	0	0	0	-1
normalized size	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	6.371	180.000	0.000	0.599	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	728	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.758	7.811	180.000	0.000	0.664	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	366	0	0	1679	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	4.72	0.00	0.00	-0.00
time (sec)	N/A	1.160	1.602	4.740	0.000	1.847	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	406	0	0	1389	0	0	-1
normalized size	1	1.00	1.62	0.00	0.00	5.53	0.00	0.00	-0.00
time (sec)	N/A	0.330	1.317	4.609	0.000	1.005	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	239	0	0	1102	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	7.20	0.00	0.00	-0.01
time (sec)	N/A	0.295	0.564	2.728	0.000	0.821	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.095	3.254	1.605	0.000	0.786	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.111	23.958	1.147	0.000	0.532	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.098	7.181	3.518	0.000	0.667	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	1.066	2.061	0.000	0.689	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	501	0	0	0	0	0	-1
normalized size	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	4.378	1.540	0.000	0.576	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	612	0	0	0	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	5.030	180.000	0.000	0.654	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	436	0	0	1771	0	0	-1
normalized size	1	1.00	1.57	0.00	0.00	6.37	0.00	0.00	-0.00
time (sec)	N/A	1.117	1.498	4.654	0.000	2.588	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	249	0	0	1311	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	7.41	0.00	0.00	-0.01
time (sec)	N/A	0.272	1.776	4.328	0.000	1.801	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	135	0	0	379	0	0	-1
normalized size	1	1.00	1.55	0.00	0.00	4.36	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.635	2.418	0.000	0.609	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.116	32.022	1.406	0.000	1.039	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.133	37.542	1.102	0.000	0.667	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.111	9.907	3.915	0.000	0.563	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	4.712	3.768	0.000	0.506	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	334	0	0	0	0	0	-1
normalized size	1	1.00	3.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	1.391	2.171	0.000	0.716	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	501	0	0	0	0	0	-1
normalized size	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	4.490	1.510	0.000	0.519	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	348	0	0	2415	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	8.88	0.00	0.00	-0.00
time (sec)	N/A	1.281	1.971	4.575	0.000	1.186	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	218	0	0	786	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	4.39	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.377	4.483	0.000	0.779	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	204	0	0	692	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	4.49	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.316	2.580	0.000	1.407	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.123	43.676	1.476	0.000	0.553	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.142	54.748	1.234	0.000	0.617	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.122	14.891	4.402	0.000	1.649	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.111	13.536	3.947	0.000	0.620	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	488	0	0	0	0	0	-1
normalized size	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	2.656	3.603	0.000	0.716	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	517	0	0	0	0	0	-1
normalized size	1	1.00	1.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	5.505	2.023	0.000	0.731	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F(-2)	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	596	576	0	0	0	0	0	0	-1
normalized size	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.545	0.246	180.000	0.000	1.511	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F(-2)	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	352	0	0	0	0	0	0	-1
normalized size	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	0.161	180.000	0.000	0.710	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F(-2)	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	192	0	0	0	0	0	0	-1
normalized size	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.126	180.000	0.000	0.489	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	2.483	2.377	0.000	0.722	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	7.550	180.000	0.000	1.046	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.110	1.077	1.794	0.000	0.611	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.099	0.122	1.724	0.000	0.614	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.100	1.420	2.583	0.000	1.650	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.110	1.729	2.456	0.000	0.664	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	213	0	0	393	0	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	1.579	0.443	6.404	0.000	0.671	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	178	0	0	336	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	1.06	0.00	0.00	-0.00
time (sec)	N/A	1.383	0.383	5.727	0.000	0.579	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	140	0	0	279	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.353	3.326	0.000	0.670	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	0.414	3.198	0.000	1.083	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.102	3.539	180.000	0.000	1.244	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.00	10	0.600
2	A	5	5	1.00	10	0.500
3	A	8	6	1.00	10	0.600
4	A	4	4	1.00	8	0.500
5	A	7	5	1.00	6	0.833
6	A	6	6	1.00	10	0.600
7	A	4	3	1.00	10	0.300
8	A	4	4	1.00	10	0.400
9	A	5	5	1.00	10	0.500
10	A	14	9	1.00	10	0.900
11	A	10	10	1.00	10	1.000
12	A	11	8	1.00	10	0.800
13	A	7	7	1.00	8	0.875
14	A	9	6	1.00	6	1.000
15	A	7	7	1.00	10	0.700
16	A	5	3	1.00	10	0.300
17	A	6	6	1.00	10	0.600
18	A	8	6	1.00	10	0.600
19	A	8	6	1.00	12	0.500
20	A	6	4	1.00	12	0.333
21	A	6	6	1.00	12	0.500
22	A	4	4	1.00	12	0.333
23	A	4	4	1.00	12	0.333
24	A	2	2	1.00	10	0.200
25	A	3	2	1.00	8	0.250
26	A	6	6	1.00	12	0.500
27	A	2	2	1.00	12	0.167
28	A	5	5	1.00	12	0.417
29	A	4	4	1.00	12	0.333
30	A	7	5	1.00	12	0.417
31	A	6	4	1.00	12	0.333
32	A	9	5	1.00	12	0.417
33	A	5	5	1.00	14	0.357
34	A	8	6	1.00	14	0.429
35	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	7	5	1.00	10	0.500
37	A	6	6	1.00	14	0.429
38	A	4	3	1.00	14	0.214
39	A	4	3	1.00	14	0.214
40	A	5	5	1.00	14	0.357
41	A	5	3	1.00	14	0.214
42	A	10	10	1.00	14	0.714
43	A	11	8	1.00	14	0.571
44	A	7	7	1.00	12	0.583
45	A	9	6	1.00	10	0.600
46	A	7	7	1.00	14	0.500
47	A	5	3	1.00	14	0.214
48	A	6	6	1.00	14	0.429
49	A	8	6	1.00	14	0.429
50	A	10	6	1.00	14	0.429
51	A	0	0	0.00	0	0.000
52	A	0	0	0.00	0	0.000
53	A	0	0	0.00	0	0.000
54	A	4	4	1.00	14	0.286
55	A	6	6	1.00	14	0.429
56	A	9	5	1.00	14	0.357
57	A	0	0	0.00	0	0.000
58	A	0	0	0.00	0	0.000
59	A	0	0	0.00	0	0.000
60	A	5	5	1.00	14	0.357
61	A	7	7	1.00	14	0.500
62	A	11	6	1.00	14	0.429
63	A	0	0	0.00	0	0.000
64	A	0	0	0.00	0	0.000
65	A	0	0	0.00	0	0.000
66	A	6	5	1.00	14	0.357
67	A	8	7	1.00	14	0.500
68	A	13	6	1.00	14	0.429
69	A	0	0	0.00	0	0.000
70	A	0	0	0.00	0	0.000
71	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	0	0	0.00	0	0.000
73	A	0	0	0.00	0	0.000
74	A	9	7	1.00	16	0.438
75	A	8	7	1.00	16	0.438
76	A	7	7	1.00	14	0.500
77	A	3	2	1.00	8	0.250
78	A	4	2	1.00	16	0.125
79	A	8	7	1.00	16	0.438
80	A	11	8	1.00	16	0.500
81	A	21	12	1.00	18	0.667
82	A	14	10	1.00	18	0.556
83	A	8	8	1.00	18	0.444
84	A	5	5	1.00	18	0.278
85	A	11	10	1.00	18	0.556
86	A	18	13	1.00	18	0.722
87	A	0	0	0.00	0	0.000
88	A	6	6	1.00	19	0.316
89	A	5	6	1.00	19	0.316
90	A	4	4	1.00	16	0.250
91	A	3	4	1.00	19	0.210
92	A	4	5	1.00	19	0.263
93	A	5	6	1.00	19	0.316
94	A	6	6	1.00	19	0.316
95	A	5	5	1.00	19	0.263
96	A	5	5	1.00	19	0.263
97	A	7	6	1.00	17	0.353
98	A	12	12	1.00	19	0.632
99	A	14	14	1.00	19	0.737
100	A	6	7	1.00	21	0.333
101	A	5	6	1.00	18	0.333
102	A	5	6	1.00	21	0.286
103	A	5	6	1.00	21	0.286
104	A	5	6	1.00	21	0.286
105	A	6	7	1.00	21	0.333
106	A	5	6	1.00	21	0.286
107	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	13	14	1.00	21	0.667
109	A	15	16	1.00	21	0.762
110	A	24	11	1.00	21	0.524
111	A	26	9	0.96	19	0.474
112	A	19	7	1.00	18	0.389
113	A	19	7	1.00	21	0.333
114	A	24	10	1.00	21	0.476
115	A	32	15	0.97	21	0.714
116	A	30	13	0.97	21	0.619
117	A	8	6	1.00	19	0.316
118	A	25	11	1.00	21	0.524
119	A	50	13	1.00	21	0.619
120	A	27	10	1.00	21	0.476
121	A	47	11	1.00	18	0.611
122	A	50	13	1.00	21	0.619
123	A	35	14	0.98	21	0.667
124	A	6	7	1.00	21	0.333
125	A	9	7	1.00	19	0.368
126	A	30	12	1.00	21	0.571
127	A	35	11	1.00	21	0.524
128	A	63	12	1.00	21	0.571
129	A	81	12	1.00	18	0.667
130	A	12	12	1.00	23	0.522
131	A	11	12	1.00	23	0.522
132	A	10	10	1.00	21	0.476
133	A	0	0	0.00	0	0.000
134	A	0	0	0.00	0	0.000
135	A	0	0	0.00	0	0.000
136	A	0	0	0.00	0	0.000
137	A	0	0	0.00	0	0.000
138	A	9	10	1.00	23	0.435
139	A	10	11	1.00	23	0.478
140	A	12	12	1.00	23	0.522
141	A	11	11	1.00	21	0.524
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	0	0	0.00	0	0.000
148	A	10	11	1.00	23	0.478
149	A	11	11	1.00	23	0.478
150	A	11	12	1.00	23	0.522
151	A	10	12	1.00	23	0.522
152	A	9	9	1.00	21	0.429
153	A	0	0	0.00	0	0.000
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000
157	A	9	10	1.00	23	0.435
158	A	9	11	1.00	23	0.478
159	A	10	11	1.00	23	0.478
160	A	9	11	1.00	23	0.478
161	A	5	5	1.00	21	0.238
162	A	0	0	0.00	0	0.000
163	A	0	0	0.00	0	0.000
164	A	0	0	0.00	0	0.000
165	A	0	0	0.00	0	0.000
166	A	4	5	1.00	20	0.250
167	A	8	10	1.00	23	0.435
168	A	10	11	1.00	23	0.478
169	A	7	8	1.00	23	0.348
170	A	6	6	1.00	21	0.286
171	A	0	0	0.00	0	0.000
172	A	0	0	0.00	0	0.000
173	A	0	0	0.00	0	0.000
174	A	0	0	0.00	0	0.000
175	A	8	9	1.00	23	0.391
176	A	8	10	1.00	20	0.500
177	A	5	6	0.97	23	0.261
178	A	5	6	0.95	23	0.261
179	A	4	5	0.93	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	0	0	0.00	0	0.000
181	A	0	0	0.00	0	0.000
182	A	0	0	0.00	0	0.000
183	A	0	0	0.00	0	0.000
184	A	0	0	0.00	0	0.000
185	A	0	0	0.00	0	0.000
186	A	15	10	1.00	26	0.385
187	A	12	10	1.00	26	0.385
188	A	7	8	1.00	26	0.308
189	A	0	0	0.00	0	0.000
190	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^4 \text{sech}^{-1}(ax)^2\,dx$

Optimal. Leaf size=164

$$\frac{3i\text{Li}_2\left(-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}-\frac{3i\text{Li}_2\left(ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}-\frac{3\text{sech}^{-1}(ax)\tan^{-1}\left(e^{\text{sech}^{-1}(ax)}\right)}{10a^5}-\frac{3x}{20a^4}-\frac{3x\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{20a^4}-$$

```
[Out] -3/20*x/a^4-1/30*x^3/a^2+1/5*x^5*arcsech(a*x)^2-3/10*arcsech(a*x)*arctan(1/
a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^5+3/20*I*polylog(2,-I*(1/a/x+(1/a/x-
1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-3/20*I*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1
+1/a/x)^(1/2)))/a^5-3/20*x*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^
4-1/10*x^3*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6285, 5418, 4185, 4180, 2279, 2391}

$$\frac{3i\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}-\frac{3i\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}-\frac{x^3}{30a^2}-\frac{x^3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{10a^2}-\frac{3x}{20a^4}-\frac{3x\sqrt{\frac{1-ax}{ax+1}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSech[a*x]^2,x]
```

```
[Out] (-3*x)/(20*a^4) - x^3/(30*a^2) - (3*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*A
rcSech[a*x])/(20*a^4) - (x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*
x])/(10*a^2) + (x^5*ArcSech[a*x]^2)/5 - (3*ArcSech[a*x]*ArcTan[E^ArcSech[a*
x]])/(10*a^5) + (((3*I)/20)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5 - (((3*I)/
20)*PolyLog[2, I*E^ArcSech[a*x]])/a^5
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int x^4 \operatorname{sech}^{-1}(ax)^2\,dx &= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}^5(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^5} \\
&= \frac{1}{5}x^5\operatorname{sech}^{-1}(ax)^2 - \frac{2\operatorname{Subst}\left(\int x\operatorname{sech}^5(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)}{5a^5} \\
&= -\frac{x^3}{30a^2} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\operatorname{sech}^{-1}(ax)^2 - \frac{3\operatorname{Subst}\left(\int x\operatorname{sech}^3(x)\,dx, x, \operatorname{se}\right.}{10a^5} \\
&= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\operatorname{sech} \\
&= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\operatorname{sech} \\
&= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\operatorname{sech} \\
&= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\operatorname{sech}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 182, normalized size = 1.11

$$\frac{12a^5x^5\operatorname{sech}^{-1}(ax)^2 - 2a^3x^3 - 6a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax) + 9i\operatorname{Li}_2\left(-ie^{-\operatorname{sech}^{-1}(ax)}\right) - 9i\operatorname{Li}_2\left(ie^{-\operatorname{sech}^{-1}(ax)}\right) - 9ax}{60a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcSech[a*x]^2,x]
```

```
[Out] (-9*a*x - 2*a^3*x^3 - 9*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x
] - 6*a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + 12*a^5*x^5
*ArcSech[a*x]^2 + (9*I)*ArcSech[a*x]*Log[1 - I/E^ArcSech[a*x]] - (9*I)*ArcS
ech[a*x]*Log[1 + I/E^ArcSech[a*x]] + (9*I)*PolyLog[2, (-I)/E^ArcSech[a*x]]
- (9*I)*PolyLog[2, I/E^ArcSech[a*x]])/(60*a^5)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\,\text{arsech}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*arcsech(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,\text{arsech}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsech(a*x)^2, x)
```

maple [A] time = 0.95, size = 289, normalized size = 1.76

$$\frac{x^5\text{arcsech}\,(ax)^2}{5}-\frac{\text{arcsech}\,(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,x^4}{10a}-\frac{3\,\text{arcsech}\,(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,x^2}{20a^3}-\frac{x^3}{30a^2}-\frac{3x}{20a^4}+\frac{3i\text{arcsech}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsech(a*x)^2,x)
```

```
[Out] 1/5*x^5*arcsech(a*x)^2-1/10/a*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/
x)^(1/2)*x^4-3/20/a^3*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)
*x^2-1/30*x^3/a^2-3/20*x/a^4+3/20*I/a^5*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1
)^(1/2)*(1+1/a/x)^(1/2)))-3/20*I/a^5*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(
1/2)*(1+1/a/x)^(1/2)))+3/20*I/a^5*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x
)^(1/2)))-3/20*I/a^5*dilog(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acosh(1/(a*x))^2,x)
```

```
[Out] int(x^4*acosh(1/(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{asech}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asech(a*x)**2,x)
[Out] Integral(x**4*asech(a*x)**2, x)
```

3.2 $\int x^3 \mathbf{sech}^{-1}(ax)^2\,dx$

Optimal. Leaf size=104

$$-\frac{\log(x)}{3a^4}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\mathrm{sech}^{-1}(ax)}{3a^4}-\frac{x^2}{12a^2}-\frac{x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\mathrm{sech}^{-1}(ax)}{6a^2}+\frac{1}{4}x^4\mathrm{sech}^{-1}(ax)^2$$

```
[Out] -1/12*x^2/a^2+1/4*x^4*arcsech(a*x)^2-1/3*ln(x)/a^4-1/3*(a*x+1)*arcsech(a*x)
*((-a*x+1)/(a*x+1))^(1/2)/a^4-1/6*x^2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1
))^(1/2)/a^2
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6285, 5418, 4185, 4184, 3475}

$$-\frac{x^2}{12a^2}-\frac{x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\mathrm{sech}^{-1}(ax)}{6a^2}-\frac{\log(x)}{3a^4}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\mathrm{sech}^{-1}(ax)}{3a^4}+\frac{1}{4}x^4\mathrm{sech}^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[a*x]^2,x]
```

```
[Out] -x^2/(12*a^2) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(3*a^4)
- (x^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(6*a^2) + (x^4*Arc
Sech[a*x]^2)/4 - Log[x]/(3*a^4)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3\text{sech}^{-1}(ax)^2\,dx = -\frac{\text{Subst}\left(\int x^2\text{sech}^4(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^4}$$
$$= \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 - \frac{\text{Subst}\left(\int x\text{sech}^4(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{2a^4}$$
$$= -\frac{x^2}{12a^2} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 - \frac{\text{Subst}\left(\int x\text{sech}^2(x)\,dx, x, \text{sech}\right.}{3a^4}$$
$$= -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 + \cdots$$
$$= -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 - \cdots$$

Mathematica [A] time = 0.12, size = 77, normalized size = 0.74

$$-\frac{-3a^4x^4\text{sech}^{-1}(ax)^2 + a^2x^2 + 2\sqrt{\frac{1-ax}{ax+1}}\left(a^3x^3 + a^2x^2 + 2ax + 2\right)\text{sech}^{-1}(ax) + 4\log(x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[a*x]^2,x]
```

```
[Out] -1/12*(a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(2 + 2*a*x + a^2*x^2 + a^3*x^3
)*ArcSech[a*x] - 3*a^4*x^4*ArcSech[a*x]^2 + 4*Log[x])/a^4
```

fricas [A] time = 0.61, size = 125, normalized size = 1.20

$$\frac{3\,a^4x^4\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - a^2x^2 - 2\left(a^3x^3 + 2\,ax\right)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) - 4\,\log(x)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*x^4*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - a^2
*x^2 - 2*(a^3*x^3 + 2*a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a
^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) - 4*log(x))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\text{arsech}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(a*x)^2, x)
```

maple [A] time = 0.80, size = 151, normalized size = 1.45

$$-\frac{\text{arcsech}\,(ax)}{3a^4} + \frac{x^4\text{arcsech}\,(ax)^2}{4} - \frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,\text{arcsech}\,(ax)\,x^3}{6a} - \frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,\text{arcsech}\,(ax)\,x}{3a^3} - \frac{x^2}{12a^2} + \frac{\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(a*x)^2,x)
```

```
[Out] -1/3/a^4*arcsech(a*x)+1/4*x^4*arcsech(a*x)^2-1/6/a*(-(a*x-1)/a/x)^(1/2)*((a
*x+1)/a/x)^(1/2)*arcsech(a*x)*x^3-1/3/a^3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x
)^(1/2)*arcsech(a*x)*x-1/12*x^2/a^2+1/3/a^4*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+
1/a/x)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arsech}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsech(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/(a*x))^2,x)
```

```
[Out] int(x^3*acosh(1/(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asech}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(a*x)**2,x)
```

```
[Out] Integral(x**3*asech(a*x)**2, x)
```

3.3 $\int x^2\text{sech}^{-1}(ax)^2\,dx$

Optimal. Leaf size=117

$$\frac{i\text{Li}_2\left(-ie^{\text{sech}^{-1}(ax)}\right)}{3a^3}-\frac{i\text{Li}_2\left(ie^{\text{sech}^{-1}(ax)}\right)}{3a^3}-\frac{2\text{sech}^{-1}(ax)\tan^{-1}\left(e^{\text{sech}^{-1}(ax)}\right)}{3a^3}-\frac{x}{3a^2}-\frac{x\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{3a^2}+\frac{1}{3}x^3\text{sec}$$

```
[Out] -1/3*x/a^2+1/3*x^3*arcsech(a*x)^2-2/3*arcsech(a*x)*arctan(1/a/x+(1/a/x-1)^(
1/2)*(1+1/a/x)^(1/2))/a^3+1/3*I*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/
x)^(1/2)))/a^3-1/3*I*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
^3-1/3*x*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6285, 5418, 4185, 4180, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{3a^3}-\frac{i\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{3a^3}-\frac{x}{3a^2}-\frac{x\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{3a^2}-\frac{2\text{sech}^{-1}(ax)\tan^{-1}\left(e^{\text{sec}}\right.}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a*x]^2,x]
```

```
[Out] -x/(3*a^2) - (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(3*a^2) +
 (x^3*ArcSech[a*x]^2)/3 - (2*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/(3*a^3) +
 ((I/3)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^3 - ((I/3)*PolyLog[2, I*E^ArcSec
h[a*x]])/a^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
```

```
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^2\text{sech}^{-1}(ax)^2\,dx = -\frac{\text{Subst}\left(\int x^2\text{sech}^3(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}$$
$$= \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\,\text{Subst}\left(\int x\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{\text{Subst}\left(\int x\text{sech}(x)\,dx, x, \text{sech}\right.}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{sech}^{-1}(ax)\tan^{-1}\left(e^{\text{sech}^{-1}(ax)}\right.}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{sech}^{-1}(ax)\tan^{-1}\left(e^{\text{sech}^{-1}(ax)}\right.}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{sech}^{-1}(ax)\tan^{-1}\left(e^{\text{sech}^{-1}(ax)}\right.}{3a^3}$$

Mathematica [A] time = 0.25, size = 138, normalized size = 1.18

$$\frac{a^3x^3\text{sech}^{-1}(ax)^2 + i\text{Li}_2\left(-ie^{-\text{sech}^{-1}(ax)}\right) - i\text{Li}_2\left(ie^{-\text{sech}^{-1}(ax)}\right) - ax - ax\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax) + i\text{sech}^{-1}(ax)}{3a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSech[a*x]^2,x]
```

```
[Out] (-(a*x) - a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + a^3*x^3*Ar
cSech[a*x]^2 + I*ArcSech[a*x]*Log[1 - I/E^ArcSech[a*x]] - I*ArcSech[a*x]*Lo
g[1 + I/E^ArcSech[a*x]] + I*PolyLog[2, (-I)/E^ArcSech[a*x]] - I*PolyLog[2,
I/E^ArcSech[a*x]])/(3*a^3)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{arsech}\,(ax)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsech(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{arsech}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(a*x)^2, x)
```

maple [A] time = 0.72, size = 240, normalized size = 2.05

$$\frac{x^3 \mathrm{arcsech}(ax)^2}{3} - \frac{\mathrm{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,x^2}{3a} - \frac{x}{3a^2} + \frac{i\mathrm{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^3} - \frac{i\mathrm{arcs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(a*x)^2,x)
```

```
[Out] 1/3*x^3*arcsech(a*x)^2-1/3/a*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x
)^(1/2)*x^2-1/3*x/a^2+1/3*I/a^3*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^(1/2)*
(1+1/a/x)^(1/2)))-1/3*I/a^3*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1
/a/x)^(1/2)))+1/3*I/a^3*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-
1/3*I/a^3*dilog(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsech}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsech(a*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/(a*x))^2,x)
```

```
[Out] int(x^2*acosh(1/(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asech}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(a*x)**2,x)
```

```
[Out] Integral(x**2*asech(a*x)**2, x)
```

3.4 $\int x\text{sech}^{-1}(ax)^2\,dx$

Optimal. Leaf size=53

$$-\frac{\log(x)}{a^2}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{a^2}+\frac{1}{2}x^2\text{sech}^{-1}(ax)^2$$

```
[Out] 1/2*x^2*arcsech(a*x)^2-ln(x)/a^2-(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1
/2)/a^2
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6285, 5418, 4184, 3475}

$$-\frac{\log(x)}{a^2}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{a^2}+\frac{1}{2}x^2\text{sech}^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a*x]^2,x]
```

```
[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/a^2) + (x^2*ArcSech[a*
x]^2)/2 - Log[x]/a^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\text{sech}^{-1}(ax)^2\,dx = -\frac{\text{Subst}\left(\int x^2\text{sech}^2(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}$$
$$= \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 - \frac{\text{Subst}\left(\int x\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}$$
$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 + \frac{\text{Subst}\left(\int \tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}$$
$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 - \frac{\log(x)}{a^2}$$

Mathematica [**A**] time = 0.06, size = 53, normalized size = 1.00

$$-\frac{\log(x)}{a^2} - \frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[a*x]^2,x]
[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/a^2) + (x^2*ArcSech[a*
x]^2)/2 - Log[x]/a^2
```

fricas [**B**] time = 0.63, size = 106, normalized size = 2.00

$$\frac{a^2x^2\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - 2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) - 2\,\log(x)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(a*x)^2,x, algorithm="fricas")
[Out] 1/2*(a^2*x^2*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 2*a*x*
sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)
/(a*x)) - 2*log(x))/a^2
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{arsech}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(a*x)^2,x, algorithm="giac")
[Out] integrate(x*arcsech(a*x)^2, x)
```

maple [**B**] time = 0.61, size = 101, normalized size = 1.91

$$-\frac{\text{arcsech}\,(ax)}{a^2}+\frac{x^2\text{arcsech}\,(ax)^2}{2}-\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\text{arcsech}\,(ax)\,x}{a}+\frac{\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(a*x)^2,x)
```

```
[Out] -1/a^2*arcsech(a*x)+1/2*x^2*arcsech(a*x)^2-1/a*(-(a*x-1)/a/x)^(1/2)*((a*x+1
)/a/x)^(1/2)*arcsech(a*x)*x+1/a^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)
```

maxima [A] time = 0.33, size = 40, normalized size = 0.75

$$\frac{1}{2}\,x^{2}\,\mathrm{arsech}\left(ax\right)^{2}-\frac{x\sqrt{\frac{1}{a^{2}x^{2}}-1}\,\mathrm{arsech}\left(ax\right)}{a}-\frac{\log(x)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsech(a*x)^2 - x*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)/a - log(x)/a^
2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a*x))^2,x)
```

```
[Out] int(x*acosh(1/(a*x))^2, x)
```

sympy [A] time = 0.99, size = 42, normalized size = 0.79

$$\begin{cases} \frac{x^{2}\,\mathrm{asech}^{2}\left(ax\right)}{2}-\frac{\sqrt{-a^{2}x^{2}+1}\,\mathrm{asech}\left(ax\right)}{a^{2}}-\frac{\log\left(x\right)}{a^{2}} & \text{for } a\neq 0\\ \infty x^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(a*x)**2,x)
```

```
[Out] Piecewise((x**2*asech(a*x)**2/2 - sqrt(-a**2*x**2 + 1)*asech(a*x)/a**2 - lo
g(x)/a**2, Ne(a, 0)), (oo*x**2, True))
```

3.5 $\int \operatorname{sech}^{-1}(ax)^2\,dx$

Optimal. Leaf size=63

$$\frac{2i\text{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}-\frac{2i\text{Li}_2\left(ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}+x\operatorname{sech}^{-1}(ax)^2-\frac{4\operatorname{sech}^{-1}(ax)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

```
[Out] x*arcsech(a*x)^2-4*arcsech(a*x)*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2
))/a+2*I*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a-2*I*polylo
g(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6279, 5418, 4180, 2279, 2391}

$$\frac{2i\text{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}-\frac{2i\text{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}+x\operatorname{sech}^{-1}(ax)^2-\frac{4\operatorname{sech}^{-1}(ax)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^2,x]
```

```
[Out] x*ArcSech[a*x]^2 - (4*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/a + ((2*I)*PolyL
og[2, (-I)*E^ArcSech[a*x]])/a - ((2*I)*PolyLog[2, I*E^ArcSech[a*x]])/a
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = -\frac{\operatorname{Subst}\left(\int x^2\operatorname{sech}(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)}{a}$$
$$= x\operatorname{sech}^{-1}(ax)^2 - \frac{2\operatorname{Subst}\left(\int x\operatorname{sech}(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)}{a}$$
$$= x\operatorname{sech}^{-1}(ax)^2 - \frac{4\operatorname{sech}^{-1}(ax)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{(2i)\operatorname{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \operatorname{sech}^{-1}\right.}{a}$$
$$= x\operatorname{sech}^{-1}(ax)^2 - \frac{4\operatorname{sech}^{-1}(ax)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{(2i)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(ax)}\right)}{a} -$$
$$= x\operatorname{sech}^{-1}(ax)^2 - \frac{4\operatorname{sech}^{-1}(ax)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{2i\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{2i\operatorname{Li}_2\left(ie^{\operatorname{sech}^{-1}(ax)}\right.}{a}$$

Mathematica [A] time = 0.20, size = 90, normalized size = 1.43

$$\frac{i\left(2\operatorname{Li}_2\left(-ie^{-\operatorname{sech}^{-1}(ax)}\right) - 2\operatorname{Li}_2\left(ie^{-\operatorname{sech}^{-1}(ax)}\right) + \operatorname{sech}^{-1}(ax)\left(-iax\operatorname{sech}^{-1}(ax) + 2\log\left(1-ie^{-\operatorname{sech}^{-1}(ax)}\right) - 2\log\left(1\right.\right.\right.}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a*x]^2,x]
```

```
[Out] (I*(ArcSech[a*x]*((-I)*a*x*ArcSech[a*x] + 2*Log[1 - I/E^ArcSech[a*x]] - 2*L
og[1 + I/E^ArcSech[a*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*PolyLog[2
, I/E^ArcSech[a*x]]))/a
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{arsech}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsech(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsech}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^2, x)
```

maple [A] time = 0.34, size = 190, normalized size = 3.02

$$x\operatorname{arcsech}(ax)^2 + \frac{2i\operatorname{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)}{a} - \frac{2i\operatorname{arcsech}(ax)\ln\left(1-i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1\right.\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^2,x)
```

```
[Out] x*arcsech(a*x)^2+2*I/a*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)
^(1/2)))-2*I/a*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))
+2*I/a*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-2*I/a*dilog(1-I*(
1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(\sqrt{ax+1}\sqrt{-ax+1}+1\right)^2 - \int -\frac{a^2x^2\log(a)^2 + \left(a^2x^2-1\right)\log(x)^2 + \left(a^2x^2\log(a)^2 + \left(a^2x^2-1\right)\log(x)^2 - \log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - integrate(-(a^2*x^2*log(a)^2 +
(a^2*x^2 - 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log(a
)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - 2*
(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*x^2 - 1)*log(x) - log(a))*sq
rt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log(x) - log(a))*log(sqrt(a*x +
1)*sqrt(-a*x + 1) + 1) - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))/(a^
2*x^2 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^2,x)
```

```
[Out] int(acosh(1/(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**2,x)
```

```
[Out] Integral(asech(a*x)**2, x)
```

3.6 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x} \, dx$

Optimal. Leaf size=64

$$-\operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right)+\frac{1}{2}\operatorname{Li}_3\left(-e^{2\operatorname{sech}^{-1}(ax)}\right)+\frac{1}{3}\operatorname{sech}^{-1}(ax)^3-\operatorname{sech}^{-1}(ax)^2\log\left(e^{2\operatorname{sech}^{-1}(ax)}+1\right)$$

```
[Out] 1/3*arcsech(a*x)^3-arcsech(a*x)^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)-arcsech(a*x)*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)+1
/2*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

Rubi [A] time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6285, 3718, 2190, 2531, 2282, 6589}

$$-\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(ax)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2\operatorname{sech}^{-1}(ax)}\right)+\frac{1}{3}\operatorname{sech}^{-1}(ax)^3-\operatorname{sech}^{-1}(ax)^2\log\left(e^{2\operatorname{sech}^{-1}(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^2/x,x]
```

```
[Out] ArcSech[a*x]^3/3 - ArcSech[a*x]^2*Log[1 + E^(2*ArcSech[a*x])] - ArcSech[a*x
]*PolyLog[2, -E^(2*ArcSech[a*x])] + PolyLog[3, -E^(2*ArcSech[a*x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
```

```
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx &= -\operatorname{Subst}\left(\int x^2\tanh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)\\
&= \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - 2\operatorname{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(ax)\right)\\
&= \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) + 2\operatorname{Subst}\left(\int x\log\left(1+e^{2x}\right)\,dx, x, \operatorname{sech}^{-1}\right.\\
&= \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \operatorname{Subst}\left(\right.\\
&= \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2}\operatorname{Subst}\\
&= \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2}\operatorname{Li}_3\left(-\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.98

$$\operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-e^{-2\operatorname{sech}^{-1}(ax)}\right)+\frac{1}{2}\operatorname{Li}_3\left(-e^{-2\operatorname{sech}^{-1}(ax)}\right)-\frac{1}{3}\operatorname{sech}^{-1}(ax)^3-\operatorname{sech}^{-1}(ax)^2\log\left(e^{-2\operatorname{sech}^{-1}(ax)}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a*x]^2/x,x]
[Out] -1/3*ArcSech[a*x]^3 - ArcSech[a*x]^2*Log[1 + E^(-2*ArcSech[a*x])] + ArcSech
[a*x]*PolyLog[2, -E^(-2*ArcSech[a*x])] + PolyLog[3, -E^(-2*ArcSech[a*x])]/2
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arsech}(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x,x, algorithm="fricas")
[Out] integral(arcsech(a*x)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^2/x, x)
```

maple [A] time = 0.14, size = 136, normalized size = 2.12

$$\frac{\operatorname{arcsech}(ax)^3}{3}-\operatorname{arcsech}(ax)^2\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right)-\operatorname{arcsech}(ax)\operatorname{polylog}\left(2,-\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^2/x,x)
```

```
[Out] 1/3*arcsech(a*x)^3-arcsech(a*x)^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)-arcsech(a*x)*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)+1
/2*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(a*x)^2/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^2/x,x)
```

```
[Out] int(acosh(1/(a*x))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**2/x,x)
```

```
[Out] Integral(asech(a*x)**2/x, x)
```

3.7 $\int \frac{\text{sech}^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$-\frac{\text{sech}^{-1}(ax)^2}{x}+\frac{2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{x}-\frac{2}{x}$$

```
[Out] -2/x-arcsech(a*x)^2/x+2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/x
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6285, 3296, 2638}

$$-\frac{\text{sech}^{-1}(ax)^2}{x}+\frac{2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{x}-\frac{2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^2/x^2,x]
```

```
[Out] -2/x + (2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/x - ArcSech[a*x
]^2/x
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^{-1}(ax)^2}{x^2}\,dx &= -\left(a\,\text{Subst}\left(\int x^2\sinh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\right)\\
&= -\frac{\text{sech}^{-1}(ax)^2}{x}+(2a)\,\text{Subst}\left(\int x\cosh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= \frac{2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{x}-\frac{\text{sech}^{-1}(ax)^2}{x}-(2a)\,\text{Subst}\left(\int \sinh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= -\frac{2}{x}+\frac{2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)}{x}-\frac{\text{sech}^{-1}(ax)^2}{x}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 42, normalized size = 0.86

$$-\frac{\text{sech}^{-1}(ax)^2-2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)+2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^2/x^2,x]
```

```
[Out] -((2 - 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + ArcSech[a*x]^2)
/x)
```

fricas [B] time = 0.47, size = 97, normalized size = 1.98

$$\frac{2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)-\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2
)) + 1)/(a*x)) - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 2)
/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{arsech}\,(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^2/x^2, x)
```

maple [A] time = 0.12, size = 61, normalized size = 1.24

$$a\left(-\frac{\mathrm{arcsech}\,(ax)^2}{ax}+2\,\mathrm{arcsech}\,(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}-\frac{2}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^2/x^2,x)
```

```
[Out] a*(-arcsech(a*x)^2/a/x+2*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)-2/a/x)
```

maxima [A] time = 0.33, size = 35, normalized size = 0.71

$$2\,a\sqrt{\frac{1}{a^2x^2}-1}\;\mathrm{arsech}\,(ax)-\frac{\mathrm{arsech}\,(ax)^2}{x}-\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x) - arcsech(a*x)^2/x - 2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{acosh}\left(\frac{1}{a\,x}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^2/x^2,x)
```

```
[Out] int(acosh(1/(a*x))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**2/x**2,x)
```

```
[Out] Integral(asech(a*x)**2/x**2, x)
```

3.8 $\int \frac{\text{sech}^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=90

$$-\frac{1}{4}a^2\text{sech}^{-1}(ax)^2-\frac{(1-ax)(ax+1)}{4x^2}-\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2x^2}+\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{2x^2}$$

```
[Out] -1/4*(-a*x+1)*(a*x+1)/x^2-1/4*a^2*arcsech(a*x)^2-1/2*(-a*x+1)*(a*x+1)*arcse
ch(a*x)^2/x^2+1/2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6285, 5372, 3310, 30}

$$-\frac{1}{4}a^2\text{sech}^{-1}(ax)^2-\frac{(1-ax)(ax+1)}{4x^2}-\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2x^2}+\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^2/x^3,x]
```

```
[Out] -((1 - a*x)*(1 + a*x))/(4*x^2) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSe
ch[a*x])/(2*x^2) - (a^2*ArcSech[a*x]^2)/4 - ((1 - a*x)*(1 + a*x)*ArcSech[a*
x]^2)/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^3}\,dx = -\left(a^2\,\text{Subst}\left(\int x^2\cosh(x)\sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)\right)$$
$$= -\frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^2}{2x^2} + a^2\,\text{Subst}\left(\int x\sinh^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$
$$= -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{2x^2} - \frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^2\,\text{Subst}$$
$$= -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{2x^2} - \frac{1}{4}a^2\text{sech}^{-1}(ax)^2 - \frac{(1-ax)(1+ax)\text{sech}^{-1}}{2x^2}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.60

$$\frac{\left(a^2x^2-2\right)\text{sech}^{-1}(ax)^2 + 2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax) - 1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^2/x^3,x]
```

```
[Out] (-1 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + (-2 + a^2*x^2)*A
rcSech[a*x]^2)/(4*x^2)
```

fricas [A] time = 0.56, size = 106, normalized size = 1.18

$$\frac{2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) + \left(a^2x^2-2\right)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - 1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2
*x^2)) + 1)/(a*x)) + (a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))
+ 1)/(a*x))^2 - 1)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsech}\,(ax)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^2/x^3, x)
```

maple [A] time = 0.12, size = 77, normalized size = 0.86

$$a^2\left(-\frac{\text{arcsech}\,(ax)^2}{2a^2x^2} + \frac{\text{arcsech}\,(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{2ax} + \frac{\text{arcsech}\,(ax)^2}{4} - \frac{1}{4x^2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^2/x^3,x)
```

[Out] a^2*(-1/2*arcsech(a*x)^2/a^2/x^2+1/2*arcsech(a*x)/a/x*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+1/4*arcsech(a*x)^2-1/4/x^2/a^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^2/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(1/(a*x))^2/x^3,x)

[Out] int(acosh(1/(a*x))^2/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**2/x**3,x)

[Out] Integral(asech(a*x)**2/x**3, x)

3.9 $\int \frac{\text{sech}^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=102

$$-\frac{4a^2}{9x}+\frac{4a^2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{9x}-\frac{\text{sech}^{-1}(ax)^2}{3x^3}+\frac{2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{9x^3}-\frac{2}{27x^3}$$

```
[Out] -2/27/x^3-4/9*a^2/x-1/3*arcsech(a*x)^2/x^3+2/9*(a*x+1)*arcsech(a*x)*((-a*x+
1)/(a*x+1))^(1/2)/x^3+4/9*a^2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)
/x
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6285, 5373, 3310, 3296, 2638}

$$-\frac{4a^2}{9x}+\frac{4a^2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{9x}-\frac{\text{sech}^{-1}(ax)^2}{3x^3}+\frac{2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)}{9x^3}-\frac{2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^2/x^4,x]
```

```
[Out] -2/(27*x^3) - (4*a^2)/(9*x) + (2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSec
h[a*x])/(9*x^3) + (4*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/
(9*x) - ArcSech[a*x]^2/(3*x^3)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx &= -\left(a^3\,\text{Subst}\left(\int x^2\cosh^2(x)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)\right)\\
&= -\frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{1}{3}\left(2a^3\right)\text{Subst}\left(\int x\cosh^3(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)\\
&= -\frac{2}{27x^3} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{1}{9}\left(4a^3\right)\text{Subst}\left(\int x\cosh(x)\,dx, x\right.\\
&= -\frac{2}{27x^3} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} - \\
&= -\frac{2}{27x^3} - \frac{4a^2}{9x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x} - \frac{\operatorname{sech}^{-1}(a}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 0.72

$$\frac{-2\left(6a^2x^2+1\right)+6\sqrt{\frac{1-ax}{ax+1}}\left(2a^3x^3+2a^2x^2+ax+1\right)\operatorname{sech}^{-1}(ax)-9\operatorname{sech}^{-1}(ax)^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^2/x^4,x]
```

```
[Out] (-2*(1 + 6*a^2*x^2) + 6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2 + 2*
a^3*x^3)*ArcSech[a*x] - 9*ArcSech[a*x]^2)/(27*x^3)
```

fricas [A] time = 0.95, size = 116, normalized size = 1.14

$$-\frac{12\,a^2x^2-6\left(2\,a^3x^3+ax\right)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)+9\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2+2}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(12*a^2*x^2 - 6*(2*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log(
(a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) + 9*log((a*x*sqrt(-(a^2*x^2
 - 1)/(a^2*x^2)) + 1)/(a*x))^2 + 2)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^2/x^4, x)
```

maple [A] time = 0.42, size = 112, normalized size = 1.10

$$a^3\left(-\frac{\operatorname{arcsech}(ax)^2}{3a^3x^3}+\frac{4\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{9}+\frac{2\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{9a^2x^2}-\frac{4}{9ax}-\frac{2}{27a^3x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^2/x^4,x)
```

```
[Out] a^3*(-1/3*arcsech(a*x)^2/a^3/x^3+4/9*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*
x+1)/a/x)^(1/2)+2/9*arcsech(a*x)/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)
^(1/2)-4/9/a/x-2/27/a^3/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(a*x)^2/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^2/x^4,x)
```

```
[Out] int(acosh(1/(a*x))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**2/x**4,x)
```

```
[Out] Integral(asech(a*x)**2/x**4, x)
```

3.10 $\int x^4 \operatorname{sech}^{-1}(ax)^3 \, dx$

Optimal. Leaf size=297

$$\frac{9i\operatorname{sech}^{-1}(ax)\text{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5}-\frac{9i\operatorname{sech}^{-1}(ax)\text{Li}_2\left(ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5}-\frac{9i\text{Li}_3\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5}+\frac{9i\text{Li}_3\left(ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5}+\frac{\tan^{-1}}{}$$

```
[Out] -9/20*x*arcsech(a*x)/a^4-1/10*x^3*arcsech(a*x)/a^2+1/5*x^5*arcsech(a*x)^3-9
/20*arcsech(a*x)^2*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^5+1/2*ar
ctan((a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a/x)/a^5+9/20*I*arcsech(a*x)*polylog(
2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-9/20*I*arcsech(a*x)*polyl
og(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-9/20*I*polylog(3,-I*(1/
a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5+9/20*I*polylog(3,I*(1/a/x+(1/a/x-
1)^(1/2)*(1+1/a/x)^(1/2)))/a^5+1/20*x*(a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a^4-
9/40*x*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^4-3/20*x^3*(a*x+1)
*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] time = 0.20, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6285, 5418, 4186, 3768, 3770, 4180, 2531, 2282, 6589}

$$\frac{9i\operatorname{sech}^{-1}(ax)\text{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5}-\frac{9i\operatorname{sech}^{-1}(ax)\text{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5}-\frac{9i\text{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSech[a*x]^3,x]
```

```
[Out] (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(20*a^4) - (9*x*ArcSech[a*x])/(20*a
^4) - (x^3*ArcSech[a*x])/(10*a^2) - (9*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x
)*ArcSech[a*x]^2)/(40*a^4) - (3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*Arc
Sech[a*x]^2)/(20*a^2) + (x^5*ArcSech[a*x]^3)/5 - (9*ArcSech[a*x]^2*ArcTan[E
^ArcSech[a*x]])/(20*a^5) + ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*
x)]/(2*a^5) + (((9*I)/20)*ArcSech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5
 - (((9*I)/20)*ArcSech[a*x]*PolyLog[2, I*E^ArcSech[a*x]])/a^5 - (((9*I)/20)
*PolyLog[3, (-I)*E^ArcSech[a*x]])/a^5 + (((9*I)/20)*PolyLog[3, I*E^ArcSech[
a*x]])/a^5
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^4\text{sech}^{-1}(ax)^3\,dx = -\frac{\text{Subst}\left(\int x^3\text{sech}^5(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^5}$$
$$= \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 - \frac{3\,\text{Subst}\left(\int x^2\text{sech}^5(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{5a^5}$$
$$= -\frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 + \frac{\text{Subst}\left(\int \text{sech}^3(\right.}{}$$
$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3}{}$$
$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3}{}$$
$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3}{}$$
$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3}{}$$
$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3}{}$$

Mathematica [A] time = 0.64, size = 281, normalized size = 0.95

$$\frac{8a^5x^5\text{sech}^{-1}(ax)^3 - 6a^3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2 - 4a^3x^3\text{sech}^{-1}(ax) + 18i\text{sech}^{-1}(ax)\text{Li}_2\left(-ie^{-\text{sech}^{-1}(ax)}\right) -}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcSech[a*x]^3,x]
```

```
[Out] (2*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 18*a*x*ArcSech[a*x] - 4*a^3*x^
3*ArcSech[a*x] - 9*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 -
 6*a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 8*a^5*x^5*A
rcSech[a*x]^3 + 40*ArcTan[Tanh[ArcSech[a*x]/2]] + (9*I)*ArcSech[a*x]^2*Log[
1 - I/E^ArcSech[a*x]] - (9*I)*ArcSech[a*x]^2*Log[1 + I/E^ArcSech[a*x]] + (1
8*I)*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - (18*I)*ArcSech[a*x]*Pol
yLog[2, I/E^ArcSech[a*x]] + (18*I)*PolyLog[3, (-I)/E^ArcSech[a*x]] - (18*I)
*PolyLog[3, I/E^ArcSech[a*x]])/(40*a^5)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\,\text{arsech}\,(ax)^3\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4*arcsech(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,\text{arsech}\,(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsech(a*x)^3, x)
```

maple **[F]** time = 1.34, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{arcsech}\,(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsech(a*x)^3,x)
```

```
[Out] int(x^4*arcsech(a*x)^3,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,\mathrm{arsech}\,(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsech(a*x)^3, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acosh(1/(a*x))^3,x)
```

```
[Out] int(x^4*acosh(1/(a*x))^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,\mathrm{asech}^3\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asech(a*x)**3,x)
```

```
[Out] Integral(x**4*asech(a*x)**3, x)
```

3.11 $\int x^3 \operatorname{sech}^{-1}(ax)^3\,dx$

Optimal. Leaf size=184

$$\frac{\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right)}{2a^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{4a^4}-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\operatorname{sech}^{-1}(ax)^2}{2a^4}-\frac{\operatorname{sech}^{-1}(ax)^2}{2a^4}+\frac{\operatorname{sech}^{-1}(ax)\log\left(e^{2\operatorname{sech}^{-1}(ax)}+\right.}{a^4}$$

```
[Out] -1/4*x^2*arcsech(a*x)/a^2-1/2*arcsech(a*x)^2/a^4+1/4*x^4*arcsech(a*x)^3+arc
sech(a*x)*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^4+1/2*polylog(2
,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^4+1/4*(a*x+1)*((-a*x+1)/(a*x
+1))^(1/2)/a^4-1/2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^4-1/4*
x^2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6285, 5418, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391}

$$\frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(ax)}\right)}{2a^4}-\frac{x^2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\operatorname{sech}^{-1}(ax)^2}{4a^2}-\frac{x^2\operatorname{sech}^{-1}(ax)}{4a^2}+\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{4a^4}-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\operatorname{sec}}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[a*x]^3,x]
```

```
[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(4*a^4) - (x^2*ArcSech[a*x])/(4*a^2)
- ArcSech[a*x]^2/(2*a^4) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x
]^2)/(2*a^4) - (x^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(4*
a^2) + (x^4*ArcSech[a*x]^3)/4 + (ArcSech[a*x]*Log[1 + E^(2*ArcSech[a*x])])/
a^4 + PolyLog[2, -E^(2*ArcSech[a*x])]/(2*a^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3\text{sech}^{-1}(ax)^3\,dx = -\frac{\text{Subst}\left(\int x^3\text{sech}^4(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^4}$$
$$= \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 - \frac{3\,\text{Subst}\left(\int x^2\text{sech}^4(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 + \frac{\text{Subst}\left(\int \text{sech}^2(x\right.}{4}$$
$$= -\frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{se}$$
$$= \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-}{1+}}}{}$$
$$= \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-}{1+}}}{}$$
$$= \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-}{1+}}}{}$$
$$= \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-}{1+}}}{}$$

Mathematica [A] time = 0.64, size = 188, normalized size = 1.02

$$\frac{a^4x^4\text{sech}^{-1}(ax)^3 + \text{sech}^{-1}(ax)\left(4\log\left(e^{-2\text{sech}^{-1}(ax)}+1\right) - a^2x^2\right) - \left(a^3x^3\sqrt{\frac{1-ax}{ax+1}} + a^2x^2\sqrt{\frac{1-ax}{ax+1}} + 2ax\sqrt{\frac{1-ax}{ax+1}} + 2\right.}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcSech[a*x]^3,x]
```

```
[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - (-2 + 2*Sqrt[(1 - a*x)/(1 + a*x)] +
2*a*x*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 - a*x)/(1 + a*x)] + a^3*x
^3*Sqrt[(1 - a*x)/(1 + a*x)])*ArcSech[a*x]^2 + a^4*x^4*ArcSech[a*x]^3 + Arc
Sech[a*x]*(-(a^2*x^2) + 4*Log[1 + E^(-2*ArcSech[a*x])]) - 2*PolyLog[2, -E^(
-2*ArcSech[a*x])])/(4*a^4)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\,\text{arsech}\,(ax)^3\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsech(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\text{arsech}\,(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(a*x)^3, x)
```

maple [A] time = 0.86, size = 246, normalized size = 1.34

$$\frac{x^4\operatorname{arcsech}(ax)^3}{4}-\frac{\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,x^3}{4a}-\frac{\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,x}{2a^3}-\frac{x^2\operatorname{arcsech}(ax)}{4a^2}+\frac{\sqrt{-\frac{ax-1}{ax}}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(a*x)^3,x)
```

```
[Out] 1/4*x^4*arcsech(a*x)^3-1/4/a*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a
/x)^(1/2)*x^3-1/2/a^3*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/
2)*x-1/4*x^2*arcsech(a*x)/a^2+1/4/a^3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)*x-1/2*arcsech(a*x)^2/a^4-1/4/a^4+arcsech(a*x)*ln(1+(1/a/x+(1/a/x-1)^(1/
2)*(1+1/a/x)^(1/2))^2)/a^4+1/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^
(1/2))^2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arsech}(ax)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsech(a*x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/(a*x))^3,x)
```

```
[Out] int(x^3*acosh(1/(a*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asech}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(a*x)**3,x)
```

```
[Out] Integral(x**3*asech(a*x)**3, x)
```

3.12 $\int x^2\text{sech}^{-1}(ax)^3\,dx$

Optimal. Leaf size=198

$$\frac{i\text{sech}^{-1}(ax)\text{Li}_2\left(-ie^{\text{sech}^{-1}(ax)}\right)}{a^3}-\frac{i\text{sech}^{-1}(ax)\text{Li}_2\left(ie^{\text{sech}^{-1}(ax)}\right)}{a^3}-\frac{i\text{Li}_3\left(-ie^{\text{sech}^{-1}(ax)}\right)}{a^3}+\frac{i\text{Li}_3\left(ie^{\text{sech}^{-1}(ax)}\right)}{a^3}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{a}}\right.}{a^3}$$

```
[Out] -x*arcsech(a*x)/a^2+1/3*x^3*arcsech(a*x)^3-arcsech(a*x)^2*arctan(1/a/x+(1/a
/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^3+arctan((a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a/
x)/a^3+I*arcsech(a*x)*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))
/a^3-I*arcsech(a*x)*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^
3-I*polylog(3,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^3+I*polylog(3,I
*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^3-1/2*x*(a*x+1)*arcsech(a*x)^2*
((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] time = 0.14, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6285, 5418, 4186, 3770, 4180, 2531, 2282, 6589}

$$\frac{i\text{sech}^{-1}(ax)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3}-\frac{i\text{sech}^{-1}(ax)\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a^3}-\frac{i\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3}+\frac{i\text{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a*x]^3,x]
```

```
[Out] -((x*ArcSech[a*x])/a^2) - (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*
x]^2)/(2*a^2) + (x^3*ArcSech[a*x]^3)/3 - (ArcSech[a*x]^2*ArcTan[E^ArcSech[a
*x]])/a^3 + ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x)]/a^3 + (I*Ar
cSech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^3 - (I*ArcSech[a*x]*PolyLog[2
, I*E^ArcSech[a*x]])/a^3 - (I*PolyLog[3, (-I)*E^ArcSech[a*x]])/a^3 + (I*Pol
yLog[3, I*E^ArcSech[a*x]])/a^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{sech}^{-1}(ax)^3\,dx = -\frac{\text{Subst}\left(\int x^3\text{sech}^3(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}$$
$$= \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{Subst}\left(\int x^2\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{Subst}\left(\int x^2\text{sech}(x)\right.}{2a}$$
$$= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{sech}^{-1}(ax)^2\tan^{-1}\left(\right.}{a^3}$$
$$= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{sech}^{-1}(ax)^2\tan^{-1}\left(\right.}{a^3}$$
$$= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{sech}^{-1}(ax)^2\tan^{-1}\left(\right.}{a^3}$$
$$= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{sech}^{-1}(ax)^2\tan^{-1}\left(\right.}{a^3}$$

Mathematica [A] time = 0.50, size = 199, normalized size = 1.01

$$\frac{2a^3x^3\text{sech}^{-1}(ax)^3 + 3i\left(2\text{sech}^{-1}(ax)\text{Li}_2\left(-ie^{-\text{sech}^{-1}(ax)}\right) - 2\text{sech}^{-1}(ax)\text{Li}_2\left(ie^{-\text{sech}^{-1}(ax)}\right) + 2\text{Li}_3\left(-ie^{-\text{sech}^{-1}(ax)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSech[a*x]^3,x]
```

```
[Out] (-6*a*x*ArcSech[a*x] - 3*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*
x]^2 + 2*a^3*x^3*ArcSech[a*x]^3 + (3*I)*((-4*I)*ArcTan[Tanh[ArcSech[a*x]/2]
] + ArcSech[a*x]^2*Log[1 - I/E^ArcSech[a*x]] - ArcSech[a*x]^2*Log[1 + I/E^A
rcSech[a*x]] + 2*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*ArcSech[a
*x]*PolyLog[2, I/E^ArcSech[a*x]] + 2*PolyLog[3, (-I)/E^ArcSech[a*x]] - 2*Po
lyLog[3, I/E^ArcSech[a*x]]))/(6*a^3)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{arsech}\,(ax)^3\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsech(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{arsech}\,(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(a*x)^3, x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsech}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(a*x)^3,x)
```

```
[Out] int(x^2*arcsech(a*x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsech}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsech(a*x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/(a*x))^3,x)
```

```
[Out] int(x^2*acosh(1/(a*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asech}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(a*x)**3,x)
```

```
[Out] Integral(x**2*asech(a*x)**3, x)
```

3.13 $\int x\text{sech}^{-1}(ax)^3\,dx$

Optimal. Leaf size=102

$$\frac{3\text{Li}_2\left(-e^{2\text{sech}^{-1}(ax)}\right)}{2a^2}-\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)^2}{2a^2}-\frac{3\text{sech}^{-1}(ax)^2}{2a^2}+\frac{3\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)}{a^2}+\frac{1}{2}x^2\text{sech}$$

```
[Out] -3/2*arcsech(a*x)^2/a^2+1/2*x^2*arcsech(a*x)^3+3*arcsech(a*x)*ln(1+(1/a/x+(
1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^2+3/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2
)*(1+1/a/x)^(1/2))^2)/a^2-3/2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/
2)/a^2
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6285, 5418, 4184, 3718, 2190, 2279, 2391}

$$\frac{3\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)}{2a^2}-\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)^2}{2a^2}-\frac{3\text{sech}^{-1}(ax)^2}{2a^2}+\frac{3\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)}{a^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a*x]^3,x]
```

```
[Out] (-3*ArcSech[a*x]^2)/(2*a^2) - (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSec
h[a*x]^2)/(2*a^2) + (x^2*ArcSech[a*x]^3)/2 + (3*ArcSech[a*x]*Log[1 + E^(2*A
rcSech[a*x])])/a^2 + (3*PolyLog[2, -E^(2*ArcSech[a*x])])/(2*a^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\text{sech}^{-1}(ax)^3\,dx = -\frac{\text{Subst}\left(\int x^3\text{sech}^2(x)\tanh(x)\,dx,x,\text{sech}^{-1}(ax)\right)}{a^2}$$

$$= \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 - \frac{3\,\text{Subst}\left(\int x^2\text{sech}^2(x)\,dx,x,\text{sech}^{-1}(ax)\right)}{2a^2}$$

$$= -\frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\,\text{Subst}\left(\int x\tanh(x)\,dx,x,\text{sech}^{-1}(ax)\right.}{a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{6\,\text{Subst}\left(\int\frac{e^{2x}x}{1+e^{2x}}\,dx,x,\right.}{a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{sech}^{-1}(ax)\log\left(1+e\right.}{a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{sech}^{-1}(ax)\log\left(1+e\right.}{a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{sech}^{-1}(ax)\log\left(1+e\right.}{a^2}$$

Mathematica [A] time = 0.40, size = 101, normalized size = 0.99

$$\frac{\text{sech}^{-1}(ax)\left(a^2x^2\text{sech}^{-1}(ax)^2 - 3\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} - 1\right)\text{sech}^{-1}(ax) + 6\log\left(e^{-2\text{sech}^{-1}(ax)}+1\right)\right) - 3\text{Li}_2\left(-e^{-2\text{sech}}\right.}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSech[a*x]^3,x]
```

```
[Out] (ArcSech[a*x]*(-3*(-1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 +
 a*x)])*ArcSech[a*x] + a^2*x^2*ArcSech[a*x]^2 + 6*Log[1 + E^(-2*ArcSech[a*x
])]) - 3*PolyLog[2, -E^(-2*ArcSech[a*x])])/(2*a^2)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{arsech}\,(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arcsech(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arsech}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(a*x)^3,x, algorithm="giac")
[Out] integrate(x*arcsech(a*x)^3, x)
```

maple [A] time = 0.64, size = 152, normalized size = 1.49

$$\frac{x^2 \operatorname{arcsech}(ax)^3}{2} - \frac{3 \operatorname{arcsech}(ax)^2 \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \, x}{2a} - \frac{3 \operatorname{arcsech}(ax)^2}{2a^2} + \frac{3 \operatorname{arcsech}(ax) \ln\left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{\right.\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(a*x)^3,x)
[Out] 1/2*x^2*arcsech(a*x)^3-3/2/a*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a
/x)^(1/2)*x-3/2*arcsech(a*x)^2/a^2+3*arcsech(a*x)*ln(1+(1/a/x+(1/a/x-1)^(1/
2)*(1+1/a/x)^(1/2))^2)/a^2+3/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^
(1/2))^2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arsech}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(a*x)^3,x, algorithm="maxima")
[Out] integrate(x*arcsech(a*x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a*x))^3,x)
[Out] int(x*acosh(1/(a*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asech}^3(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(a*x)**3,x)
[Out] Integral(x*asech(a*x)**3, x)
```

3.14 $\int \operatorname{sech}^{-1}(ax)^3\,dx$

Optimal. Leaf size=111

$$\frac{6i\operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}-\frac{6i\operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}-\frac{6i\operatorname{Li}_3\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}+\frac{6i\operatorname{Li}_3\left(ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}+x\operatorname{sech}^{-1}($$

```
[Out] x*arcsech(a*x)^3-6*arcsech(a*x)^2*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1
/2))/a+6*I*arcsech(a*x)*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)
))/a-6*I*arcsech(a*x)*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/
a-6*I*polylog(3,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a+6*I*polylog(3
,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6279, 5418, 4180, 2531, 2282, 6589}

$$\frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}-\frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}-\frac{6i\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}+\frac{6i\operatorname{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^3,x]
```

```
[Out] x*ArcSech[a*x]^3 - (6*ArcSech[a*x]^2*ArcTan[E^ArcSech[a*x]])/a + ((6*I)*Arc
Sech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a - ((6*I)*ArcSech[a*x]*PolyLog[
2, I*E^ArcSech[a*x]])/a - ((6*I)*PolyLog[3, (-I)*E^ArcSech[a*x]])/a + ((6*I
)*PolyLog[3, I*E^ArcSech[a*x]])/a
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
```

```
[q, 1]
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{-1}(ax)^3\,dx &= -\frac{\operatorname{Subst}\left(\int x^3\operatorname{sech}(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)}{a}\\
&= x\operatorname{sech}^{-1}(ax)^3 - \frac{3\operatorname{Subst}\left(\int x^2\operatorname{sech}(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)}{a}\\
&= x\operatorname{sech}^{-1}(ax)^3 - \frac{6\operatorname{sech}^{-1}(ax)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{(6i)\operatorname{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \operatorname{sech}\right.}{a}\\
&= x\operatorname{sech}^{-1}(ax)^3 - \frac{6\operatorname{sech}^{-1}(ax)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{sec}}{}\\
&= x\operatorname{sech}^{-1}(ax)^3 - \frac{6\operatorname{sech}^{-1}(ax)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{sec}}{}\\
&= x\operatorname{sech}^{-1}(ax)^3 - \frac{6\operatorname{sech}^{-1}(ax)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{sec}}{}\end{aligned}$$

Mathematica [A] time = 0.13, size = 128, normalized size = 1.15

$$x\operatorname{sech}^{-1}(ax)^3 - \frac{3i\left(-2\operatorname{sech}^{-1}(ax)\left(\operatorname{Li}_2\left(-ie^{-\operatorname{sech}^{-1}(ax)}\right)-\operatorname{Li}_2\left(ie^{-\operatorname{sech}^{-1}(ax)}\right)\right)-2\left(\operatorname{Li}_3\left(-ie^{-\operatorname{sech}^{-1}(ax)}\right)-\operatorname{Li}_3\left(ie^{-\operatorname{sech}}\right.\right.\right.}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a*x]^3,x]
```

```
[Out] x*ArcSech[a*x]^3 - ((3*I)*(-(ArcSech[a*x]^2*(Log[1 - I/E^ArcSech[a*x]] - Lo
g[1 + I/E^ArcSech[a*x]])) - 2*ArcSech[a*x]*(PolyLog[2, (-I)/E^ArcSech[a*x]]
 - PolyLog[2, I/E^ArcSech[a*x]]) - 2*(PolyLog[3, (-I)/E^ArcSech[a*x]] - Pol
yLog[3, I/E^ArcSech[a*x]])))/a
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{arsech}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsech(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsech}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^3, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \operatorname{arcsech}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^3,x)
```

```
[Out] int(arcsech(a*x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \log\left(\sqrt{ax+1}\,\sqrt{-ax+1}+1\right)^3 - \int \frac{a^2x^2\log(a)^3 + \left(a^2x^2-1\right)\log(x)^3 + 3\left(a^2x^2\log(a) + \left(a^2x^2\left(\log(a)+1\right) + \left(a^2x\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^3 - integrate((a^2*x^2*log(a)^3 + (
a^2*x^2 - 1)*log(x)^3 + 3*(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*x^
2 - 1)*log(x) - log(a))*sqrt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log(x)
 - log(a))*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - log(a)^3 + 3*(a^2*x^2*
log(a) - log(a))*log(x)^2 + (a^2*x^2*log(a)^3 + (a^2*x^2 - 1)*log(x)^3 - lo
g(a)^3 + 3*(a^2*x^2*log(a) - log(a))*log(x)^2 + 3*(a^2*x^2*log(a)^2 - log(a
)^2)*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - 3*(a^2*x^2*log(a)^2 + (a^2*x^2
- 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log(a)^2 + 2*(
a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - log(a)^2 +
2*(a^2*x^2*log(a) - log(a))*log(x))*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1) +
 3*(a^2*x^2*log(a)^2 - log(a)^2)*log(x))/(a^2*x^2 + (a^2*x^2 - 1)*sqrt(a*x
+ 1)*sqrt(-a*x + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^3,x)
```

```
[Out] int(acosh(1/(a*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**3,x)
```

```
[Out] Integral(asech(a*x)**3, x)
```

3.15 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} \, dx$

Optimal. Leaf size=88

$$-\frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right)+\frac{3}{2}\operatorname{sech}^{-1}(ax)\operatorname{Li}_3\left(-e^{2\operatorname{sech}^{-1}(ax)}\right)-\frac{3}{4}\operatorname{Li}_4\left(-e^{2\operatorname{sech}^{-1}(ax)}\right)+\frac{1}{4}\operatorname{sech}^{-1}(ax)^4-\operatorname{sech}^{-1}($$

```
[Out] 1/4*arcsech(a*x)^4-arcsech(a*x)^3*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)-3/2*arcsech(a*x)^2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)
)^2)+3/2*arcsech(a*x)*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
-3/4*polylog(4,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6285, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(ax)}\right)+\frac{3}{2}\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(3,-e^{2\operatorname{sech}^{-1}(ax)}\right)-\frac{3}{4}\operatorname{PolyLog}\left(4,-e^{2\operatorname{sech}^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^3/x,x]
```

```
[Out] ArcSech[a*x]^4/4 - ArcSech[a*x]^3*Log[1 + E^(2*ArcSech[a*x])] - (3*ArcSech[
a*x]^2*PolyLog[2, -E^(2*ArcSech[a*x])])/2 + (3*ArcSech[a*x]*PolyLog[3, -E^(
2*ArcSech[a*x])])/2 - (3*PolyLog[4, -E^(2*ArcSech[a*x])])/4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx &= -\text{Subst}\left(\int x^3\tanh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)\\
&= \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - 2\,\text{Subst}\left(\int \frac{e^{2x}x^3}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(ax)\right)\\
&= \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) + 3\,\text{Subst}\left(\int x^2\log\left(1+e^{2x}\right)\,dx, x, \operatorname{sech}\right.\\
&= \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + 3\,\text{Sub}\\
&= \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2}\operatorname{sech}\\
&= \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2}\operatorname{sech}\\
&= \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3\log\left(1+e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2}\operatorname{sech}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 84, normalized size = 0.95

$$\frac{1}{4}\left(6\operatorname{sech}^{-1}(ax)^2\operatorname{Li}_2\left(-e^{-2\operatorname{sech}^{-1}(ax)}\right) + 6\operatorname{sech}^{-1}(ax)\operatorname{Li}_3\left(-e^{-2\operatorname{sech}^{-1}(ax)}\right) + 3\operatorname{Li}_4\left(-e^{-2\operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax)^4 - 4\operatorname{sec}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a*x]^3/x,x]
```

```
[Out] (-ArcSech[a*x]^4 - 4*ArcSech[a*x]^3*Log[1 + E^(-2*ArcSech[a*x])] + 6*ArcSec
h[a*x]^2*PolyLog[2, -E^(-2*ArcSech[a*x])] + 6*ArcSech[a*x]*PolyLog[3, -E^(-
2*ArcSech[a*x])] + 3*PolyLog[4, -E^(-2*ArcSech[a*x])])/4
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsech}(ax)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x,x, algorithm="fricas")
```

[Out] integral(arcsech(a*x)^3/x, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^3/x, x)

maple [A] time = 0.15, size = 181, normalized size = 2.06

$$\frac{\operatorname{arcsech}(ax)^4}{4} - \operatorname{arcsech}(ax)^3 \ln\left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1}\,\sqrt{1 + \frac{1}{ax}}\right)^2\right) - \frac{3\operatorname{arcsech}(ax)^2 \operatorname{polylog}\left(2, -\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} -\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^3/x,x)

[Out] 1/4*arcsech(a*x)^4-arcsech(a*x)^3*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)-3/2*arcsech(a*x)^2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)+3/2*arcsech(a*x)*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)-3/4*polylog(4,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^3/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(1/(a*x))^3/x,x)

[Out] int(acosh(1/(a*x))^3/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**3/x,x)

[Out] Integral(asech(a*x)**3/x, x)

3.16 $\int \frac{\text{sech}^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=83

$$\frac{6\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{x}-\frac{\text{sech}^{-1}(ax)^3}{x}+\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)^2}{x}-\frac{6\text{sech}^{-1}(ax)}{x}$$

```
[Out] -6*arcsech(a*x)/x-arcsech(a*x)^3/x+6*(a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/x+3*(
a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6285, 3296, 2637}

$$\frac{6\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{x}-\frac{\text{sech}^{-1}(ax)^3}{x}+\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(ax)^2}{x}-\frac{6\text{sech}^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^3/x^2,x]
```

```
[Out] (6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/x - (6*ArcSech[a*x])/x + (3*Sqrt[(1
 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/x - ArcSech[a*x]^3/x
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^{-1}(ax)^3}{x^2}\,dx &= -\left(a\,\text{Subst}\left(\int x^3\sinh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\right)\\
&= -\frac{\text{sech}^{-1}(ax)^3}{x}+(3a)\,\text{Subst}\left(\int x^2\cosh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{x}-\frac{\text{sech}^{-1}(ax)^3}{x}-(6a)\,\text{Subst}\left(\int x\sinh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= -\frac{6\text{sech}^{-1}(ax)}{x}+\frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{x}-\frac{\text{sech}^{-1}(ax)^3}{x}+(6a)\,\text{Subst}\left(\int \cosh(x)\,dx,x
\right.\\
&= \frac{6\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{x}-\frac{6\text{sech}^{-1}(ax)}{x}+\frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\text{sech}^{-1}(ax)^2}{x}-\frac{\text{sech}^{-1}(ax)^3}{x}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.90

$$\frac{6\sqrt{\frac{1-ax}{ax+1}}(ax+1)-\text{sech}^{-1}(ax)^3+3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2-6\text{sech}^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^3/x^2,x]
```

```
[Out] (6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 3*Sqrt[(1 - a*x)/
(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 - ArcSech[a*x]^3)/x
```

fricas [A] time = 0.71, size = 155, normalized size = 1.87

$$\frac{3\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3+6\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}-6\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] (3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2
)) + 1)/(a*x))^2 - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^3 +
6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) - 6*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*
x^2)) + 1)/(a*x)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsech}\,(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^3/x^2, x)
```

maple [A] time = 0.13, size = 98, normalized size = 1.18

$$a\left(-\frac{\text{arcsech}\,(ax)^3}{ax}+3\text{arcsech}\,(ax)^2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}-\frac{6\,\text{arcsech}\,(ax)}{ax}+6\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^3/x^2,x)
```

```
[Out] a*(-arcsech(a*x)^3/a/x+3*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^
(1/2)-6/a/x*arcsech(a*x)+6*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2))
```

maxima [A] time = 0.32, size = 55, normalized size = 0.66

$$3\,a\sqrt{\frac{1}{a^2x^2}-1}\,\text{arsech}\,(ax)^2-\frac{\text{arsech}\,(ax)^3}{x}+6\,a\sqrt{\frac{1}{a^2x^2}-1}-\frac{6\,\text{arsech}\,(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] 3*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)^2 - arcsech(a*x)^3/x + 6*a*sqrt(1/(a
^2*x^2) - 1) - 6*arcsech(a*x)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^3/x^2,x)
```

```
[Out] int(acosh(1/(a*x))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**3/x**2,x)
```

```
[Out] Integral(asech(a*x)**3/x**2, x)
```

3.17 $\int \frac{\text{sech}^{-1}(ax)^3}{x^3} dx$

Optimal. Leaf size=137

$$-\frac{1}{4}a^2\text{sech}^{-1}(ax)^3-\frac{3}{8}a^2\text{sech}^{-1}(ax)+\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{8x^2}-\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2x^2}+\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(a}{4x^2}$$

```
[Out] -3/8*a^2*arcsech(a*x)-3/4*(-a*x+1)*(a*x+1)*arcsech(a*x)/x^2-1/4*a^2*arcsech
(a*x)^3-1/2*(-a*x+1)*(a*x+1)*arcsech(a*x)^3/x^2+3/8*(a*x+1)*((-a*x+1)/(a*x+
1))^(1/2)/x^2+3/4*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6285, 5372, 3311, 30, 2635, 8}

$$-\frac{1}{4}a^2\text{sech}^{-1}(ax)^3-\frac{3}{8}a^2\text{sech}^{-1}(ax)+\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{8x^2}-\frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^3}{2x^2}+\frac{3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\text{sech}^{-1}(a}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^3/x^3,x]
```

```
[Out] (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(8*x^2) - (3*a^2*ArcSech[a*x])/8 -
(3*(1 - a*x)*(1 + a*x)*ArcSech[a*x])/(4*x^2) + (3*Sqrt[(1 - a*x)/(1 + a*x)]
*(1 + a*x)*ArcSech[a*x]^2)/(4*x^2) - (a^2*ArcSech[a*x]^3)/4 - ((1 - a*x)*(1
 + a*x)*ArcSech[a*x]^3)/(2*x^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = -\left(a^2\,\text{Subst}\left(\int x^3\cosh(x)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)\right)$$
$$= -\frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^3}{2x^2} + \frac{1}{2}\left(3a^2\right)\text{Subst}\left(\int x^2\sinh^2(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)$$
$$= -\frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{2x^2}$$
$$= \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{8x^2} - \frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \frac{1}{4}a^2\operatorname{sech}$$
$$= \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{8x^2} - \frac{3}{8}a^2\operatorname{sech}^{-1}(ax) - \frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\operatorname{sech}^{-1}}{4x^2}$$

Mathematica [A] time = 0.15, size = 147, normalized size = 1.07

$$\frac{-3a^2x^2\log(x) + 3a^2x^2\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) + 2\left(a^2x^2-2\right)\operatorname{sech}^{-1}(ax)^3 + 3\sqrt{\frac{1-ax}{ax+1}}\,(ax+1) + 6\sqrt{\frac{1-ax}{ax+1}}\,(ax}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^3/x^3,x]
```

```
[Out] (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 6*Sqrt[(1 - a*x)/
(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 2*(-2 + a^2*x^2)*ArcSech[a*x]^3 - 3*a
^2*x^2*Log[x] + 3*a^2*x^2*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 -
 a*x)/(1 + a*x)]])/(8*x^2)
```

fricas [A] time = 0.63, size = 174, normalized size = 1.27

$$\frac{6\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 + 2\left(a^2x^2-2\right)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3 + 3\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}} + 3\left(a^2x^2-2\right)\log\left(\frac{ax\sqrt{-\frac{a}{}}}{}\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2
*x^2)) + 1)/(a*x))^2 + 2*(a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^
2)) + 1)/(a*x))^3 + 3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 3*(a^2*x^2 - 2)*
log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^3/x^3, x)
```

maple [A] time = 0.13, size = 126, normalized size = 0.92

$$a^2\left(-\frac{\operatorname{arcsech}(ax)^3}{2a^2x^2}+\frac{3\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{4ax}+\frac{\operatorname{arcsech}(ax)^3}{4}-\frac{3\operatorname{arcsech}(ax)}{4a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{8ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^3/x^3,x)
```

```
[Out] a^2*(-1/2*arcsech(a*x)^3/a^2/x^2+3/4*arcsech(a*x)^2/a/x*(-(a*x-1)/a/x)^(1/2
)*((a*x+1)/a/x)^(1/2)+1/4*arcsech(a*x)^3-3/4/a^2/x^2*arcsech(a*x)+3/8/a/x*(
-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+3/8*arcsech(a*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(a*x)^3/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^3/x^3,x)
```

```
[Out] int(acosh(1/(a*x))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**3/x**3,x)
```

```
[Out] Integral(asech(a*x)**3/x**3, x)
```

3.18 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=179

$$\frac{14a^2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{9x}+\frac{2a^2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\operatorname{sech}^{-1}(ax)^2}{3x}-\frac{4a^2\operatorname{sech}^{-1}(ax)}{3x}+\frac{2\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1)^3}{27x^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\operatorname{sech}$$

```
[Out] 2/27*((-a*x+1)/(a*x+1))^(3/2)*(a*x+1)^3/x^3-2/9*arcsech(a*x)/x^3-4/3*a^2*ar
csech(a*x)/x-1/3*arcsech(a*x)^3/x^3+14/9*a^2*(a*x+1)*((-a*x+1)/(a*x+1))^(1/
2)/x+1/3*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x^3+2/3*a^2*(a*x+1
)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x
```

Rubi [A] time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6285, 5373, 3311, 3296, 2637, 2633}

$$\frac{14a^2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{9x}+\frac{2a^2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\operatorname{sech}^{-1}(ax)^2}{3x}-\frac{4a^2\operatorname{sech}^{-1}(ax)}{3x}+\frac{2\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1)^3}{27x^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)\operatorname{sech}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^3/x^4,x]
```

```
[Out] (14*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(9*x) + (2*((1 - a*x)/(1 + a*x
))^(3/2)*(1 + a*x)^3)/(27*x^3) - (2*ArcSech[a*x])/(9*x^3) - (4*a^2*ArcSech[
a*x])/(3*x) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x^3)
+ (2*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x) - ArcSec
h[a*x]^3/(3*x^3)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
```

```
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = -\left(a^3\,\text{Subst}\left(\int x^3\cosh^2(x)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)\right)$$
$$= -\frac{\operatorname{sech}^{-1}(ax)^3}{3x^3} + a^3\,\text{Subst}\left(\int x^2\cosh^3(x)\,dx, x, \operatorname{sech}^{-1}(ax)\right)$$
$$= -\frac{2\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x^3} - \frac{\operatorname{sech}^{-1}(ax)^3}{3x^3} + \frac{1}{9}\left(2a^3\right)\text{Subst}\left(\int \cosh^3(x\right.$$
$$= -\frac{2\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{2a^2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x} - \frac{\operatorname{sech}^{-1}(}{3x^3}$$
$$= \frac{2a^2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{9x} + \frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3} - \frac{2\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{4a^2\operatorname{sech}^{-1}(ax)}{3x} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1}{}$$
$$= \frac{14a^2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{9x} + \frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3} - \frac{2\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{4a^2\operatorname{sech}^{-1}(ax)}{3x} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1}{}$$

Mathematica [A] time = 0.13, size = 120, normalized size = 0.67

$$\frac{-6\left(6a^2x^2+1\right)\operatorname{sech}^{-1}(ax) + 2\sqrt{\frac{1-ax}{ax+1}}\left(20a^3x^3+20a^2x^2+ax+1\right) + 9\sqrt{\frac{1-ax}{ax+1}}\left(2a^3x^3+2a^2x^2+ax+1\right)\operatorname{sech}^{-1}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^3/x^4,x]
```

```
[Out] (2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 20*a^2*x^2 + 20*a^3*x^3) - 6*(1 + 6
*a^2*x^2)*ArcSech[a*x] + 9*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2 +
 2*a^3*x^3)*ArcSech[a*x]^2 - 9*ArcSech[a*x]^3)/(27*x^3)
```

fricas [A] time = 0.68, size = 186, normalized size = 1.04

$$\frac{9\left(2\,a^3x^3+ax\right)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - 9\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3 - 6\left(6\,a^2x^2+1\right)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) + 2}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(2*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^
2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 9*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x
```

```
^2)) + 1)/(a*x))^3 - 6*(6*a^2*x^2 + 1)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^
2)) + 1)/(a*x)) + 2*(20*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^3/x^4, x)
```

maple [A] time = 0.43, size = 192, normalized size = 1.07

$$a^3\left(-\frac{\operatorname{arcsech}(ax)^3}{3a^3x^3}+\frac{2\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{3}+\frac{\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{3a^2x^2}-\frac{4\operatorname{arcsech}(ax)}{3ax}+\frac{40\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^3/x^4,x)
```

```
[Out] a^3*(-1/3*arcsech(a*x)^3/a^3/x^3+2/3*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((
a*x+1)/a/x)^(1/2)+1/3*arcsech(a*x)^2/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/
a/x)^(1/2)-4/3/a/x*arcsech(a*x)+40/27*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)-2/9*arcsech(a*x)/a^3/x^3+2/27/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(a*x)^3/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x))^3/x^4,x)
```

```
[Out] int(acosh(1/(a*x))^3/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**3/x**4,x)
```

```
[Out] Integral(asech(a*x)**3/x**4, x)
```

3.19 $\int x^6 \left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=142

$$\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{5b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\sin^{-1}(cx)}{112c^7}-\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{cx+1}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{cx+1}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/7*x^7*(a+b*arcsech(c*x))-5/112*b*x*(-c*x+1)^(1/2)/c^6/(1/(c*x+1))^(1/2)-5
/168*b*x^3*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/42*b*x^5*(-c*x+1)^(1/2)/c
^2/(1/(c*x+1))^(1/2)+5/112*b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^
7
```

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6283, 100, 12, 90, 41, 216}

$$\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{cx+1}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{cx+1}}}-\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{cx+1}}}+\frac{5b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\sin^{-1}(cx)}{112c^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-5*b*x*Sqrt[1 - c*x])/(112*c^6*Sqrt[(1 + c*x)^(-1)]) - (5*b*x^3*Sqrt[1 - c
*x])/(168*c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^5*Sqrt[1 - c*x])/(42*c^2*Sqrt[(1
 + c*x)^(-1)]) + (x^7*(a + b*ArcSech[c*x]))/7 + (5*b*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*ArcSin[c*x])/(112*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^6\left(a+b\text{sech}^{-1}(cx)\right)dx=\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^6}{\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$=-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{5x^4}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{42c^2}$$

$$=-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^4}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{42c^2}$$

$$=-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)-\frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-}{168c^4}$$

$$=-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int}{56c^4}$$

$$=-\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(5b\sqrt{\frac{1}{1+cx}}\right.}{}$$

$$=-\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(5b\sqrt{\frac{1}{1+cx}}\right.}{}$$

$$=-\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{5b\sqrt{\frac{1}{1+cx}}}{}$$

Mathematica [C] time = 0.22, size = 143, normalized size = 1.01

$$\frac{ax^7}{7}+\frac{5ib\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2icx\right)}{112c^7}+b\sqrt{\frac{1-cx}{cx+1}}\left(-\frac{5x}{112c^6}-\frac{5x^2}{112c^5}-\frac{5x^3}{168c^4}-\frac{5x^4}{168c^3}-\frac{x^5}{42c^2}-\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\text{sech}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(1 - c*x)/(1 + c*x)]*((-5*x)/(112*c^6) - (5*x^2)/(112*c^
5) - (5*x^3)/(168*c^4) - (5*x^4)/(168*c^3) - x^5/(42*c^2) - x^6/(42*c)) + (
```

```
b*x^7*ArcSech[c*x])/7 + (((5*I)/112)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x)])/c^7
```

fricas [A] time = 0.59, size = 183, normalized size = 1.29

$$\frac{48\,ac^{7}x^{7}-48\,bc^{7}\log\left(\frac{cx\sqrt{-\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}-1}{x}\right)-30\,b\arctan\left(\frac{cx\sqrt{-\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}-1}{cx}\right)+48\left(bc^{7}x^{7}-bc^{7}\right)\log\left(\frac{cx\sqrt{-\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}+1}{cx}\right)-\left(8\,b\right.}{336\,c^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 - 48*b*c^7*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)
/x) - 30*b*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 48*(b*c
^7*x^7 - b*c^7)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*
c^6*x^6 + 10*b*c^4*x^4 + 15*b*c^2*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsech}\,(cx)+a)x^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^6, x)
```

maple [A] time = 0.09, size = 138, normalized size = 0.97

$$\frac{\frac{c^{7}x^{7}a}{7}+b\left(\frac{c^{7}x^{7}\operatorname{arcsech}(cx)}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(8c^{5}x^{5}\sqrt{-c^{2}x^{2}+1}+10c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}+15cx\sqrt{-c^{2}x^{2}+1}-15\arcsin(cx)\right)}{336\sqrt{-c^{2}x^{2}+1}}\right)}{c^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^7*(1/7*c^7*x^7*a+b*(1/7*c^7*x^7*arcsech(c*x)-1/336*(-(c*x-1)/c/x)^(1/2)
*c*x*((c*x+1)/c/x)^(1/2)*(8*c^5*x^5*(-c^2*x^2+1)^(1/2)+10*c^3*x^3*(-c^2*x^2
+1)^(1/2)+15*c*x*(-c^2*x^2+1)^(1/2)-15*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.41, size = 135, normalized size = 0.95

$$\frac{1}{7}\,ax^{7}+\frac{1}{336}\left(48\,x^{7}\,\text{arsech}\,(cx)-\frac{\frac{15\left(\frac{1}{c^{2}x^{2}}-1\right)^{\frac{5}{2}}+40\left(\frac{1}{c^{2}x^{2}}-1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^{2}x^{2}}-1}}{c^{6}\left(\frac{1}{c^{2}x^{2}}-1\right)^{3}+3\,c^{6}\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}+3\,c^{6}\left(\frac{1}{c^{2}x^{2}}-1\right)+c^{6}}+\frac{15\,\arctan\left(\sqrt{\frac{1}{c^{2}x^{2}}-1}\right)}{c^{6}}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7 + 1/336*(48*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*
(1/(c^2*x^2) - 1)^(3/2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^
3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(
sqrt(1/(c^2*x^2) - 1))/c^6)/c)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*acosh(1/(c*x))),x)
[Out] int(x^6*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{6} \left(a + b \operatorname{asech}{\left(c x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asech(c*x)),x)
[Out] Integral(x**6*(a + b*asech(c*x)), x)
```

3.20 $\int x^5 \left(a + b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=109

$$\frac{1}{6}x^6\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{4b\sqrt{1-cx}}{45c^6\sqrt{\frac{1}{cx+1}}}-\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{cx+1}}}-\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/6*x^6*(a+b*arcsech(c*x))-4/45*b*(-c*x+1)^(1/2)/c^6/(1/(c*x+1))^(1/2)-2/45
*b*x^2*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/30*b*x^4*(-c*x+1)^(1/2)/c^2/(
1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6283, 100, 12, 74}

$$\frac{1}{6}x^6\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{cx+1}}}-\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{cx+1}}}-\frac{4b\sqrt{1-cx}}{45c^6\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-4*b*Sqrt[1 - c*x])/(45*c^6*Sqrt[(1 + c*x)^(-1)]) - (2*b*x^2*Sqrt[1 - c*x]
)/(45*c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^4*Sqrt[1 - c*x])/(30*c^2*Sqrt[(1 + c
*x)^(-1)]) + (x^6*(a + b*ArcSech[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)dx = \frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^5}{\sqrt{1-cx}\sqrt{1+cx}}dx$$

$$= -\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{4x^3}{\sqrt{1-cx}\sqrt{1+cx}}dx}{30c^2}$$

$$= -\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{\sqrt{1-cx}\sqrt{1+cx}}dx}{15c^2}$$

$$= -\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int - \cdots}{45c^4}$$

$$= -\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \cdots}{45c^4}$$

$$= -\frac{4b\sqrt{1-cx}}{45c^6\sqrt{\frac{1}{1+cx}}}-\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right)$$

Mathematica [A] time = 0.11, size = 97, normalized size = 0.89

$$\frac{ax^6}{6}+b\sqrt{\frac{1-cx}{cx+1}}\left(-\frac{4}{45c^6}-\frac{4x}{45c^5}-\frac{2x^2}{45c^4}-\frac{2x^3}{45c^3}-\frac{x^4}{30c^2}-\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^6)/6 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-4/(45*c^6) - (4*x)/(45*c^5) - (2*
x^2)/(45*c^4) - (2*x^3)/(45*c^3) - x^4/(30*c^2) - x^5/(30*c)) + (b*x^6*ArcS
ech[c*x])/6
```

fricas [A] time = 0.51, size = 100, normalized size = 0.92

$$\frac{15\,bc^5x^6\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+15\,ac^5x^6-\left(3\,bc^4x^5+4\,bc^2x^3+8\,bx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{90\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^5*x^6*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 15
*a*c^5*x^6 - (3*b*c^4*x^5 + 4*b*c^2*x^3 + 8*b*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)))/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsech}\,(cx)+a)x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5, x)
```

maple [A] time = 0.06, size = 81, normalized size = 0.74

$$\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6x^6\operatorname{arcsech}(cx)}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(3c^4x^4+4c^2x^2+8)}{90}\right)}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a+b*(1/6*c^6*x^6*arcsech(c*x)-1/90*(-(c*x-1)/c/x)^(1/2)*
c*x*((c*x+1)/c/x)^(1/2)*(3*c^4*x^4+4*c^2*x^2+8)))
```

maxima [A] time = 0.31, size = 78, normalized size = 0.72

$$\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\operatorname{arsech}(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2)
- 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^5*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 5.57, size = 94, normalized size = 0.86

$$\begin{cases}\frac{ax^6}{6}+\frac{bx^6\operatorname{asech}(cx)}{6}-\frac{bx^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{2bx^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{4b\sqrt{-c^2x^2+1}}{45c^6} & \text{for } c\neq 0\\ \frac{x^6(a+\infty b)}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*asech(c*x)/6 - b*x**4*sqrt(-c**2*x**2 + 1)/(30
*c**2) - 2*b*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*sqrt(-c**2*x**2 + 1)
/(45*c**6), Ne(c, 0)), (x**6*(a + oo*b)/6, True))
```

3.21 $\int x^4 \left(a + b\,\mathbf{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=110

$$\frac{1}{5}x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{3b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{40c^5}-\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{cx+1}}}-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/5*x^5*(a+b*arcsech(c*x))-3/40*b*x*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/
20*b*x^3*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)+3/40*b*arcsin(c*x)*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)/c^5
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6283, 100, 12, 90, 41, 216}

$$\frac{1}{5}x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{cx+1}}}-\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{cx+1}}}+\frac{3b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{40c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-3*b*x*Sqrt[1 - c*x])/(40*c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^3*Sqrt[1 - c*x]
)/(20*c^2*Sqrt[(1 + c*x)^(-1)]) + (x^5*(a + b*ArcSech[c*x]))/5 + (3*b*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(40*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx=\frac{1}{5}x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{1}{5}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^4}{\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$=-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{5}x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{3x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{20c^2}$$

$$=-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{5}x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{\left(3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{20c^2}$$

$$=-\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{5}x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{\left(3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int}{40c^4}$$

$$=-\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{5}x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{\left(3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int}{40c^4}$$

$$=-\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{5}x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sin^{-1}}{40c^5}$$

Mathematica [C] time = 0.14, size = 123, normalized size = 1.12

$$\frac{ax^5}{5}+\frac{3ib\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2icx\right)}{40c^5}+b\sqrt{\frac{1-cx}{cx+1}}\left(-\frac{3x}{40c^4}-\frac{3x^2}{40c^3}-\frac{x^3}{20c^2}-\frac{x^4}{20c}\right)+\frac{1}{5}bx^5\mathrm{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(1 - c*x)/(1 + c*x)]*((-3*x)/(40*c^4) - (3*x^2)/(40*c^3)
 - x^3/(20*c^2) - x^4/(20*c)) + (b*x^5*ArcSech[c*x])/5 + (((3*I)/40)*b*Log[
(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^5
```

fricas [B] time = 0.55, size = 174, normalized size = 1.58

$$\frac{8\,ac^5x^5-8\,bc^5\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-6\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)+8\left(bc^5x^5-bc^5\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(2\,bc^4x^4\right.}{40\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 - 8*b*c^5*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x)
 - 6*b*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 8*(b*c^5*x^
5 - b*c^5)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*b*c^4*x
^4 + 3*b*c^2*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsech}(cx) + a)x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4, x)
```

maple [A] time = 0.06, size = 118, normalized size = 1.07

$$\frac{\frac{c^5x^5a}{5} + b\left(\frac{c^5x^5\operatorname{arcsech}(cx)}{5} + \frac{\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}\left(-2c^3x^3\sqrt{-c^2x^2+1} - 3cx\sqrt{-c^2x^2+1} + 3\arcsin(cx)\right)}{40\sqrt{-c^2x^2+1}}\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a+b*(1/5*c^5*x^5*arcsech(c*x)+1/40*(-(c*x-1)/c/x)^(1/2)*
c*x*((c*x+1)/c/x)^(1/2)*(-2*c^3*x^3*(-c^2*x^2+1)^(1/2)-3*c*x*(-c^2*x^2+1)^(
1/2)+3*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.41, size = 106, normalized size = 0.96

$$\frac{1}{5}ax^5 + \frac{1}{40}\left(8x^5 \operatorname{arsech}(cx) - \frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2 + 2c^4\left(\frac{1}{c^2x^2}-1\right) + c^4} + \frac{3\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt
(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4
) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^4*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \operatorname{asech}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**4*(a + b*asech(c*x)), x)
```

3.22 $\int x^3 \left(a + b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=77

$$\frac{1}{4}x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{cx+1}}}-\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/4*x^4*(a+b*arcsech(c*x))-1/6*b*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/12*
b*x^2*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{6283, 100, 12, 74\}$

$$\frac{1}{4}x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{cx+1}}}-\frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*Sqrt[1 - c*x])/(6*c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^2*Sqrt[1 - c*x])/(12
*c^2*Sqrt[(1 + c*x)^(-1)]) + (x^4*(a + b*ArcSech[c*x]))/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)dx=\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{\sqrt{1-cx}\sqrt{1+cx}}dx$$
$$=-\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{2x}{\sqrt{1-cx}\sqrt{1+cx}}dx}{12c^2}$$
$$=-\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x}{\sqrt{1-cx}\sqrt{1+cx}}dx}{6c^2}$$
$$=-\frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)$$

Mathematica [A] time = 0.08, size = 77, normalized size = 1.00

$$\frac{ax^4}{4}+b\sqrt{\frac{1-cx}{cx+1}}\left(-\frac{1}{6c^4}-\frac{x}{6c^3}-\frac{x^2}{12c^2}-\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcSech[c*x]),x]
[Out] (a*x^4)/4 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*1/c^4 - x/(6*c^3) - x^2/(12*c
^2) - x^3/(12*c)) + (b*x^4*ArcSech[c*x])/4
```

fricas [A] time = 0.47, size = 90, normalized size = 1.17

$$\frac{3\,bc^3x^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+3\,ac^3x^4-\left(bc^2x^3+2\,bx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] 1/12*(3*b*c^3*x^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 3*a
*c^3*x^4 - (b*c^2*x^3 + 2*b*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\,\text{arsech}\,(cx)+a)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^3, x)
```

maple [A] time = 0.06, size = 72, normalized size = 0.94

$$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\text{arcsech}(cx)}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(c^2x^2+2)}{12}\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a+b*(1/4*c^4*x^4*arcsech(c*x)-1/12*(-(c*x-1)/c/x)^(1/2)*
c*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2)))
```

maxima [A] time = 0.31, size = 57, normalized size = 0.74

$$\frac{1}{4}ax^4 + \frac{1}{12}\left(3x^4 \operatorname{arsech}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3
*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^3*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 1.97, size = 68, normalized size = 0.88

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{asech}(cx)}{4} - \frac{bx^2\sqrt{-c^2x^2+1}}{12c^2} - \frac{b\sqrt{-c^2x^2+1}}{6c^4} & \text{for } c \neq 0 \\ \frac{x^4(a+\infty b)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*asech(c*x)/4 - b*x**2*sqrt(-c**2*x**2 + 1)/(12
*c**2) - b*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), (x**4*(a + oo*b)/4, Tr
ue))
```

3.23 $\int x^2 \left(a + b\text{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=78

$$\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{6c^3}-\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/3*x^3*(a+b*arcsech(c*x))-1/6*b*x*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)+1/6
*b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6283, 90, 41, 216}

$$\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*x*Sqrt[1 - c*x])/(6*c^2*Sqrt[(1 + c*x)^(-1)]) + (x^3*(a + b*ArcSech[c*x
]))/3 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\text{sech}^{-1}(cx)\right)dx=\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}}dx$$
$$=-\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-cx}\sqrt{1+cx}}dx}{6c^2}$$
$$=-\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{6c^2}$$
$$=-\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sin^{-1}(cx)}{6c^3}$$

Mathematica [C] time = 0.09, size = 103, normalized size = 1.32

$$\frac{ax^3}{3}+\frac{ib\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2icx\right)}{6c^3}+b\sqrt{\frac{1-cx}{cx+1}}\left(-\frac{x}{6c^2}-\frac{x^2}{6c}\right)+\frac{1}{3}bx^3\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^3)/3 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*x/c^2 - x^2/(6*c)) + (b*x^3*A
rcSech[c*x])/3 + ((I/6)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)])/c^3
```

fricas [B] time = 0.64, size = 162, normalized size = 2.08

$$\frac{2\,ac^3x^3-bc^2x^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-2\,bc^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)+2\left(bc^3x^3-bc^3\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{cx}\right.}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 - b*c^2*x^2*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*b*c^3*log((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan((c*x*sqrt(-(c^2*x^2
 - 1)/(c^2*x^2)) - 1)/(c*x)) + 2*(b*c^3*x^3 - b*c^3)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2, x)
```

maple [A] time = 0.06, size = 96, normalized size = 1.23

$$\frac{\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\text{arcsech}(cx)}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(-cx\sqrt{-c^2x^2+1}+\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a+b*(1/3*c^3*x^3*arcsech(c*x)+1/6*(-(c*x-1)/c/x)^(1/2)*c
*x*((c*x+1)/c/x)^(1/2)*(-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/(-c^2*x^2+1)^(
1/2)))
```

maxima [A] time = 0.40, size = 73, normalized size = 0.94

$$\frac{1}{3}ax^3 + \frac{1}{6}\left(2\,x^3\,\mathrm{arsech}\,(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x
^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a + b\,\mathrm{asech}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x)), x)
```

3.24 $\int x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/2*x^2*(a+b*arcsech(c*x))-1/2*b*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6283, 74}

$$\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*Sqrt[1 - c*x])/(2*c^2*Sqrt[(1 + c*x)^(-1)]) + (x^2*(a + b*ArcSech[c*x])
)/2
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx&=\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{1}{2}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx\\&=-\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 1.27

$$\frac{ax^2}{2}+b\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{cx+1}}+\frac{1}{2}bx^2\,\mathrm{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^2)/2 + b*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)] + (b*x^2*Arc
Sech[c*x])/2
```

fricas [B] time = 0.58, size = 73, normalized size = 1.62

$$\frac{bcx^2 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + acx^2 - bx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2
- b*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsech}(cx) + a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x, x)
```

maple [A] time = 0.06, size = 63, normalized size = 1.40

$$\frac{\frac{c^2x^2a}{2} + b\left(\frac{\operatorname{arcsech}(cx)c^2x^2}{2} - \frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx}{2}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a+b*(1/2*arcsech(c*x)*c^2*x^2-1/2*(-(c*x-1)/c/x)^(1/2)*(
(c*x+1)/c/x)^(1/2)*c*x))
```

maxima [A] time = 0.30, size = 36, normalized size = 0.80

$$\frac{1}{2}ax^2 + \frac{1}{2}\left(x^2 \operatorname{arsech}(cx) - \frac{x\sqrt{\frac{1}{c^2x^2} - 1}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b
```

mupad [B] time = 1.39, size = 50, normalized size = 1.11

$$\frac{a\,x^2}{2} + \frac{b\,x^2\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{2} - \frac{b\,x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(1/(c*x))),x)
```

```
[Out] (a*x^2)/2 + (b*x^2*acosh(1/(c*x)))/2 - (b*x*(1/(c*x) - 1)^(1/2)*(1/(c*x) +
1)^(1/2))/(2*c)
```

sympy [A] time = 0.53, size = 46, normalized size = 1.02

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{asech}(cx)}{2} - \frac{b\sqrt{-c^2x^2+1}}{2c^2} & \text{for } c \neq 0 \\ \frac{x^2(a+\infty b)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*asech(c*x)/2 - b*sqrt(-c**2*x**2 + 1)/(2*c**2)
, Ne(c, 0)), (x**2*(a + oo*b)/2, True))
```

3.25 $\int \left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=40

$$ax+\frac{b\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sin^{-1}(cx)}{c}+bx\,\mathrm{sech}^{-1}(cx)$$

```
[Out] a*x+b*x*arcsech(c*x)+b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6277, 216}

$$ax+\frac{b\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sin^{-1}(cx)}{c}+bx\,\mathrm{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSech[c*x],x]
```

```
[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])
/c
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6277

```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx &= ax+b\int \mathrm{sech}^{-1}(cx)\,dx\\
&= ax+bx\,\mathrm{sech}^{-1}(cx)+\left(b\sqrt{\frac{1}{1+cx}}\,\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx\\
&= ax+bx\,\mathrm{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1}{1+cx}}\,\sqrt{1+cx}\,\sin^{-1}(cx)}{c}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 60, normalized size = 1.50

$$ax-\frac{b\sqrt{\frac{1-cx}{cx+1}}\,\sqrt{1-c^2x^2}\,\sin^{-1}(cx)}{c(cx-1)}+bx\,\mathrm{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSech[c*x],x]
```

```
[Out] a*x + b*x*ArcSech[c*x] - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Arc
Sin[c*x])/(c*(-1 + c*x))
```

fricas [B] time = 0.54, size = 119, normalized size = 2.98

$$\frac{acx - bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx - bc)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x - b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*log((c*x*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int b\,\mathrm{arsech}\,(cx) + a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arcsech(c*x) + a, x)
```

maple [A] time = 0.04, size = 42, normalized size = 1.05

$$ax + bx\,\mathrm{arcsech}\,(cx) - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsech(c*x),x)
```

```
[Out] a*x+b*x*arcsech(c*x)-b/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
```

maxima [A] time = 0.30, size = 31, normalized size = 0.78

$$ax + \frac{\left(cx\,\mathrm{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b/c
```

mupad [B] time = 1.34, size = 44, normalized size = 1.10

$$a\,x + b\,x\,\mathrm{acosh}\left(\frac{1}{c\,x}\right) + \frac{b\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(1/(c*x)),x)
```

```
[Out] a*x + b*x*acosh(1/(c*x)) + (b*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/
2))))/c
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asech}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asech(c*x),x)
```

```
[Out] Integral(a + b*asech(c*x), x)
```

3.26 $\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=56

$$-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b}-\log\left(e^{-2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{2}b\text{Li}_2\left(-e^{-2\text{sech}^{-1}(cx)}\right)$$

```
[Out] -1/2*(a+b*arcsech(c*x))^2/b-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))^2)
```

Rubi [A] time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}b\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)+\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b}-\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSech[c*x])/x,x]
```

```
[Out] (a + b*ArcSech[c*x])^2/(2*b) - (a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*
x])] - (b*PolyLog[2, -E^(2*ArcSech[c*x])])/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\\ &= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b} - 2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(cx)\right)\\ &= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right) + b\,\text{Subst}\left(\int \log\left(1+e\right.\right.\\ &= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right) + \frac{1}{2}b\,\text{Subst}\left(\int \frac{\log(1+}{x}\right.\\ &= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right) - \frac{1}{2}b\text{Li}_2\left(-e^{2\text{sech}^{-1}(cx)}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.84

$$a\log(x)+\frac{1}{2}b\left(\text{Li}_2\left(-e^{-2\text{sech}^{-1}(cx)}\right)-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx)+2\log\left(e^{-2\text{sech}^{-1}(cx)}+1\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/x,x]
```

```
[Out] a*Log[x] + (b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])
])) + PolyLog[2, -E^(-2*ArcSech[c*x])]))/2
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsech}\,(cx)+a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x, x)
```

maple [A] time = 0.16, size = 100, normalized size = 1.79

$$a\ln(cx)+\frac{b\text{arcsech}\,(cx)^2}{2}-b\,\text{arcsech}\,(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\right)-\frac{b\,\text{polylog}\left(2,-\left(\frac{1}{cx}+\sqrt{-1+\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x,x)
```

```
[Out] a*ln(c*x)+1/2*b*arcsech(c*x)^2-b*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))^2)-1/2*b*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{x}\,dx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x) + a*lo
g(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/x,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\,(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x,x)
```

```
[Out] Integral((a + b*asech(c*x))/x, x)
```

3.27 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2} dx$

Optimal. Leaf size=40

$$\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{cx+1}}} - \frac{a+b\text{sech}^{-1}(cx)}{x}$$

```
[Out] (-a-b*arcsech(c*x))/x+b*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6283, 95}

$$\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{cx+1}}} - \frac{a+b\text{sech}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/x^2,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/x
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{x^2} dx &= -\frac{a+b\text{sech}^{-1}(cx)}{x} - \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}} dx\\ &= \frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{x}\end{aligned}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 1.05

$$-\frac{a}{x} + b\left(c+\frac{1}{x}\right)\sqrt{\frac{1-cx}{cx+1}} - \frac{b\text{sech}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/x^2,x]
```

```
[Out] -(a/x) + b*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/x
```

fricas [A] time = 0.49, size = 66, normalized size = 1.65

$$\frac{bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
[Out] (b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2)) + 1)/(c*x)) - a)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/x^2, x)
```

maple [A] time = 0.06, size = 58, normalized size = 1.45

$$c\left(-\frac{a}{cx} + b\left(-\frac{\operatorname{arcsech}(cx)}{cx} + \sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^2,x)
[Out] c*(-a/c/x+b*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)))
```

maxima [A] time = 0.31, size = 32, normalized size = 0.80

$$\left(c\sqrt{\frac{1}{c^2x^2} - 1} - \frac{\operatorname{arsech}(cx)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b - a/x
```

mupad [B] time = 1.48, size = 46, normalized size = 1.15

$$b\,c\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1} - \frac{b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/x^2,x)
[Out] b*c*(1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - (b*acosh(1/(c*x)))/x - a/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asech}(cx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))/x**2, x)
```

3.28 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=94

$$-\frac{a+b\,\text{sech}^{-1}(cx)}{2x^2}+\frac{1}{4}bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/x^2+1/4*b*(-c*x+1)^(1/2)/x^2/(1/(c*x+1))^(1/2)+1/4*
b*c^2*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6283, 103, 12, 92, 208}

$$-\frac{a+b\,\text{sech}^{-1}(cx)}{2x^2}+\frac{1}{4}bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/x^3,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(4*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(2*x^
2) + (b*c^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1
 + c*x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
```

```
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{2x^2} - \frac{1}{2}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} - \frac{1}{4}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{c^2}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} - \frac{1}{4}\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}\left(bc^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{c-cx^2}\,dx, x, \sqrt{}\right.$$

$$= \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

Mathematica [A] time = 0.08, size = 117, normalized size = 1.24

$$-\frac{a}{2x^2}-\frac{1}{4}bc^2\log(x)+\frac{1}{4}bc^2\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\frac{1-cx}{cx+1}}+1\right)+b\left(\frac{c}{4x}+\frac{1}{4x^2}\right)\sqrt{\frac{1-cx}{cx+1}}-\frac{b\text{sech}^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 + b*(1/(4*x^2) + c/(4*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech
[c*x])/(2*x^2) - (b*c^2*Log[x])/4 + (b*c^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)
] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/4
```

fricas [A] time = 0.43, size = 77, normalized size = 0.82

$$\frac{bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+\left(bc^2x^2-2\,b\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (b*c^2*x^2 - 2*b)*log((c*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^3, x)
```

maple [A] time = 0.07, size = 112, normalized size = 1.19

$$c^{2}\left(-\frac{a}{2c^{2}x^{2}}+b\left(-\frac{\operatorname{arcsech}\left(cx\right)}{2c^{2}x^{2}}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)c^{2}x^{2}+\sqrt{-c^{2}x^{2}+1}\right)}{4cx\sqrt{-c^{2}x^{2}+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^3,x)
```

```
[Out] c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c
/x*((c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^
(1/2))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.31, size = 105, normalized size = 1.12

$$-\frac{1}{8}\,b\left(\frac{\frac{2\,c^{4}x\sqrt{\frac{1}{c^{2}x^{2}}-1}}{c^{2}x^{2}\left(\frac{1}{c^{2}x^{2}}-1\right)-1}-c^{3}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}-1}+1\right)+c^{3}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}-1}-1\right)}{c}+\frac{4\,\operatorname{arsech}\left(cx\right)}{x^{2}}\right)-\frac{a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) -
1))/c + 4*arcsech(c*x)/x^2) - 1/2*a/x^2
```

mupad [B] time = 1.46, size = 61, normalized size = 0.65

$$\frac{b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\,\left(\frac{c^{2}\,x}{4}-\frac{1}{2\,x}\right)}{x}-\frac{a}{2\,x^{2}}+\frac{b\,c\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/x^3,x)
```

```
[Out] (b*acosh(1/(c*x))*((c^2*x)/4 - 1/(2*x)))/x - a/(2*x^2) + (b*c*(1/(c*x) - 1)
^(1/2)*(1/(c*x) + 1)^(1/2))/(4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{asech}\left(cx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))/x**3, x)
```

3.29 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=77

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{3x^3}+\frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/3*(-a-b*arcsech(c*x))/x^3+1/9*b*(-c*x+1)^(1/2)/x^3/(1/(c*x+1))^(1/2)+2/9*
b*c^2*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6283, 103, 12, 95}

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{3x^3}+\frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/x^4,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(9*x^3*Sqrt[(1 + c*x)^(-1)]) + (2*b*c^2*Sqrt[1 - c*x])/(9
*x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^4}\,dx = -\frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3} - \frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx$$
$$= \frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3} + \frac{1}{9}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx$$
$$= \frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3} - \frac{1}{9}\left(2bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx$$
$$= \frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}} + \frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3}$$

Mathematica [A] time = 0.07, size = 74, normalized size = 0.96

$$-\frac{a}{3x^3} + b\left(\frac{2c^3}{9} + \frac{2c^2}{9x} + \frac{c}{9x^2} + \frac{1}{9x^3}\right)\sqrt{\frac{1-cx}{cx+1}} - \frac{b\operatorname{sech}^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*((2*c^3)/9 + 1/(9*x^3) + c/(9*x^2) + (2*c^2)/(9*x))*Sqrt[(1
 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(3*x^3)
```

fricas [A] time = 0.58, size = 79, normalized size = 1.03

$$-\frac{3\,b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - \left(2\,bc^3x^3 + bcx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + 3\,a}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*b*c^3*x^
3 + b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 3*a)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx) + a}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^4, x)
```

maple [A] time = 0.06, size = 77, normalized size = 1.00

$$c^3\left(-\frac{a}{3c^3x^3} + b\left(-\frac{\mathrm{arcsech}\,(cx)}{3c^3x^3} + \frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2c^2x^2+1\right)}{9c^2x^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^4,x)
```

```
[Out] c^3*(-1/3/c^3/x^3*a+b*(-1/3*arcsech(c*x)/c^3/x^3+1/9*(-(c*x-1)/c/x)^(1/2)/c
^2/x^2*((c*x+1)/c/x)^(1/2)*(2*c^2*x^2+1)))
```

maxima [A] time = 0.30, size = 56, normalized size = 0.73

$$\frac{1}{9}b\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\,\operatorname{arsech}\left(cx\right)}{x^3}\right)-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*ar
csech(c*x)/x^3) - 1/3*a/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/x^4,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\operatorname{asech}\left(cx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))/x**4, x)
```

3.30 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=126

$$-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/4*(-a-b*arcsech(c*x))/x^4+1/16*b*(-c*x+1)^(1/2)/x^4/(1/(c*x+1))^(1/2)+3/3
2*b*c^2*(-c*x+1)^(1/2)/x^2/(1/(c*x+1))^(1/2)+3/32*b*c^4*arctanh((-c*x+1)^(1
/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6283, 103, 12, 92, 208}

$$-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}+\frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{cx+1}}}+\frac{3}{32}bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/x^5,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(16*x^4*Sqrt[(1 + c*x)^(-1)]) + (3*b*c^2*Sqrt[1 - c*x])/(
32*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(4*x^4) + (3*b*c^4*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
```

```
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{4x^4} - \frac{1}{4}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^5\sqrt{1-cx}\sqrt{1+cx}}\,dx$$
$$= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} + \frac{1}{16}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{3c^2}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,d$$
$$= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} - \frac{1}{16}\left(3bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}}$$
$$= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} - \frac{1}{32}\left(3bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{}{x\sqrt{}}$$
$$= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} - \frac{1}{32}\left(3bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{}{x\sqrt{}}$$
$$= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} + \frac{1}{32}\left(3bc^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}$$
$$= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} + \frac{3}{32}bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{}\right.$$

Mathematica [A] time = 0.11, size = 137, normalized size = 1.09

$$-\frac{a}{4x^4}-\frac{3}{32}bc^4\log(x)+\frac{3}{32}bc^4\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\frac{1-cx}{cx+1}}+1\right)+b\left(\frac{3c^3}{32x}+\frac{3c^2}{32x^2}+\frac{c}{16x^3}+\frac{1}{16x^4}\right)\sqrt{\frac{1-cx}{cx+1}}-\frac{bs}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(1/(16*x^4) + c/(16*x^3) + (3*c^2)/(32*x^2) + (3*c^3)/(32*x)
)*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(4*x^4) - (3*b*c^4*Log[x])/3
2 + (3*b*c^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*
x)]])/32
```

fricas [A] time = 0.60, size = 90, normalized size = 0.71

$$\frac{\left(3\,bc^4x^4-8\,b\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(3\,bc^3x^3+2\,bcx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-8\,a}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x
)) + (3*b*c^3*x^3 + 2*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 8*a)/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^5, x)
```

maple [A] time = 0.07, size = 135, normalized size = 1.07

$$c^{4}\left(-\frac{a}{4c^{4}x^{4}}+b\left(-\frac{\operatorname{arcsech}(cx)}{4c^{4}x^{4}}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)c^{4}x^{4}+3c^{2}x^{2}\sqrt{-c^{2}x^{2}+1}+2\sqrt{-c^{2}x^{2}+1}\right)}{32c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^5,x)
```

```
[Out] c^4*(-1/4*a/c^4/x^4+b*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/
c^3/x^3*((c*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*c^2*
x^2*(-c^2*x^2+1)^(1/2)+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.31, size = 147, normalized size = 1.17

$$\frac{1}{64}\,b\left(\frac{3\,c^{5}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}-1}+1\right)-3\,c^{5}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}-1}-1\right)-\frac{2\left(3\,c^{8}x^{3}\left(\frac{1}{c^{2}x^{2}}-1\right)^{\frac{3}{2}}-5\,c^{6}x\sqrt{\frac{1}{c^{2}x^{2}}-1}\right)}{c^{4}x^{4}\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}-2\,c^{2}x^{2}\left(\frac{1}{c^{2}x^{2}}-1\right)+1}}{c}-\frac{16\,\operatorname{arsech}(cx)}{x^{4}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1/(c
^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt(1/(
c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) +
 1))/c - 16*arcsech(c*x)/x^4) - 1/4*a/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/x^5,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\,(cx)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**5,x)
```

```
[Out] Integral((a + b*asech(c*x))/x**5, x)
```

3.31 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=109

$$-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}+\frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{cx+1}}}+\frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/5*(-a-b*arcsech(c*x))/x^5+1/25*b*(-c*x+1)^(1/2)/x^5/(1/(c*x+1))^(1/2)+4/7
5*b*c^2*(-c*x+1)^(1/2)/x^3/(1/(c*x+1))^(1/2)+8/75*b*c^4*(-c*x+1)^(1/2)/x/(1
/(c*x+1))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6283, 103, 12, 95}

$$-\frac{a+b\text{sech}^{-1}(cx)}{5x^5}+\frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{cx+1}}}+\frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/x^6,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(25*x^5*Sqrt[(1 + c*x)^(-1)]) + (4*b*c^2*Sqrt[1 - c*x])/(
75*x^3*Sqrt[(1 + c*x)^(-1)]) + (8*b*c^4*Sqrt[1 - c*x])/(75*x*Sqrt[(1 + c*x)
^(-1)]) - (a + b*ArcSech[c*x])/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{5x^5} - \frac{1}{5}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^6\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5} + \frac{1}{25}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{4c^2}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5} - \frac{1}{25}\left(4bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5} + \frac{1}{75}\left(4bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{\ldots}{x^2\sqrt{1-\ldots}}$$

$$= \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5} - \frac{1}{75}\left(8bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\ldots}{x^2\sqrt{1-\ldots}}$$

$$= \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}} + \frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.86

$$-\frac{a}{5x^5} + b\left(\frac{8c^5}{75} + \frac{8c^4}{75x} + \frac{4c^3}{75x^2} + \frac{4c^2}{75x^3} + \frac{c}{25x^4} + \frac{1}{25x^5}\right)\sqrt{\frac{1-cx}{cx+1}} - \frac{b\text{sech}^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 + b*((8*c^5)/75 + 1/(25*x^5) + c/(25*x^4) + (4*c^2)/(75*x^3) + (
4*c^3)/(75*x^2) + (8*c^4)/(75*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*
x])/(5*x^5)
```

fricas [A] time = 0.63, size = 89, normalized size = 0.82

$$-\frac{15\,b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - \left(8\,bc^5x^5 + 4\,bc^3x^3 + 3\,bcx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + 15\,a}{75\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*(15*b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5*
x^5 + 4*b*c^3*x^3 + 3*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*a)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx) + a}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^6, x)
```

maple [A] time = 0.07, size = 85, normalized size = 0.78

$$c^{5}\left(-\frac{a}{5c^{5}x^{5}}+b\left(-\frac{\operatorname{arcsech}(cx)}{5c^{5}x^{5}}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(8c^{4}x^{4}+4c^{2}x^{2}+3\right)}{75c^{4}x^{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^6,x)
```

```
[Out] c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*arcsech(c*x)+1/75*(-(c*x-1)/c/x)^(1/2)/
c^4/x^4*((c*x+1)/c/x)^(1/2)*(8*c^4*x^4+4*c^2*x^2+3)))
```

maxima [A] time = 0.30, size = 73, normalized size = 0.67

$$\frac{1}{75}\,b\left(\frac{3\,c^{6}\left(\frac{1}{c^{2}x^{2}}-1\right)^{\frac{5}{2}}+10\,c^{6}\left(\frac{1}{c^{2}x^{2}}-1\right)^{\frac{3}{2}}+15\,c^{6}\sqrt{\frac{1}{c^{2}x^{2}}-1}}{c}-\frac{15\,\operatorname{arsech}(cx)}{x^{5}}\right)-\frac{a}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 1
5*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) - 1/5*a/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/x^6,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}(cx)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**6,x)
```

```
[Out] Integral((a + b*asech(c*x))/x**6, x)
```

3.32 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^7}\,dx$

Optimal. Leaf size=158

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6}+\frac{5}{96}bc^6\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{cx+1}}}+\frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{cx+1}}}$$

```
[Out] 1/6*(-a-b*arcsech(c*x))/x^6+1/36*b*(-c*x+1)^(1/2)/x^6/(1/(c*x+1))^(1/2)+5/1
44*b*c^2*(-c*x+1)^(1/2)/x^4/(1/(c*x+1))^(1/2)+5/96*b*c^4*(-c*x+1)^(1/2)/x^2
/(1/(c*x+1))^(1/2)+5/96*b*c^6*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6283, 103, 12, 92, 208}

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6}+\frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{cx+1}}}+\frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{cx+1}}}+\frac{5}{96}bc^6\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/x^7,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(36*x^6*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^2*Sqrt[1 - c*x])/(
144*x^4*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^4*Sqrt[1 - c*x])/(96*x^2*Sqrt[(1 + c
*x)^(-1)]) - (a + b*ArcSech[c*x])/(6*x^6) + (5*b*c^6*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/96
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
```

```
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^7}\,dx = -\frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} - \frac{1}{6}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^7\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} + \frac{1}{36}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{5c^2}{x^5\sqrt{1-cx}\sqrt{1+cx}}\,d$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} - \frac{1}{36}\left(5bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^5\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} + \frac{1}{144}\left(5bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} - \frac{1}{48}\left(5bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{}{x^3}$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} - \frac{1}{96}\left(5bc^4\sqrt{\frac{1}{1+cx}}\right.$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} - \frac{1}{96}\left(5bc^6\sqrt{\frac{1}{1+cx}}\right.$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} + \frac{1}{96}\left(5bc^7\sqrt{\frac{1}{1+cx}}\right.$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{6x^6} + \frac{5}{96}bc^6\sqrt{\frac{1}{1+cx}}\sqrt{1}$$

Mathematica [A] time = 0.16, size = 157, normalized size = 0.99

$$-\frac{a}{6x^6}-\frac{5}{96}bc^6\log(x)+\frac{5}{96}bc^6\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\frac{1-cx}{cx+1}}+1\right)+b\left(\frac{5c^5}{96x}+\frac{5c^4}{96x^2}+\frac{5c^3}{144x^3}+\frac{5c^2}{144x^4}+\frac{c}{36x^5}+\frac{}{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(1/(36*x^6) + c/(36*x^5) + (5*c^2)/(144*x^4) + (5*c^3)/(144*
x^3) + (5*c^4)/(96*x^2) + (5*c^5)/(96*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*Ar
cSech[c*x])/(6*x^6) - (5*b*c^6*Log[x])/96 + (5*b*c^6*Log[1 + Sqrt[(1 - c*x)
/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/96
```

fricas [A] time = 0.59, size = 100, normalized size = 0.63

$$\frac{3\left(5\,bc^6x^6-16\,b\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(15\,bc^5x^5+10\,bc^3x^3+8\,bcx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-48\,a}{288\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/288*(3*(5*b*c^6*x^6 - 16*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x)) + (15*b*c^5*x^5 + 10*b*c^3*x^3 + 8*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) - 48*a)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^7, x)
```

maple [A] time = 0.07, size = 155, normalized size = 0.98

$$c^6\left(-\frac{a}{6c^6x^6}+b\left(-\frac{\operatorname{arcsech}(cx)}{6c^6x^6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(15\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^6x^6+15\sqrt{-c^2x^2+1}\,c^4x^4+10c^2x^2\sqrt{-\right.}{288c^5x^5\sqrt{-c^2x^2+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^7,x)
```

```
[Out] c^6*(-1/6*a/c^6/x^6+b*(-1/6/c^6/x^6*arcsech(c*x)+1/288*(-(c*x-1)/c/x)^(1/2)
/c^5/x^5*((c*x+1)/c/x)^(1/2)*(15*arctanh(1/(-c^2*x^2+1)^(1/2))*c^6*x^6+15*(
-c^2*x^2+1)^(1/2)*c^4*x^4+10*c^2*x^2*(-c^2*x^2+1)^(1/2)+8*(-c^2*x^2+1)^(1/2
))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.32, size = 185, normalized size = 1.17

$$\frac{1}{576}b\left(\frac{15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right)-15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)-\frac{2\left(15\,c^{12}x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-40\,c^{10}x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+33\,c^8x\sqrt{\frac{1}{c^2x^2}-1}\right.}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] 1/576*b*((15*c^7*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 15*c^7*log(c*x*sqrt(1
/(c^2*x^2) - 1) - 1) - 2*(15*c^12*x^5*(1/(c^2*x^2) - 1)^(5/2) - 40*c^10*x^3
*(1/(c^2*x^2) - 1)^(3/2) + 33*c^8*x*sqrt(1/(c^2*x^2) - 1))/(c^6*x^6*(1/(c^2
*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) - 1)^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1)
- 1))/c - 96*arcsech(c*x)/x^6) - 1/6*a/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/x^7,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/x^7, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asech}(cx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**7,x)
```

```
[Out] Integral((a + b*asech(c*x))/x**7, x)
```

3.33 $\int x^3 \left(a + b\,\mathbf{sech}^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=124

$$-\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3c^4}-\frac{bx^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6c^2}+\frac{1}{4}x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{3c^4}-$$

```
[Out] -1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arcsech(c*x))^2-1/3*b^2*ln(x)/c^4-1/3*b*(c*x
+1)*(a+b*arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/6*b*x^2*(c*x+1)*(a+b*
arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6285, 5451, 4185, 4184, 3475}

$$-\frac{bx^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6c^2}-\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3c^4}+\frac{1}{4}x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2-\frac{b^2x^2}{12c^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -(b^2*x^2)/(12*c^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech
[c*x]))/(3*c^4) - (b*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech
[c*x]))/(6*c^2) + (x^4*(a + b*ArcSech[c*x])^2)/4 - (b^2*Log[x])/(3*c^4)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx = -\frac{\text{Subst}\left(\int (a+bx)^2\text{sech}^4(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^4}$$
$$= \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{b\,\text{Subst}\left(\int (a+bx)\text{sech}^4(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{2c^4}$$
$$= -\frac{b^2x^2}{12c^2} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{6c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{b\,\text{Su}}{}$$
$$= -\frac{b^2x^2}{12c^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(\right.}{6c^2}$$
$$= -\frac{b^2x^2}{12c^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(\right.}{6c^2}$$

Mathematica [A] time = 0.35, size = 212, normalized size = 1.71

$$-\frac{-3a^2c^4x^4 + 2abc^3x^3\sqrt{\frac{1-cx}{cx+1}} + 2abc^2x^2\sqrt{\frac{1-cx}{cx+1}} + 2b\text{sech}^{-1}(cx)\left(b\sqrt{\frac{1-cx}{cx+1}}\left(c^3x^3+c^2x^2+2cx+2\right) - 3ac^4x^4\right) + 4}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -1/12*(b^2*c^2*x^2 - 3*a^2*c^4*x^4 + 4*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*a*
b*c*x*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] +
 2*a*b*c^3*x^3*Sqrt[(1 - c*x)/(1 + c*x)] + 2*b*(-3*a*c^4*x^4 + b*Sqrt[(1 -
c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3))*ArcSech[c*x] - 3*b^2*c^4*x
^4*ArcSech[c*x]^2 + 4*b^2*Log[x])/c^4
```

fricas [B] time = 0.53, size = 244, normalized size = 1.97

$$\frac{3\,b^2c^4x^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 + 3\,a^2c^4x^4 - 6\,abc^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - b^2c^2x^2 - 4\,b^2\log(x) + 2\left(3\,abc^4x^4 - 3\,ab\right.}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 +
 3*a^2*c^4*x^4 - 6*a*b*c^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x)
- b^2*c^2*x^2 - 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 - (b^2*c^3*x^3
+ 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)) - 2*(a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)))/c^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsech}\,(cx) + a)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2*x^3, x)
```

maple [B] time = 0.72, size = 264, normalized size = 2.13

$$\frac{a^2x^4}{4}-\frac{b^2\mathrm{arcsech}\,(cx)}{3c^4}+\frac{b^2\mathrm{arcsech}\,(cx)^2\,x^4}{4}-\frac{b^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\mathrm{arcsech}\,(cx)\,x^3}{6c}-\frac{b^2\mathrm{arcsech}\,(cx)\,\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,x}{3c^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))^2,x)
```

```
[Out] 1/4*a^2*x^4-1/3/c^4*b^2*arcsech(c*x)+1/4*b^2*arcsech(c*x)^2*x^4-1/6/c*b^2*(
-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)*x^3-1/3/c^3*b^2*arcsec
h(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x-1/12*b^2*x^2/c^2+1/3/c^4*
b^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+1/2*a*b*x^4*arcsech(c*
x)-1/6/c*a*b*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3-1/3/c^3*a*b*(-(c*
x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a^2x^4+\frac{1}{6}\left(3\,x^4\,\mathrm{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)ab+b^2\int x^3\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/6*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a*b + b^2*integrate(x^3*log(sqrt(1/(c*x) +
1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int(x^3*(a + b*acosh(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,(a+b\,\mathrm{asech}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))**2, x)
```

3.34 $\int x^2 \left(a + b\,\text{sech}^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=140

$$-\frac{2b\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^3}-\frac{bx\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2+\frac{ib^2\text{Li}}{}$$

```
[Out] -1/3*b^2*x/c^2+1/3*x^3*(a+b*arcsech(c*x))^2-2/3*b*(a+b*arcsech(c*x))*arctan
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c^3+1/3*I*b^2*polylog(2,-I*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/3*I*b^2*polylog(2,I*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/3*b*x*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1
)/(c*x+1))^(1/2)/c^2
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6285, 5451, 4185, 4180, 2279, 2391}

$$\frac{ib^2\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{3c^3}-\frac{bx\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}-\frac{2b\tan^{-1}\left(e^{\text{sec}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -(b^2*x)/(3*c^2) - (b*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[
c*x]))/(3*c^2) + (x^3*(a + b*ArcSech[c*x])^2)/3 - (2*b*(a + b*ArcSech[c*x])
*ArcTan[E^ArcSech[c*x]])/(3*c^3) + ((I/3)*b^2*PolyLog[2, (-I)*E^ArcSech[c*x
]])/c^3 - ((I/3)*b^2*PolyLog[2, I*E^ArcSech[c*x]])/c^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^2\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=-\frac{\text{Subst}\left(\int(a+bx)^2\text{sech}^3(x)\tanh(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{c^3}$$
$$=\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{(2b)\,\text{Subst}\left(\int(a+bx)\text{sech}^3(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{3c^3}$$
$$=-\frac{b^2x}{3c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{b\,\text{Subst}\left(\right.}{}$$
$$=-\frac{b^2x}{3c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{2b\left(a+b\right.}{}$$
$$=-\frac{b^2x}{3c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{2b\left(a+b\right.}{}$$
$$=-\frac{b^2x}{3c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{2b\left(a+b\right.}{}$$

Mathematica [A] time = 1.27, size = 224, normalized size = 1.60

$$\frac{1}{3}\left(a^2x^3+ab\left(2x^3\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}\left(c^3x^3+\sqrt{1-c^2x^2}\,\sin^{-1}(cx)-cx\right)}{c^3(cx-1)}\right)+\frac{b^2\left(c^3x^3\text{sech}^{-1}(cx)^2+i\text{Li}_2\left(-ie^{-\text{sech}}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] (a^2*x^3 + a*b*(2*x^3*ArcSech[c*x] - (Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*x) + c
^3*x^3 + Sqrt[1 - c^2*x^2]*ArcSin[c*x]))/(c^3*(-1 + c*x))) + (b^2*(-(c*x) -
 c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x] + c^3*x^3*ArcSech[c*x
]^2 + I*ArcSech[c*x]*Log[1 - I/E^ArcSech[c*x]] - I*ArcSech[c*x]*Log[1 + I/E
^ArcSech[c*x]] + I*PolyLog[2, (-I)/E^ArcSech[c*x]] - I*PolyLog[2, I/E^ArcSe
ch[c*x]]))/c^3)/3
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\,\text{arsech}\,(cx)^2+2\,abx^2\,\text{arsech}\,(cx)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsech}(cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2*x^2, x)
```

maple [A] time = 0.64, size = 372, normalized size = 2.66

$$\frac{x^3a^2}{3}+\frac{x^3b^2\operatorname{arcsech}(cx)^2}{3}-\frac{b^2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,x^2}{3c}-\frac{b^2x}{3c^2}+\frac{ib^2\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\right.\right.}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*x^3*b^2*arcsech(c*x)^2-1/3/c*b^2*arcsech(c*x)*(-(c*x-1)/c/x
)^(1/2)*((c*x+1)/c/x)^(1/2)*x^2-1/3*b^2*x/c^2+1/3*I/c^3*b^2*arcsech(c*x)*ln
(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/3*I/c^3*b^2*arcsech(c*x)*l
n(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/3*I/c^3*b^2*dilog(1+I*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/3*I/c^3*b^2*dilog(1-I*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2)))+2/3*a*b*x^3*arcsech(c*x)-1/3/c*a*b*(-(c*x-1)/
c/x)^(1/2)*x^2*((c*x+1)/c/x)^(1/2)+1/3/c^2*a*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x
+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)*arcsin(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2x^3+\frac{1}{3}\left(2x^3\operatorname{arsech}(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)ab+b^2\int x^2\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2
*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*a*b + b^2*integra
te(x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int(x^2*(a + b*acosh(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\operatorname{asech}(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))**2, x)
```

3.35 $\int x \left(a + b\,\mathrm{sech}^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=65

$$-\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^2}+\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arcsech(c*x))^2-b^2*ln(x)/c^2-b*(c*x+1)*(a+b*arcsech(c*x))*((-
c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6285, 5451, 4184, 3475}

$$-\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^2}+\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/c^2) + (x^2*
(a + b*ArcSech[c*x])^2)/2 - (b^2*Log[x])/c^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 dx = -\frac{\text{Subst}\left(\int(a+bx)^2\operatorname{sech}^2(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^2}$$
$$= \frac{1}{2}x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 - \frac{b\,\text{Subst}\left(\int(a+bx)\operatorname{sech}^2(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^2}$$
$$= -\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^2} + \frac{1}{2}x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 + \frac{b^2\,\text{Subst}\left(\int \tanh\right.}{}$$
$$= -\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^2} + \frac{1}{2}x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{c^2}$$

Mathematica [A] time = 0.25, size = 112, normalized size = 1.72

$$\frac{a\left(ac^2x^2-2b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\right)-2b\operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1-cx}{cx+1}}(cx+1)-ac^2x^2\right)+b^2c^2x^2\operatorname{sech}^{-1}(cx)^2-2b^2\log(cx)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] (a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) - 2*b*(-(a*c^2*x^2
) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*c^2*x^2*ArcSe
ch[c*x]^2 - 2*b^2*Log[c*x])/(2*c^2)
```

fricas [B] time = 0.70, size = 205, normalized size = 3.15

$$\frac{b^2c^2x^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 + a^2c^2x^2 - 2\,abc^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\,b^2\log(x) + 2\left(abc^2x^2 - b^2\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + a^
2*c^2*x^2 - 2*a*b*c^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*a
*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*b^2*log(x) + 2*(a*b*c^2*x^2 - b^2
*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) + 1)/(c*x)))/c^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arsech}(cx)+a)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2*x, x)
```

maple [B] time = 0.58, size = 168, normalized size = 2.58

$$\frac{a^2x^2}{2} - \frac{b^2\operatorname{arcsech}(cx)}{c^2} + \frac{x^2b^2\operatorname{arcsech}(cx)^2}{2} - \frac{b^2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,x}{c} + \frac{b^2\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right.\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2-1/c^2*b^2*arcsech(c*x)+1/2*x^2*b^2*arcsech(c*x)^2-1/c*b^2*arcse
ch(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x+1/c^2*b^2*ln(1+(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+a*b*arcsech(c*x)*x^2-1/c*a*b*(-(c*x-1)/c
/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x
```

maxima [A] time = 0.32, size = 84, normalized size = 1.29

$$\frac{1}{2}\,b^2x^2\,\mathrm{arsech}\,(cx)^2+\frac{1}{2}\,a^2x^2+\left(x^2\,\mathrm{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)ab-\left(\frac{x\sqrt{\frac{1}{c^2x^2}-1}\,\mathrm{arsech}\,(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arcsech(c*x)^2 + 1/2*a^2*x^2 + (x^2*arcsech(c*x) - x*sqrt(1/(c^
2*x^2) - 1)/c)*a*b - (x*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x)/c + log(x)/c^2)*
b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int(x*(a + b*acosh(1/(c*x)))^2, x)
```

sympy [A] time = 1.17, size = 99, normalized size = 1.52

$$\begin{cases} \frac{a^2x^2}{2}+abx^2\,\mathrm{asech}\,(cx)-\frac{ab\sqrt{-c^2x^2+1}}{c^2}+\frac{b^2x^2\,\mathrm{asech}^2\,(cx)}{2}-\frac{b^2\sqrt{-c^2x^2+1}\,\mathrm{asech}\,(cx)}{c^2}-\frac{b^2\log\,(x)}{c^2} & \text{for } c\neq 0\\ \frac{x^2(a+\infty b)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*asech(c*x) - a*b*sqrt(-c**2*x**2 + 1)/c**
2 + b**2*x**2*asech(c*x)**2/2 - b**2*sqrt(-c**2*x**2 + 1)*asech(c*x)/c**2 -
 b**2*log(x)/c**2, Ne(c, 0)), (x**2*(a + oo*b)**2/2, True))
```

3.36 $\int \left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=78

$$x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2-\frac{4b\tan^{-1}\left(e^{\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c}+\frac{2ib^2\mathrm{Li}_2\left(-ie^{\mathrm{sech}^{-1}(cx)}\right)}{c}-\frac{2ib^2\mathrm{Li}_2\left(ie^{\mathrm{sech}^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arcsech(c*x))^2-4*b*(a+b*arcsech(c*x))*arctan(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/c+2*I*b^2*polylog(2,-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2)))/c-2*I*b^2*polylog(2,I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6279, 5451, 4180, 2279, 2391}

$$\frac{2ib^2\text{PolyLog}\left(2,-ie^{\mathrm{sech}^{-1}(cx)}\right)}{c}-\frac{2ib^2\text{PolyLog}\left(2,ie^{\mathrm{sech}^{-1}(cx)}\right)}{c}+x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2-\frac{4b\tan^{-1}\left(e^{\mathrm{sech}^{-1}(cx)}\right)\left(a+b\mathrm{se}\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^2,x]
```

```
[Out] x*(a + b*ArcSech[c*x])^2 - (4*b*(a + b*ArcSech[c*x])*ArcTan[E^ArcSech[c*x]]
)/c + ((2*I)*b^2*PolyLog[2, (-I)*E^ArcSech[c*x]])/c - ((2*I)*b^2*PolyLog[2,
 I*E^ArcSech[c*x]])/c
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = -\frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{(2b)\,\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{4b\left(a+b\text{sech}^{-1}(cx)\right)\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{\left(2ib^2\right)\text{Subst}\left(\int\right.}{}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{4b\left(a+b\text{sech}^{-1}(cx)\right)\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{\left(2ib^2\right)\text{Subst}\left(\int\right.}{}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{4b\left(a+b\text{sech}^{-1}(cx)\right)\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{2ib^2\text{Li}_2\left(-ie^{\text{sech}^{-1}}\right.}{c}$$

Mathematica [A] time = 0.27, size = 126, normalized size = 1.62

$$a^2x+\frac{2ab\left(cx\text{sech}^{-1}(cx)-2\tan^{-1}\left(\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)\right)\right)}{c}+\frac{ib^2\left(2\text{Li}_2\left(-ie^{-\text{sech}^{-1}(cx)}\right)-2\text{Li}_2\left(ie^{-\text{sech}^{-1}(cx)}\right)+\text{sec}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]))/c + (I*
b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x] + 2*Log[1 - I/E^ArcSech[c*x]] - 2*
Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[c*x]] - 2*PolyLog[
2, I/E^ArcSech[c*x]]))/c
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\,\text{arsech}\,(cx)^2+2\,ab\,\text{arsech}\,(cx)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{arsech}\,(cx)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2, x)
```

maple [A] time = 0.30, size = 250, normalized size = 3.21

$$x\,b^2\text{arcsech}\,(cx)^2-\frac{2i\text{arcsech}\,(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)b^2}{c}+\frac{2i\text{arcsech}\,(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1\right.\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^2,x)
```

```
[Out] x*b^2*arcsech(c*x)^2-2*I/c*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2)))*b^2+2*I/c*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2)))*b^2+2*x*a*b*arcsech(c*x)+2*I/c*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2)))*b^2-2*I/c*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2)))*b^2+a^2*x-2/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*a*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(x\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)^2-\int-\frac{c^2x^2\log(c)^2+\left(c^2x^2-1\right)\log(x)^2+\left(c^2x^2\log(c)^2+\left(c^2x^2-1\right)\log(x)^2-\text{lo}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] (x*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 - integrate(-(c^2*x^2*log(c)^2 +
 (c^2*x^2 - 1)*log(x)^2 + (c^2*x^2*log(c)^2 + (c^2*x^2 - 1)*log(x)^2 - log(
c)^2 + 2*(c^2*x^2*log(c) - log(c))*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2
*(c^2*x^2*log(c) + (c^2*x^2*(log(c) + 1) + (c^2*x^2 - 1)*log(x) - log(c))*s
qrt(c*x + 1)*sqrt(-c*x + 1) + (c^2*x^2 - 1)*log(x) - log(c))*log(sqrt(c*x +
 1)*sqrt(-c*x + 1) + 1) - log(c)^2 + 2*(c^2*x^2*log(c) - log(c))*log(x))/(c
^2*x^2 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 1), x))*b^2 + a^2*x +
 2*(c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{asech}\,(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**2,x)
```

```
[Out] Integral((a + b*asech(c*x))**2, x)
```

3.37 $\int \frac{\left(a+b\,\mathbf{sech}^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=83

$$-b\mathrm{Li}_2\left(-e^{2\mathrm{sech}^{-1}(cx)}\right)\left(a+b\mathrm{sech}^{-1}(cx)\right)+\frac{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}{3b}-\log\left(e^{2\mathrm{sech}^{-1}(cx)}+1\right)\left(a+b\mathrm{sech}^{-1}(cx)\right)^2+\frac{1}{2}b^2\mathrm{Li}_3$$

```
[Out] 1/3*(a+b*arcsech(c*x))^3/b-(a+b*arcsech(c*x))^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)-b*(a+b*arcsech(c*x))*polylog(2,-(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)+1/2*b^2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))^2)
```

Rubi [A] time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 3718, 2190, 2531, 2282, 6589}

$$-b\mathrm{PolyLog}\left(2,-e^{2\mathrm{sech}^{-1}(cx)}\right)\left(a+b\mathrm{sech}^{-1}(cx)\right)+\frac{1}{2}b^2\mathrm{PolyLog}\left(3,-e^{2\mathrm{sech}^{-1}(cx)}\right)+\frac{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}{3b}-\log\left(e^{2\mathrm{sech}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^2/x,x]
```

```
[Out] (a + b*ArcSech[c*x])^3/(3*b) - (a + b*ArcSech[c*x])^2*Log[1 + E^(2*ArcSech[
c*x])] - b*(a + b*ArcSech[c*x])*PolyLog[2, -E^(2*ArcSech[c*x])] + (b^2*Poly
Log[3, -E^(2*ArcSech[c*x])])/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x}\,dx &= -\operatorname{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3b} - 2\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) + (2b)\operatorname{Subst}\left(\int (a+b\right.\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) - b\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) - b\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) - b\left(a+b\operatorname{sech}^{-1}(cx)\right)
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 116, normalized size = 1.40

$$a^2\log(cx)+ab\left(\operatorname{Li}_2\left(-e^{-2\operatorname{sech}^{-1}(cx)}\right)-\operatorname{sech}^{-1}(cx)\left(\operatorname{sech}^{-1}(cx)+2\log\left(e^{-2\operatorname{sech}^{-1}(cx)}+1\right)\right)\right)+b^2\left(\operatorname{sech}^{-1}(cx)\operatorname{Li}_2\left(-e^{-2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x,x]
```

```
[Out] a^2*Log[c*x] + a*b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[
c*x])])) + PolyLog[2, -E^(-2*ArcSech[c*x])]) + b^2*(-1/3*ArcSech[c*x]^3 - A
rcSech[c*x]^2*Log[1 + E^(-2*ArcSech[c*x])] + ArcSech[c*x]*PolyLog[2, -E^(-2
*ArcSech[c*x])] + PolyLog[3, -E^(-2*ArcSech[c*x])]/2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsech}(cx)^2+2\,ab\operatorname{arsech}(cx)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2/x, x)
```

maple [A] time = 0.13, size = 250, normalized size = 3.01

$$a^2\ln(cx)+\frac{b^2\mathrm{arcsech}\,(cx)^3}{3}-b^2\mathrm{arcsech}\,(cx)^2\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\right)-b^2\mathrm{arcsech}\,(cx)\,\mathrm{polylog}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^2/x,x)
```

```
[Out] a^2*ln(c*x)+1/3*b^2*arcsech(c*x)^3-b^2*arcsech(c*x)^2*ln(1+(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))^2)-b^2*arcsech(c*x)*polylog(2,-(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2)+1/2*b^2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))^2)+a*b*arcsech(c*x)^2-2*a*b*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2)-a*b*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\log(x)+\int\frac{b^2\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^2}{x}+\frac{2\,ab\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x
))^2/x + 2*a*b*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^2/x,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\mathrm{asech}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**2/x,x)
```

```
[Out] Integral((a + b*asech(c*x))**2/x, x)
```

3.38 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=61

$$\frac{2b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}-\frac{2b^2}{x}$$

```
[Out] -2*b^2/x-(a+b*arcsech(c*x))^2/x+2*b*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1)/(c
*x+1))^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6285, 3296, 2638}

$$\frac{2b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}-\frac{2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^2/x^2,x]
```

```
[Out] (-2*b^2)/x + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))
/x - (a + b*ArcSech[c*x])^2/x
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^2}\,dx &= -\left(c\,\mathrm{Subst}\left(\int (a+bx)^2\sinh(x)\,dx,x,\mathrm{sech}^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}+(2bc)\,\mathrm{Subst}\left(\int (a+bx)\cosh(x)\,dx,x,\mathrm{sech}^{-1}(cx)\right)\\
&= \frac{2b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}-\left(2b^2c\right)\mathrm{Subst}\left(\int \sinh(x\right.\\
&= -\frac{2b^2}{x}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 87, normalized size = 1.43

$$-\frac{a^2-2ab\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2b\operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1-cx}{cx+1}}(cx+1)-a\right)+b^2\operatorname{sech}^{-1}(cx)^2+2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^2,x]
[Out] -((a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 2*b*(-a + b*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*ArcSech[c*x]^2)/x)
```

fricas [B] time = 0.59, size = 143, normalized size = 2.34

$$\frac{2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-b^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2-a^2-2\,b^2+2\left(b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-ab\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="fricas")
[Out] (2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b^2*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) - a*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsech}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^2/x^2, x)
```

maple [B] time = 0.14, size = 124, normalized size = 2.03

$$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{cx}+2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}-\frac{2}{cx}\right)+2ab\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\sqrt{-\frac{cx-1}{cx}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^2/x^2,x)
[Out] c*(-a^2/c/x+b^2*(-1/c/x*arcsech(c*x)^2+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*
((c*x+1)/c/x)^(1/2)-2/c/x)+2*a*b*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)*
((c*x+1)/c/x)^(1/2)))
```

maxima [A] time = 0.32, size = 78, normalized size = 1.28

$$2\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\operatorname{arsech}(cx)}{x}\right)ab+2\left(c\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arsech}(cx)-\frac{1}{x}\right)b^2-\frac{b^2\operatorname{arsech}(cx)^2}{x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="maxima")
[Out] 2*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) -
1)*arcsech(c*x) - 1/x)*b^2 - b^2*arcsech(c*x)^2/x - a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^2/x^2,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{asech}\,(cx)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))**2/x**2, x)
```

3.39 $\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=118

$$-\frac{1}{2}abc^2\text{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2}-\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2x^2}-\frac{1}{4}b^2c^2\text{sech}^{-1}(cx)$$

```
[Out] -1/4*b^2*(-c*x+1)*(c*x+1)/x^2-1/2*a*b*c^2*arcsech(c*x)-1/4*b^2*c^2*arcsech(
c*x)^2-1/2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))^2/x^2+1/2*b*(c*x+1)*(a+b*arc
sech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6285, 5446, 3310}

$$-\frac{1}{2}abc^2\text{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2}-\frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2x^2}-\frac{1}{4}b^2c^2\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^2/x^3,x]
```

```
[Out] -(b^2*(1 - c*x)*(1 + c*x))/(4*x^2) - (a*b*c^2*ArcSech[c*x])/2 - (b^2*c^2*Ar
cSech[c*x]^2)/4 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x
]))/(2*x^2) - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*x^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^3}\,dx = -\left(c^2\operatorname{Subst}\left(\int (a+bx)^2\cosh(x)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$
$$= -\frac{(1-cx)(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2x^2} + \left(bc^2\right)\operatorname{Subst}\left(\int (a+bx)\sinh^2(x)\,dx, x, \operatorname{sech}^{-1}\right.$$
$$= -\frac{b^2(1-cx)(1+cx)}{4x^2} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2} - \frac{(1-cx)(1+cx)\left(a+b\operatorname{se}\right.}{2x^2}$$
$$= -\frac{b^2(1-cx)(1+cx)}{4x^2} - \frac{1}{2}abc^2\operatorname{sech}^{-1}(cx) - \frac{1}{4}b^2c^2\operatorname{sech}^{-1}(cx)^2 + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\right.}{2x^2}$$

Mathematica [A] time = 0.18, size = 183, normalized size = 1.55

$$\frac{-2a^2 - 2abc^2x^2\log(x) + 2abc^2x^2\log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1\right) + 2ab\sqrt{\frac{1-cx}{cx+1}} + 2abcx\sqrt{\frac{1-cx}{cx+1}} + 2b\operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1}{c}}\right.}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^3,x]
```

```
[Out] (-2*a^2 - b^2 + 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c*x*Sqrt[(1 - c*x)/
(1 + c*x)] + 2*b*(-2*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x
] + b^2*(-2 + c^2*x^2)*ArcSech[c*x]^2 - 2*a*b*c^2*x^2*Log[x] + 2*a*b*c^2*x^
2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(4*x^
2)
```

fricas [A] time = 0.58, size = 165, normalized size = 1.40

$$\frac{2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + \left(b^2c^2x^2 - 2\,b^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 - 2\,a^2 - b^2 + 2\left(abc^2x^2 + b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\,ab\right)\log\left(\frac{cx\sqrt{-\right.}{}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (b^2*c^2*x^2 - 2*b^2)*log((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 - 2*a^2 - b^2 + 2*(a*b*c^2
*x^2 + b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*a*b)*log((c*x*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arsech}(cx) + a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2/x^3, x)
```

maple [A] time = 0.14, size = 192, normalized size = 1.63

$$c^2\left(-\frac{a^2}{2c^2x^2} + b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{2c^2x^2} + \frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{2cx} + \frac{\operatorname{arcsech}(cx)^2}{4} - \frac{1}{4c^2x^2}\right) + 2ab\left(-\frac{\operatorname{arcsech}(cx)}{2c^2x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^2/x^3,x)
```

```
[Out] c^2*(-1/2*a^2/c^2/x^2+b^2*(-1/2/c^2/x^2*arcsech(c*x)^2+1/2*arcsech(c*x)/c/x
*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/4*arcsech(c*x)^2-1/4/c^2/x^2)+2
*a*b*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((c*x+1)/c/x)^
(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+
1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\,ab\left(\frac{\frac{2\,c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right)+c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)}{c}+\frac{4\,\mathrm{arsech}\,(cx)}{x^2}\right)+b^2\int\frac{\log\left(\sqrt{\frac{1}{cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1)
- 1))/c + 4*arcsech(c*x)/x^2) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/
(c*x) - 1) + 1/(c*x))^2/x^3, x) - 1/2*a^2/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^2/x^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asech}{\left(c x \right)}\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))**2/x**3, x)
```

3.40 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=122

$$\frac{4bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{9x}+\frac{2b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{9x^3}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{3x^3}-\frac{4b^2c^2}{9x}-\frac{2b^2}{27x^3}$$

```
[Out] -2/27*b^2/x^3-4/9*b^2*c^2/x-1/3*(a+b*arcsech(c*x))^2/x^3+2/9*b*(c*x+1)*(a+b
*arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/x^3+4/9*b*c^2*(c*x+1)*(a+b*arcsech(
c*x))*((-c*x+1)/(c*x+1))^(1/2)/x
```

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6285, 5447, 3310, 3296, 2638}

$$\frac{4bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{9x}+\frac{2b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{9x^3}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{3x^3}-\frac{4b^2c^2}{9x}-\frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^2/x^4,x]
```

```
[Out] (-2*b^2)/(27*x^3) - (4*b^2*c^2)/(9*x) + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(a + b*ArcSech[c*x]))/(9*x^3) + (4*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(a + b*ArcSech[c*x]))/(9*x) - (a + b*ArcSech[c*x])^2/(3*x^3)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx = -\left(c^3\,\text{Subst}\left(\int (a+bx)^2\cosh^2(x)\sinh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)\cosh^3(x)\,dx, x, \text{sech}^{-1}(cx)\right)$$
$$= -\frac{2b^2}{27x^3}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3}+\frac{1}{9}\left(4bc^3\right)\text{Su}$$
$$= -\frac{2b^2}{27x^3}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3}+\frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}\right.}{9x}$$
$$= -\frac{2b^2}{27x^3}-\frac{4b^2c^2}{9x}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3}+\frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+\right.}{9x}$$

Mathematica [A] time = 0.29, size = 134, normalized size = 1.10

$$\frac{-9a^2+6ab\sqrt{\frac{1-cx}{cx+1}}\left(2c^3x^3+2c^2x^2+cx+1\right)+6b\text{sech}^{-1}(cx)\left(b\sqrt{\frac{1-cx}{cx+1}}\left(2c^3x^3+2c^2x^2+cx+1\right)-3a\right)-2b^2\left(6c\right.}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^4,x]
```

```
[Out] (-9*a^2 - 2*b^2*(1 + 6*c^2*x^2) + 6*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
+ 2*c^2*x^2 + 2*c^3*x^3) + 6*b*(-3*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
 + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x] - 9*b^2*ArcSech[c*x]^2)/(27*x^3)
```

fricas [A] time = 0.52, size = 181, normalized size = 1.48

$$-\frac{12\,b^2c^2x^2+9\,b^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2+9\,a^2+2\,b^2+6\left(3\,ab-\left(2\,b^2c^3x^3+b^2cx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right.}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(12*b^2*c^2*x^2 + 9*b^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x))^2 + 9*a^2 + 2*b^2 + 6*(3*a*b - (2*b^2*c^3*x^3 + b^2*c*x)*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) -
6*(2*a*b*c^3*x^3 + a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsech}\,(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2/x^4, x)
```

maple [A] time = 0.44, size = 192, normalized size = 1.57

$$c^{3}\left(-\frac{a^{2}}{3c^{3}x^{3}}+b^{2}\left(-\frac{\operatorname{arcsech}(cx)^{2}}{3c^{3}x^{3}}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{9}+\frac{2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{9c^{2}x^{2}}-\frac{4}{9cx}-\frac{2}{27c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^2/x^4,x)
```

```
[Out] c^3*(-1/3*a^2/c^3/x^3+b^2*(-1/3/c^3/x^3*arcsech(c*x)^2+4/9*arcsech(c*x)*(-(
c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+2/9*arcsech(c*x)/c^2/x^2*(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2)-4/9/c/x-2/27/c^3/x^3)+2*a*b*(-1/3*arcsech(c*x)
/c^3/x^3+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2)*(2*c^2*x^2+1)
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{9}\,ab\left(\frac{c^{4}\left(\frac{1}{c^{2}x^{2}}-1\right)^{\frac{3}{2}}+3\,c^{4}\sqrt{\frac{1}{c^{2}x^{2}}-1}}{c}-\frac{3\,\operatorname{arsech}(cx)}{x^{3}}\right)+b^{2}\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^{2}}{x^{4}}\,dx-\frac{a^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*
arcsech(c*x)/x^3) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) +
 1/(c*x))^2/x^4, x) - 1/3*a^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^2/x^4,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{asech}(cx)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))**2/x**4, x)
```

3.41 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=151

$$\frac{3}{16}abc^4\mathrm{sech}^{-1}(cx)+\frac{3bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\mathrm{sech}^{-1}(cx)\right)}{16x^2}-\frac{\left(a+b\mathrm{sech}^{-1}(cx)\right)^2}{4x^4}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\mathrm{sech}^{-1}\right.}{8x^4}$$

```
[Out] -1/32*b^2/x^4-3/32*b^2*c^2/x^2+3/16*a*b*c^4*arcsech(c*x)+3/32*b^2*c^4*arcse
ch(c*x)^2-1/4*(a+b*arcsech(c*x))^2/x^4+1/8*b*(c*x+1)*(a+b*arcsech(c*x))*((-
c*x+1)/(c*x+1))^(1/2)/x^4+3/16*b*c^2*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1)/(
c*x+1))^(1/2)/x^2
```

Rubi [A] time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6285, 5447, 3310}

$$\frac{3bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\mathrm{sech}^{-1}(cx)\right)}{16x^2}+\frac{3}{16}abc^4\mathrm{sech}^{-1}(cx)-\frac{\left(a+b\mathrm{sech}^{-1}(cx)\right)^2}{4x^4}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\mathrm{sech}^{-1}\right.}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^2/x^5,x]
```

```
[Out] -b^2/(32*x^4) - (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcSech[c*x])/16 + (3*b^2
*c^4*ArcSech[c*x]^2)/32 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Arc
Sech[c*x]))/(8*x^4) + (3*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*A
rcSech[c*x]))/(16*x^2) - (a + b*ArcSech[c*x])^2/(4*x^4)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5}\,dx = -\left(c^4\,\text{Subst}\left(\int (a+bx)^2\cosh^3(x)\sinh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right)\text{Subst}\left(\int (a+bx)\cosh^4(x)\,dx, x, \text{sech}^{-1}(cx)\right)$$
$$= -\frac{b^2}{32x^4} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{8x^4} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{8}\left(3bc^4\right)\text{Subst}$$
$$= -\frac{b^2}{32x^4} - \frac{3b^2c^2}{32x^2} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{8x^4} + \frac{3bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sec}\right.}{16x^2}$$
$$= -\frac{b^2}{32x^4} - \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\text{sech}^{-1}(cx) + \frac{3}{32}b^2c^4\text{sech}^{-1}(cx)^2 + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{s}\right.}{8x^4}$$

Mathematica [A] time = 0.28, size = 268, normalized size = 1.77

$$\frac{-8a^2 - 6abc^4x^4\log(x) + 6abc^4x^4\log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1\right) + 6abc^3x^3\sqrt{\frac{1-cx}{cx+1}} + 6abc^2x^2\sqrt{\frac{1-cx}{cx+1}} + 2b\text{sech}^{-1}(cx)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 - b^2 - 3*b^2*c^2*x^2 + 4*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*a*b*c*x
*Sqrt[(1 - c*x)/(1 + c*x)] + 6*a*b*c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] + 6*a*
b*c^3*x^3*Sqrt[(1 - c*x)/(1 + c*x)] + 2*b*(-8*a + b*Sqrt[(1 - c*x)/(1 + c*x
)]*(2 + 2*c*x + 3*c^2*x^2 + 3*c^3*x^3))*ArcSech[c*x] + b^2*(-8 + 3*c^4*x^4)
*ArcSech[c*x]^2 - 6*a*b*c^4*x^4*Log[x] + 6*a*b*c^4*x^4*Log[1 + Sqrt[(1 - c*
x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(32*x^4)
```

fricas [A] time = 0.62, size = 204, normalized size = 1.35

$$-\frac{3\,b^2c^2x^2 - \left(3\,b^2c^4x^4 - 8\,b^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 + 8\,a^2 + b^2 - 2\left(3\,abc^4x^4 - 8\,ab + \left(3\,b^2c^3x^3 + 2\,b^2cx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right.}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/32*(3*b^2*c^2*x^2 - (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) + 1)/(c*x))^2 + 8*a^2 + b^2 - 2*(3*a*b*c^4*x^4 - 8*a*b + (3*b^2
*c^3*x^3 + 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsech}\,(cx) + a)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2/x^5, x)
```

maple [A] time = 0.47, size = 264, normalized size = 1.75

$$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{4c^4x^4}+\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{8c^3x^3}+\frac{3\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{16cx}+\frac{3\operatorname{arcsech}}{32}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^2/x^5,x)
```

```
[Out] c^4*(-1/4*a^2/c^4/x^4+b^2*(-1/4/c^4/x^4*arcsech(c*x)^2+1/8*arcsech(c*x)/c^3
/x^3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+3/16*arcsech(c*x)/c/x*(-(c*x-
1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+3/32*arcsech(c*x)^2-1/32/c^4/x^4-3/32/c^2
/x^2)+2*a*b*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/x^3*((
c*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*c^2*x^2*(-c^2*
x^2+1)^(1/2)+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{32}ab\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right)-3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)-\frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-5\,c^6x\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}-\frac{16\operatorname{arsech}(cx}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1/
(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt(1
/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1)
 + 1))/c - 16*arcsech(c*x)/x^4) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(
1/(c*x) - 1) + 1/(c*x))^2/x^5, x) - 1/4*a^2/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^2/x^5,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^2/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{asech}(cx))^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*asech(c*x))**2/x**5, x)
```

3.42 $\int x^3 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=223

$$\frac{b^2\log\left(e^{2\mathrm{sech}^{-1}(cx)}+1\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^4}-\frac{b^2x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4c^2}-\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{b\left(a+b\right.}{}$$

```
[Out] -1/4*b^2*x^2*(a+b*arcsech(c*x))/c^2-1/2*b*(a+b*arcsech(c*x))^2/c^4+1/4*x^4*
(a+b*arcsech(c*x))^3+b^2*(a+b*arcsech(c*x))*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))^2)/c^4+1/2*b^3*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))^2)/c^4+1/4*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/2*b*(c*x+1)*(a
+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/4*b*x^2*(c*x+1)*(a+b*arcs
ech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] time = 0.24, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6285, 5451, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391}

$$\frac{b^3\mathrm{PolyLog}\left(2,-e^{2\mathrm{sech}^{-1}(cx)}\right)}{2c^4}-\frac{b^2x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4c^2}+\frac{b^2\log\left(e^{2\mathrm{sech}^{-1}(cx)}+1\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^4}-\frac{bx^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*c^4) - (b^2*x^2*(a + b*ArcSech
[c*x]))/(4*c^2) - (b*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*x^2*Sqrt[(1 - c*x)/
(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*c^2) + (x^4*(a + b*ArcSech[
c*x])^3)/4 + (b^2*(a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/c^4 + (
b^3*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*c^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=-\frac{\text{Subst}\left(\int(a+bx)^3\text{sech}^4(x)\tanh(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{c^4}$$

$$=\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{(3b)\,\text{Subst}\left(\int(a+bx)^2\text{sech}^4(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{4c^4}$$

$$=-\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2}-\frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2}+\frac{1}{4}x^4\left(a+bsec\right.$$

$$=-\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2}-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+c}{}$$

$$=\frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4}-\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+}{}$$

$$=\frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4}-\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+}{}$$

$$=\frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4}-\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+}{}$$

$$=\frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4}-\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+}{}$$

Mathematica [A] time = 2.15, size = 337, normalized size = 1.51

$$\frac{1}{4}\left(a^3x^4+a^2b\left(3x^4\text{sech}^{-1}(cx)-\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2x^2+2\right)}{c^4}\right)+\frac{ab^2\left(3c^4x^4\text{sech}^{-1}(cx)^2-c^2x^2-2\sqrt{\frac{1-cx}{cx+1}}\left(c^3x^3+c^2\right.\right.}{c^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (a^3*x^4 + b^3*x^4*ArcSech[c*x]^3 + a^2*b*(-((Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(2 + c^2*x^2))/c^4) + 3*x^4*ArcSech[c*x]) + (a*b^2*(-(c^2*x^2) - 2*S
qrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3)*ArcSech[c*x] + 3*c
^4*x^4*ArcSech[c*x]^2 + 4*Log[1/(c*x)]))/c^4 - (b^3*(-(Sqrt[(1 - c*x)/(1 +
c*x)]*(1 + c*x)) + (-2 + 2*Sqrt[(1 - c*x)/(1 + c*x)] + 2*c*x*Sqrt[(1 - c*x)
/(1 + c*x)] + c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] + c^3*x^3*Sqrt[(1 - c*x)/(1
 + c*x)])*ArcSech[c*x]^2 + ArcSech[c*x]*(c^2*x^2 - 4*Log[1 + E^(-2*ArcSech[
c*x])]) + 2*PolyLog[2, -E^(-2*ArcSech[c*x])]))/c^4)/4
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\,\text{arsech}(cx)^3+3\,ab^2x^3\,\text{arsech}(cx)^2+3\,a^2bx^3\,\text{arsech}(cx)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*x^3*
arcsech(c*x) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\,\text{arsech}(cx)+a)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3*x^3, x)
```

maple [B] time = 0.96, size = 546, normalized size = 2.45

$$\frac{x^4a^3}{4}+\frac{b^3\mathrm{arcsech}\,(cx)^3\,x^4}{4}-\frac{b^3\mathrm{arcsech}\,(cx)^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,x^3}{4c}-\frac{b^3\mathrm{arcsech}\,(cx)^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,x}{2c^3}-\frac{b^3\mathrm{arcsech}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))^3,x)
```

```
[Out] 1/4*x^4*a^3+1/4*b^3*arcsech(c*x)^3*x^4-1/4/c*b^3*arcsech(c*x)^2*(-(c*x-1)/c
/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3-1/2/c^3*b^3*arcsech(c*x)^2*(-(c*x-1)/c/x)
^(1/2)*((c*x+1)/c/x)^(1/2)*x-1/4/c^2*b^3*arcsech(c*x)*x^2+1/4/c^3*b^3*(-(c*
x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x-1/2/c^4*b^3*arcsech(c*x)^2-1/4/c^4*b^
3+1/c^4*b^3*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+1
/2*b^3*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^4-1/c^4*a*b
^2*arcsech(c*x)+3/4*a*b^2*arcsech(c*x)^2*x^4-1/2/c*a*b^2*arcsech(c*x)*(-(c*
x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3-1/c^3*a*b^2*arcsech(c*x)*(-(c*x-1)/
c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x-1/4/c^2*x^2*a*b^2+1/c^4*a*b^2*ln(1+(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/4*a^2*b*x^4*arcsech(c*x)-1/4/c*a^2*b
*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3-1/2/c^3*a^2*b*(-(c*x-1)/c/x)^
(1/2)*x*((c*x+1)/c/x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a^3x^4+\frac{1}{4}\left(3\,x^4\,\mathrm{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)a^2b+\int b^3x^3\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^3+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*x^4 + 1/4*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a^2*b + integrate(b^3*x^3*log(sqrt(1/(c*x)
+ 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3 + 3*a*b^2*x^3*log(sqrt(1/(c*x) + 1)*sqr
t(1/(c*x) - 1) + 1/(c*x))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(1/(c*x)))^3,x)
```

```
[Out] int(x^3*(a + b*acosh(1/(c*x)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\,\mathrm{asech}\,(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))**3, x)
```

3.43 $\int x^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=242

$$\frac{ib^2\mathrm{Li}_2\left(-ie^{\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^3}-\frac{ib^2\mathrm{Li}_2\left(ie^{\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^3}-\frac{b^2x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^2}-\frac{b\tan^{-1}\left(e\right.}{}$$

```
[Out] -b^2*x*(a+b*arcsech(c*x))/c^2+1/3*x^3*(a+b*arcsech(c*x))^3-b*(a+b*arcsech(c
*x))^2*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c^3+b^3*arctan((c*x+1
)*((-c*x+1)/(c*x+1))^(1/2)/c/x)/c^3+I*b^2*(a+b*arcsech(c*x))*polylog(2,-I*(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-I*b^2*(a+b*arcsech(c*x))*polyl
og(2,I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-I*b^3*polylog(3,-I*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3+I*b^3*polylog(3,I*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/2*b*x*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x
+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] time = 0.20, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6285, 5451, 4186, 3770, 4180, 2531, 2282, 6589}

$$\frac{ib^2\mathrm{PolyLog}\left(2,-ie^{\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^3}-\frac{ib^2\mathrm{PolyLog}\left(2,ie^{\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{c^3}-\frac{ib^3\mathrm{PolyLog}\left(3,-\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] -((b^2*x*(a + b*ArcSech[c*x]))/c^2) - (b*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c
*x)*(a + b*ArcSech[c*x])^2)/(2*c^2) + (x^3*(a + b*ArcSech[c*x])^3)/3 - (b*(
a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x]])/c^3 + (b^3*ArcTan[(Sqrt[(1 -
c*x)/(1 + c*x)]*(1 + c*x))/(c*x)])/c^3 + (I*b^2*(a + b*ArcSech[c*x])*PolyLo
g[2, (-I)*E^ArcSech[c*x]])/c^3 - (I*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E
^ArcSech[c*x]])/c^3 - (I*b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c^3 + (I*b^3*
PolyLog[3, I*E^ArcSech[c*x]])/c^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \left(a + b\text{sech}^{-1}(cx)\right)^3 dx = -\frac{\text{Subst}\left(\int (a+bx)^3 \text{sech}^3(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3}$$

$$= \frac{1}{3}x^3\left(a + b\text{sech}^{-1}(cx)\right)^3 - \frac{b\,\text{Subst}\left(\int (a+bx)^2 \text{sech}^3(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3}$$

$$= -\frac{b^2 x\left(a + b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a + b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a + b\text{sech}\right.$$

$$= -\frac{b^2 x\left(a + b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a + b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a + b\text{sech}\right.$$

$$= -\frac{b^2 x\left(a + b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a + b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a + b\text{sech}\right.$$

$$= -\frac{b^2 x\left(a + b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a + b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a + b\text{sech}\right.$$

$$= -\frac{b^2 x\left(a + b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a + b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a + b\text{sech}\right.$$

Mathematica [A] time = 1.11, size = 440, normalized size = 1.82

$$\frac{2a^3c^3x^3 + 6a^2bc^3x^3\text{sech}^{-1}(cx) - 3a^2bcx\sqrt{\frac{1-cx}{cx+1}}(cx+1) + 3ia^2b\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 2icx\right) - 6ab^2\left(-c^3x^3\text{sech}^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (2*a^3*c^3*x^3 - 3*a^2*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 6*a^2*b*
c^3*x^3*ArcSech[c*x] + (3*I)*a^2*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c
*x)]*(1 + c*x)] - 6*a*b^2*(c*x + c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Ar
cSech[c*x] - c^3*x^3*ArcSech[c*x]^2 - I*ArcSech[c*x]*Log[1 - I/E^ArcSech[c*
x]] + I*ArcSech[c*x]*Log[1 + I/E^ArcSech[c*x]] - I*PolyLog[2, (-I)/E^ArcSec
h[c*x]] + I*PolyLog[2, I/E^ArcSech[c*x]]) - b^3*(6*c*x*ArcSech[c*x] + 3*c*x
*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x]^2 - 2*c^3*x^3*ArcSech[c*x
]^3 - (3*I)*((-4*I)*ArcTan[Tanh[ArcSech[c*x]/2]] + ArcSech[c*x]^2*Log[1 - I
/E^ArcSech[c*x]] - ArcSech[c*x]^2*Log[1 + I/E^ArcSech[c*x]] + 2*ArcSech[c*x
]*PolyLog[2, (-I)/E^ArcSech[c*x]] - 2*ArcSech[c*x]*PolyLog[2, I/E^ArcSech[c
*x]] + 2*PolyLog[3, (-I)/E^ArcSech[c*x]] - 2*PolyLog[3, I/E^ArcSech[c*x]]))
)/(6*c^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\,\text{arsech}(cx)^3 + 3\,ab^2x^2\,\text{arsech}(cx)^2 + 3\,a^2bx^2\,\text{arsech}(cx) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arcsech(c*x)^3 + 3*a*b^2*x^2*arcsech(c*x)^2 + 3*a^2*b*x^2*
arcsech(c*x) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsech}(cx) + a)^3 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3*x^2, x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arcsech}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))^3,x)
```

```
[Out] int(x^2*(a+b*arcsech(c*x))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^3 x^3 + \frac{1}{2} \left(2 x^3 \operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2 x^2} - 1}}{c^2 \left(\frac{1}{c^2 x^2} - 1\right) + c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)}{c^2}}{c} \right) a^2 b + \int b^3 x^2 \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)^3 + 3 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2
*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*a^2*b + integrate
(b^3*x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3 + 3*a*b^2*x^2
*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(1/(c*x)))^3,x)
```

```
[Out] int(x^2*(a + b*acosh(1/(c*x)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asech}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))**3, x)
```

3.44 $\int x \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=126

$$\frac{3b^2\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2}-\frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}-\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

```
[Out] -3/2*b*(a+b*arcsech(c*x))^2/c^2+1/2*x^2*(a+b*arcsech(c*x))^3+3*b^2*(a+b*arc
sech(c*x))*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^2+3/2*b^3*pol
ylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^2-3/2*b*(c*x+1)*(a+b*
arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6285, 5451, 4184, 3718, 2190, 2279, 2391}

$$\frac{3b^3\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)}{2c^2}+\frac{3b^2\log\left(e^{2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2}-\frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (-3*b*(a + b*ArcSech[c*x])^2)/(2*c^2) - (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(a + b*ArcSech[c*x])^2)/(2*c^2) + (x^2*(a + b*ArcSech[c*x])^3)/2 + (3
*b^2*(a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/c^2 + (3*b^3*PolyLog
[2, -E^(2*ArcSech[c*x])])/(2*c^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx = -\frac{\text{Subst}\left(\int (a+bx)^3\text{sech}^2(x)\tanh(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{c^2}$$

$$= \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\text{sech}^2(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{2c^2}$$

$$= -\frac{3b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{\left(3b^2\right)\text{Subst}}{}$$

$$= -\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\text{sec}\right.$$

$$= -\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\text{sec}\right.$$

$$= -\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\text{sec}\right.$$

$$= -\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\text{sec}\right.$$

Mathematica [A] time = 0.95, size = 219, normalized size = 1.74

$$\frac{a\left(a\left(ac^2x^2-3b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\right)+6b^2\log\left(\frac{1}{cx}\right)\right)-3b^2\text{sech}^{-1}(cx)^2\left(b\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\frac{1-cx}{cx+1}}-1\right)-ac^2x^2\right)+3b\text{se}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (-3*b^2*(-(a*c^2*x^2) + b*(-1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c
*x)/(1 + c*x)]))*ArcSech[c*x]^2 + b^3*c^2*x^2*ArcSech[c*x]^3 + 3*b*ArcSech[
c*x]*(a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*b^2*Log[1
 + E^(-2*ArcSech[c*x])]) + a*(a*(a*c^2*x^2 - 3*b*Sqrt[(1 - c*x)/(1 + c*x)]*
(1 + c*x)) + 6*b^2*Log[1/(c*x)]) - 3*b^3*PolyLog[2, -E^(-2*ArcSech[c*x])])/
(2*c^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\,\text{arsech}\,(cx)^3+3\,ab^2x\,\text{arsech}\,(cx)^2+3\,a^2bx\,\text{arsech}\,(cx)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arcsech(c*x)^3 + 3*a*b^2*x*arcsech(c*x)^2 + 3*a^2*b*x*arcsec
h(c*x) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsech}(cx) + a)^3 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3*x, x)
```

maple [B] time = 0.72, size = 343, normalized size = 2.72

$$\frac{x^2 a^3}{2} + \frac{x^2 b^3 \operatorname{arcsech}(cx)^3}{2} - \frac{3b^3 \operatorname{arcsech}(cx)^2 \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} x}{2c} - \frac{3b^3 \operatorname{arcsech}(cx)^2}{2c^2} + \frac{3b^3 \operatorname{arcsech}(cx) \ln\left(1 + \left(\frac{1}{cx} + \right.\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))^3,x)
```

```
[Out] 1/2*x^2*a^3+1/2*x^2*b^3*arcsech(c*x)^3-3/2/c*b^3*arcsech(c*x)^2*(-(c*x-1)/c
/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x-3/2/c^2*b^3*arcsech(c*x)^2+3/c^2*b^3*arcsec
h(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*b^3*polylog(2,-
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^2-3/c^2*a*b^2*arcsech(c*x)+3/
2*x^2*a*b^2*arcsech(c*x)^2-3/c*a*b^2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*
x+1)/c/x)^(1/2)*x+3/c^2*a*b^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
^2)+3/2*x^2*a^2*b*arcsech(c*x)-3/2/c*a^2*b*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/
x)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2} ab^2 x^2 \operatorname{arsech}(cx)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{2}\left(x^2 \operatorname{arsech}(cx) - \frac{x\sqrt{\frac{1}{c^2x^2} - 1}}{c}\right) a^2 b - 3\left(\frac{x\sqrt{\frac{1}{c^2x^2} - 1} \operatorname{arsech}(cx)}{c} + \frac{\log(x)}{c^2}\right) ab^2 + b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arcsech(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arcsech(c*x) - x*sqrt
(1/(c^2*x^2) - 1)/c)*a^2*b - 3*(x*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x)/c + lo
g(x)/c^2)*a*b^2 + b^3*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) +
 1/(c*x))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(1/(c*x)))^3,x)
```

```
[Out] int(x*(a + b*acosh(1/(c*x)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{asech}\left(cx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*asech(c*x))**3, x)
```

3.45 $\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=140

$$\frac{6ib^2\text{Li}_2\left(-ie^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c}-\frac{6ib^2\text{Li}_2\left(ie^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c}+x\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{6b\tan^{-1}}{}$$

```
[Out] x*(a+b*arcsech(c*x))^3-6*b*(a+b*arcsech(c*x))^2*arctan(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))/c+6*I*b^2*(a+b*arcsech(c*x))*polylog(2,-I*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2)))/c-6*I*b^2*(a+b*arcsech(c*x))*polylog(2,I*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c-6*I*b^3*polylog(3,-I*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2)))/c+6*I*b^3*polylog(3,I*(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2)))/c
```

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6279, 5451, 4180, 2531, 2282, 6589}

$$\frac{6ib^2\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c}-\frac{6ib^2\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c}-\frac{6ib^3\text{PolyLog}\left(\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^3,x]
```

```
[Out] x*(a + b*ArcSech[c*x])^3 - (6*b*(a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x
]])/c + ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, (-I)*E^ArcSech[c*x]])/c
- ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E^ArcSech[c*x]])/c - ((6*I)*
b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c + ((6*I)*b^3*PolyLog[3, I*E^ArcSech[
c*x]])/c
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
```

```
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx = -\frac{\text{Subst}\left(\int (a+bx)^3\text{sech}(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{6b\left(a+b\text{sech}^{-1}(cx)\right)^2\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{\left(6ib^2\right)\text{Subst}\left(\int\right.}{}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{6b\left(a+b\text{sech}^{-1}(cx)\right)^2\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{6ib^2\left(a+b\text{sech}\right.}{}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{6b\left(a+b\text{sech}^{-1}(cx)\right)^2\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{6ib^2\left(a+b\text{sech}\right.}{}$$

$$= x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{6b\left(a+b\text{sech}^{-1}(cx)\right)^2\tan^{-1}\left(e^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{6ib^2\left(a+b\text{sech}\right.}{}$$

Mathematica [B] time = 0.44, size = 282, normalized size = 2.01

$$a^3x - \frac{3a^2b\tan^{-1}\left(\frac{cx\sqrt{\frac{1-cx}{cx+1}}}{cx-1}\right)}{c} + 3a^2bx\text{sech}^{-1}(cx) + \frac{3iab^2\left(2\text{Li}_2\left(-ie^{-\text{sech}^{-1}(cx)}\right) - 2\text{Li}_2\left(ie^{-\text{sech}^{-1}(cx)}\right) + \text{sech}^{-1}(cx)\left(-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcSech[c*x] - (3*a^2*b*ArcTan[(c*x*Sqrt[(1 - c*x)/(1 + c
*x)])/(-1 + c*x)])/c + ((3*I)*a*b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x] +
2*Log[1 - I/E^ArcSech[c*x]] - 2*Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2, (
-I)/E^ArcSech[c*x]] - 2*PolyLog[2, I/E^ArcSech[c*x]]))/c + (b^3*(c*x*ArcSec
h[c*x]^3 - (3*I)*(-(ArcSech[c*x]^2*(Log[1 - I/E^ArcSech[c*x]] - Log[1 + I/E
^ArcSech[c*x]])) - 2*ArcSech[c*x]*(PolyLog[2, (-I)/E^ArcSech[c*x]] - PolyLo
g[2, I/E^ArcSech[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSech[c*x]] - PolyLog[3, I
/E^ArcSech[c*x]]))))/c
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\,\text{arsech}\,(cx)^3 + 3\,ab^2\,\text{arsech}\,(cx)^2 + 3\,a^2b\,\text{arsech}\,(cx) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x)
 + a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{arsech}\,(cx) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{arcsech}\,(cx))^3\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^3,x)
```

```
[Out] int((a+b*arcsech(c*x))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3x\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)^3 + a^3x + \frac{3\left(cx\,\mathrm{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)a^2b}{c} - \int -\frac{b^3\log(c)^3 - 3\,ab^2\log(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^3 + a^3*x + 3*(c*x*arcsech(c*x)
 - arctan(sqrt(1/(c^2*x^2) - 1)))*a^2*b/c - integrate(-(b^3*log(c)^3 - 3*a*
b^2*log(c)^2 - (b^3*c^2*x^2 - b^3)*log(x)^3 - (b^3*c^2*log(c)^3 - 3*a*b^2*c
^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2
 + (b^3*log(c) - a*b^2 - (b^3*c^2*(log(c) + 1) - a*b^2*c^2)*x^2 - (b^3*c^2*
x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^3*c^2*x^2 - b^3)*log(x
))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 + 3*(b^3*log(c) - a*b^2 - (b^3*c
^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 - (
b^3*c^2*x^2 - b^3)*log(x)^3 - (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2
 + 3*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(
b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2
)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) -
 (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 - (b^3*c^2*x^2 - b^3)*log(x)^2
 + (b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))
*x^2 - (b^3*c^2*x^2 - b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 - (b^3*c^2*log(
c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) + 2*(b^3*log(c) -
 a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*log(sqrt(c*x + 1)*sqrt(-
c*x + 1) + 1) + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*
b^2*c^2*log(c))*x^2)*log(x))/(c^2*x^2 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c
*x + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asech}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**3,x)
```

```
[Out] Integral((a + b*asech(c*x))**3, x)
```

3.46 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x}\,dx$

Optimal. Leaf size=114

$$\frac{3}{2}b^2\mathrm{Li}_3\left(-e^{2\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{3}{2}b\mathrm{Li}_2\left(-e^{2\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2+\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^4}{4b}-\log\left(e^{2\mathrm{sec}}\right.$$

```
[Out] 1/4*(a+b*arcsech(c*x))^4/b-(a+b*arcsech(c*x))^3*ln(1+(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)-3/2*b*(a+b*arcsech(c*x))^2*polylog(2,-(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*b^2*(a+b*arcsech(c*x))*polylog(3,-(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3/4*b^3*polylog(4,-(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2)
```

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6285, 3718, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}b^2\mathrm{PolyLog}\left(3,-e^{2\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{3}{2}b\mathrm{PolyLog}\left(2,-e^{2\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2-\frac{3}{4}b^3\mathrm{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^3/x,x]
```

```
[Out] (a + b*ArcSech[c*x])^4/(4*b) - (a + b*ArcSech[c*x])^3*Log[1 + E^(2*ArcSech[
c*x])] - (3*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(2*ArcSech[c*x])])/2 + (
3*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(2*ArcSech[c*x])])/2 - (3*b^3*Poly
Log[4, -E^(2*ArcSech[c*x])])/4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x}\,dx &= -\operatorname{Subst}\left(\int (a+bx)^3\tanh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - 2\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^3\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) + (3b)\operatorname{Subst}\left(\int (a\right.\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^3\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\operatorname{sech}^{-1}(\right.\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^3\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\operatorname{sech}^{-1}(\right.\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^3\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\operatorname{sech}^{-1}(\right.\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^3\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\operatorname{sech}^{-1}(\right.
\end{aligned}$$

Mathematica [A] time = 0.25, size = 182, normalized size = 1.60

$$\frac{1}{4}\left(4a^3\log(cx) - 6a^2b\operatorname{sech}^{-1}(cx)^2 - 12a^2b\operatorname{sech}^{-1}(cx)\log\left(e^{-2\operatorname{sech}^{-1}(cx)}+1\right) + 6b^2\operatorname{Li}_3\left(-e^{-2\operatorname{sech}^{-1}(cx)}\right)\left(a+b\operatorname{sech}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x,x]
```

```
[Out] (-6*a^2*b*ArcSech[c*x]^2 - 4*a*b^2*ArcSech[c*x]^3 - b^3*ArcSech[c*x]^4 - 12
*a^2*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - 12*a*b^2*ArcSech[c*x]^2*
Log[1 + E^(-2*ArcSech[c*x])] - 4*b^3*ArcSech[c*x]^3*Log[1 + E^(-2*ArcSech[c
```

```
*x])] + 4*a^3*Log[c*x] + 6*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(-2*ArcSe
ch[c*x])] + 6*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(-2*ArcSech[c*x])] + 3
*b^3*PolyLog[4, -E^(-2*ArcSech[c*x])])/4
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{arsech}\,(cx)^3+3\,ab^2\,\text{arsech}\,(cx)^2+3\,a^2b\,\text{arsech}\,(cx)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x
) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arsech}\,(cx)+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x, x)
```

maple [B] time = 0.15, size = 454, normalized size = 3.98

$$a^3\ln(cx)+\frac{b^3\text{arcsech}\,(cx)^4}{4}-b^3\text{arcsech}\,(cx)^3\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\right)-\frac{3b^3\text{arcsech}\,(cx)^2\,\text{polylog}\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^3/x,x)
```

```
[Out] a^3*ln(c*x)+1/4*b^3*arcsech(c*x)^4-b^3*arcsech(c*x)^3*ln(1+(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))^2)-3/2*b^3*arcsech(c*x)^2*polylog(2,-(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*b^3*arcsech(c*x)*polylog(3,-(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3/4*b^3*polylog(4,-(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))^2)+a*b^2*arcsech(c*x)^3-3*a*b^2*arcsech(c*x)^2*ln(1+(1/c/x
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3*a*b^2*arcsech(c*x)*polylog(2,-(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*a*b^2*polylog(3,-(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*a^2*b*arcsech(c*x)^2-3*a^2*b*arcsech(c*x)*
ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3/2*a^2*b*polylog(2,-(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\log(x)+\int\frac{b^3\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^3}{x}+\frac{3\,ab^2\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^2}{x}+\frac{3\,a^2b\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(b^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x
))^3/x + 3*a*b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x + 3
*a^2*b*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^3/x,x)
[Out] int((a + b*acosh(1/(c*x)))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asech}\,(cx))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**3/x,x)
[Out] Integral((a + b*asech(c*x))**3/x, x)
```

3.47 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^2}\,dx$

<u>**Optimal**</u>. Leaf size=102

$$-\frac{6b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}+\frac{3b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x}+\frac{6b^3\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{x}$$

```
[Out] -6*b^2*(a+b*arcsech(c*x))/x-(a+b*arcsech(c*x))^3/x+6*b^3*(c*x+1)*((-c*x+1)/
(c*x+1))^(1/2)/x+3*b*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/
x
```

<u>**Rubi**</u> **[A]** time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6285, 3296, 2637}

$$-\frac{6b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}+\frac{3b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x}+\frac{6b^3\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^3/x^2,x]
```

```
[Out] (6*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/x - (6*b^2*(a + b*ArcSech[c*x])
)/x + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/x -
(a + b*ArcSech[c*x])^3/x
```

<u>Rule 2637</u>

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

<u>Rule 3296</u>

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

<u>Rule 6285</u>

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

<u>Rubi steps</u>

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^2}\,dx = -\left(c\operatorname{Subst}\left(\int (a+bx)^3\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x} + (3bc)\operatorname{Subst}\left(\int (a+bx)^2\cosh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)$$
$$= \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x} - \left(6b^2c\right)\operatorname{Subst}\left(\int (a\right.$$
$$= -\frac{6b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x} - \frac{\left(a+b\operatorname{sech}^{-1}(c\right.}{x}$$
$$= \frac{6b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x} - \frac{6b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}$$

Mathematica [A] time = 0.36, size = 165, normalized size = 1.62

$$-\frac{a^3+3b\operatorname{sech}^{-1}(cx)\left(a^2-2ab\sqrt{\frac{1-cx}{cx+1}}(cx+1)+2b^2\right)-3a^2b\sqrt{\frac{1-cx}{cx+1}}(cx+1)-3b^2\operatorname{sech}^{-1}(cx)^2\left(b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x^2,x]
```

```
[Out] -((a^3 + 6*a*b^2 - 3*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 6*b^3*Sqrt
[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 3*b*(a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)/(
1 + c*x)]*(1 + c*x))*ArcSech[c*x] - 3*b^2*(-a + b*Sqrt[(1 - c*x)/(1 + c*x)]
*(1 + c*x))*ArcSech[c*x]^2 + b^3*ArcSech[c*x]^3)/x)
```

fricas [B] time = 0.54, size = 228, normalized size = 2.24

$$-\frac{b^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 - 3\left(a^2b+2b^3\right)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + a^3 + 6ab^2 - 3\left(b^3cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - ab^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^3 - 3*(a^2*b + 2*
b^3)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + a^3 + 6*a*b^2 - 3*(b^3*c*x*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) - a*b^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
 1)/(c*x))^2 - 3*(2*a*b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a^2*b - 2*b^
3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsech}(cx)+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^2, x)
```

maple [B] time = 0.17, size = 227, normalized size = 2.23

$$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{cx}+3\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}-\frac{6\operatorname{arcsech}(cx)}{cx}+6\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^3/x^2,x)
```

```
[Out] c*(-a^3/c/x+b^3*(-1/c/x*arcsech(c*x)^3+3*arcsech(c*x)^2*(-(c*x-1)/c/x)^(1/2
)*((c*x+1)/c/x)^(1/2)-6/c/x*arcsech(c*x)+6*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/
x)^(1/2))+3*a*b^2*(-1/c/x*arcsech(c*x)^2+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2
)*((c*x+1)/c/x)^(1/2)-2/c/x)+3*a^2*b*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1
/2)*((c*x+1)/c/x)^(1/2)))
```

maxima [A] time = 0.33, size = 144, normalized size = 1.41

$$-\frac{b^3\operatorname{arsech}(cx)^3}{x}+3\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\operatorname{arsech}(cx)}{x}\right)a^2b+6\left(c\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arsech}(cx)-\frac{1}{x}\right)ab^2+3\left(c\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arsec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arcsech(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*a^2*b
+ 6*(c*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2*x
^2) - 1)*arcsech(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) - 1) - 2*arcsech(c*x)/x)*b^3
 - 3*a*b^2*arcsech(c*x)^2/x - a^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^3/x^2,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{asech}(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**2, x)
```

3.48 $\int \frac{\left(a+b\,\mathbf{sech}^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{3b^2(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{4x^2}-\frac{1}{4}c^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^2}-\frac{(1-c}{}$$

```
[Out] -3/8*b^3*c^2*arcsech(c*x)-3/4*b^2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))/x^2-1
/4*c^2*(a+b*arcsech(c*x))^3-1/2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))^3/x^2+3
/8*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x^2+3/4*b*(c*x+1)*(a+b*arcsech(c*x)
)^2*((-c*x+1)/(c*x+1))^(1/2)/x^2
```

Rubi [A] time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6285, 5446, 3311, 32, 2635, 8}

$$-\frac{3b^2(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{4x^2}-\frac{1}{4}c^2\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^2}-\frac{(1-c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^3/x^3,x]
```

```
[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*x^2) - (3*b^3*c^2*ArcSech[c*
x])/8 - (3*b^2*(1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x]))/(4*x^2) + (3*b*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*x^2) - (c^2*(a
+ b*ArcSech[c*x])^3)/4 - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^3)/(2*x^
2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
```

```
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx = -\left(c^2\,\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$
$$= -\frac{(1-cx)(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{2x^2} + \frac{1}{2}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx, x, \text{se}\right.$$
$$= -\frac{3b^2(1-cx)(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{4x^2} - \frac{(1}{}$$
$$= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8x^2} - \frac{3b^2(1-cx)(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+\right.}{4x^2}$$
$$= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8x^2} - \frac{3}{8}b^3c^2\operatorname{sech}^{-1}(cx) - \frac{3b^2(1-cx)(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b}{}$$

Mathematica [A] time = 0.48, size = 245, normalized size = 1.50

$$\frac{-4a^3 - 3bc^2x^2\left(2a^2+b^2\right)\log(x) + 3bc^2x^2\left(2a^2+b^2\right)\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\frac{1-cx}{cx+1}}+1\right) + 3b\left(2a^2+b^2\right)\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x^3,x]
```

```
[Out] (-4*a^3 - 6*a*b^2 + 3*b*(2*a^2 + b^2)*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) -
 6*b*(2*a^2 + b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x]
 + 6*b^2*(b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + a*(-2 + c^2*x^2))*ArcSech
[c*x]^2 + 2*b^3*(-2 + c^2*x^2)*ArcSech[c*x]^3 - 3*b*(2*a^2 + b^2)*c^2*x^2*L
og[x] + 3*b*(2*a^2 + b^2)*c^2*x^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*S
qrt[(1 - c*x)/(1 + c*x)]])/(8*x^2)
```

fricas [A] time = 0.48, size = 271, normalized size = 1.66

$$\frac{2\left(b^3c^2x^2 - 2\,b^3\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 + 3\left(2\,a^2b+b^3\right)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 4\,a^3 - 6\,ab^2 + 6\left(ab^2c^2x^2 + b^3cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - \right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b^3*c^2*x^2 - 2*b^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x))^3 + 3*(2*a^2*b + b^3)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a^3 - 6*
a*b^2 + 6*(a*b^2*c^2*x^2 + b^3*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*a*b^2
```

```
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*(4*a*b^2*c*x*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)) + (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b^3)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsech}\,(cx) + a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^3, x)
```

maple [B] time = 0.17, size = 321, normalized size = 1.97

$$c^2\left(-\frac{a^3}{2c^2x^2} + b^3\left(-\frac{\text{arcsech}\,(cx)^3}{2c^2x^2} + \frac{3\text{arcsech}\,(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4cx} + \frac{\text{arcsech}\,(cx)^3}{4} - \frac{3\,\text{arcsech}\,(cx)}{4c^2x^2} + \frac{3\sqrt{-\frac{c}{}}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^3/x^3,x)
```

```
[Out] c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arcsech(c*x)^3+3/4*arcsech(c*x)^2/c
/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/4*arcsech(c*x)^3-3/4/c^2/x^2*
arcsech(c*x)+3/8/c/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+3/8*arcsech(c
*x))+3*a*b^2*(-1/2/c^2/x^2*arcsech(c*x)^2+1/2*arcsech(c*x)/c/x*(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/4*arcsech(c*x)^2-1/4/c^2/x^2)+3*a^2*b*(-1/2/
c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((c*x+1)/c/x)^(1/2)*(arct
anh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{8}a^2b\left(\frac{\frac{2\,c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)}{c} + \frac{4\,\text{arsech}\,(cx)}{x^2}\right) - \frac{a^3}{2\,x^2} + \int \frac{b^3\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1)
- c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1
) - 1))/c + 4*arcsech(c*x)/x^2) - 1/2*a^3/x^2 + integrate(b^3*log(sqrt(1/(c
*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3/x^3 + 3*a*b^2*log(sqrt(1/(c*x) + 1)
*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^3/x^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asech}(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**3, x)
```

3.49 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=213

$$-\frac{4b^2c^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x}-\frac{2b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{9x^3}+\frac{2bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{3x}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\,(\ldots}{3\ldots}$$

```
[Out] 2/27*b^3*((-c*x+1)/(c*x+1))^(3/2)*(c*x+1)^3/x^3-2/9*b^2*(a+b*arcsech(c*x))/
x^3-4/3*b^2*c^2*(a+b*arcsech(c*x))/x-1/3*(a+b*arcsech(c*x))^3/x^3+14/9*b^3*
c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x+1/3*b*(c*x+1)*(a+b*arcsech(c*x))^2*(
(-c*x+1)/(c*x+1))^(1/2)/x^3+2/3*b*c^2*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1
)/(c*x+1))^(1/2)/x
```

Rubi [A] time = 0.17, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6285, 5447, 3311, 3296, 2637, 2633}

$$-\frac{4b^2c^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x}-\frac{2b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{9x^3}+\frac{2bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{3x}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\,(\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^3/x^4,x]
```

```
[Out] (14*b^3*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(9*x) + (2*b^3*((1 - c*x)/
(1 + c*x))^(3/2)*(1 + c*x)^3)/(27*x^3) - (2*b^2*(a + b*ArcSech[c*x]))/(9*x^
3) - (4*b^2*c^2*(a + b*ArcSech[c*x]))/(3*x) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*
(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x^3) + (2*b*c^2*Sqrt[(1 - c*x)/(1 + c*
x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x) - (a + b*ArcSech[c*x])^3/(3*x^3
)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx = -\left(c^3\,\text{Subst}\left(\int (a+bx)^3\cosh^2(x)\sinh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3x^3} + \left(bc^3\right)\text{Subst}\left(\int (a+bx)^2\cosh^3(x)\,dx, x, \text{sech}^{-1}(cx)\right)$$

$$= -\frac{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3x^3}$$

$$= -\frac{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{3}$$

$$= \frac{2b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x} + \frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3} - \frac{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3} - \frac{4b^2c^2\left(a+\right.}{}$$

$$= \frac{14b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x} + \frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3} - \frac{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3} - \frac{4b^2c^2\left(a+\right.}{}$$

Mathematica [A] time = 0.42, size = 256, normalized size = 1.20

$$\frac{-9a^3 - 3b\text{sech}^{-1}(cx)\left(9a^2 - 6ab\sqrt{\frac{1-cx}{cx+1}}\left(2c^3x^3+2c^2x^2+cx+1\right)+2b^2\left(6c^2x^2+1\right)\right)+9a^2b\sqrt{\frac{1-cx}{cx+1}}\left(2c^3x^3+2c^2x^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 - 6*a*b^2*(1 + 6*c^2*x^2) + 9*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
c*x + 2*c^2*x^2 + 2*c^3*x^3) + 2*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2
0*c^2*x^2 + 20*c^3*x^3) - 3*b*(9*a^2 + 2*b^2*(1 + 6*c^2*x^2) - 6*a*b*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x] + 9*b^2
*(-3*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3))*Arc
Sech[c*x]^2 - 9*b^3*ArcSech[c*x]^3)/(27*x^3)
```

fricas [A] time = 0.72, size = 305, normalized size = 1.43

$$-\frac{36\,ab^2c^2x^2 + 9\,b^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 + 9\,a^3 + 6\,ab^2 + 9\left(3\,ab^2 - \left(2\,b^3c^3x^3 + b^3cx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(36*a*b^2*c^2*x^2 + 9*b^3*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1
)/(c*x))^3 + 9*a^3 + 6*a*b^2 + 9*(3*a*b^2 - (2*b^3*c^3*x^3 + b^3*c*x)*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x))^2 + 3*(12*b^3*c^2*x^2 + 9*a^2*b + 2*b^3 - 6*(2*a*b^2*c^3*x^3 + a*b^2*c*
x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
+ 1)/(c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 + (9*a^2*b + 2*b^3)*c*x)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)^3}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^4, x)
```

maple [B] time = 0.51, size = 387, normalized size = 1.82

$$c^3\left(-\frac{a^3}{3c^3x^3} + b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{3c^3x^3} + \frac{2\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{3} + \frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{3c^2x^2} - \frac{4\operatorname{arcse}}{3c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^3/x^4,x)
```

```
[Out] c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arcsech(c*x)^3+2/3*arcsech(c*x)^2*(
-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/3*arcsech(c*x)^2/c^2/x^2*(-(c*x-1
)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-4/3/c/x*arcsech(c*x)+40/27*(-(c*x-1)/c/x)^
(1/2)*((c*x+1)/c/x)^(1/2)-2/9*arcsech(c*x)/c^3/x^3+2/27/c^2/x^2*(-(c*x-1)/c
/x)^(1/2)*((c*x+1)/c/x)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*arcsech(c*x)^2+4/9*arc
sech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+2/9*arcsech(c*x)/c^2/x^2
*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-4/9/c/x-2/27/c^3/x^3)+3*a^2*b*(-1
/3*arcsech(c*x)/c^3/x^3+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2
)*(2*c^2*x^2+1)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2b\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c} - \frac{3\,\operatorname{arsech}(cx)}{x^3}\right) - \frac{a^3}{3\,x^3} + \int \frac{b^3\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)^3}{x^4} + \frac{3\,ab^2\log\left(\cdot\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c -
3*arcsech(c*x)/x^3) - 1/3*a^3/x^3 + integrate(b^3*log(sqrt(1/(c*x) + 1)*sqr
t(1/(c*x) - 1) + 1/(c*x))^3/x^4 + 3*a*b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x
) - 1) + 1/(c*x))^2/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^3/x^4,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asech}(cx))^3}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**4, x)
```

3.50 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^5}\,dx$

Optimal. Leaf size=242

$$-\frac{9b^2c^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{32x^2}-\frac{3b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{32x^4}+\frac{3}{32}c^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3+\frac{9bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}\right.}{32x^2}$$

```
[Out] 45/256*b^3*c^4*arcsech(c*x)-3/32*b^2*(a+b*arcsech(c*x))/x^4-9/32*b^2*c^2*(a
+b*arcsech(c*x))/x^2+3/32*c^4*(a+b*arcsech(c*x))^3-1/4*(a+b*arcsech(c*x))^3
/x^4+3/128*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x^4+45/256*b^3*c^2*(c*x+1)*
((-c*x+1)/(c*x+1))^(1/2)/x^2+3/16*b*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/
(c*x+1))^(1/2)/x^4+9/32*b*c^2*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1
))^(1/2)/x^2
```

Rubi [A] time = 0.20, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6285, 5447, 3311, 32, 2635, 8}

$$-\frac{9b^2c^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{32x^2}-\frac{3b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{32x^4}+\frac{9bc^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{32x^2}+\frac{3}{32}c^4\left(a+b\,\mathrm{sech}^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^3/x^5,x]
```

```
[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(128*x^4) + (45*b^3*c^2*Sqrt[(1
 - c*x)/(1 + c*x)]*(1 + c*x))/(256*x^2) + (45*b^3*c^4*ArcSech[c*x])/256 - (
3*b^2*(a + b*ArcSech[c*x]))/(32*x^4) - (9*b^2*c^2*(a + b*ArcSech[c*x]))/(32
*x^2) + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(1
6*x^4) + (9*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^
2)/(32*x^2) + (3*c^4*(a + b*ArcSech[c*x])^3)/32 - (a + b*ArcSech[c*x])^3/(4
*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^5}\,dx = -\left(c^4\,\text{Subst}\left(\int (a+bx)^3\cosh^3(x)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{4}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)^2\cosh^4(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)$$

$$= -\frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^4} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{16x^4} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4x^4}$$

$$= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{128x^4} - \frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^4} - \frac{9b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}\,(1}{}$$

$$= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{128x^4} + \frac{45b^3c^2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{256x^2} - \frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^4} - \frac{9b^2c^2\left(a+b\text{se}\right.}{32x^2}$$

$$= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{128x^4} + \frac{45b^3c^2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{256x^2} + \frac{45}{256}b^3c^4\operatorname{sech}^{-1}(cx) - \frac{3b^2\left(a+b\operatorname{sech}^{-1}\right.}{32x^4}$$

Mathematica [A] time = 0.79, size = 332, normalized size = 1.37

$$\frac{-9bc^4x^4\left(8a^2+5b^2\right)\log(x)+9bc^4x^4\left(8a^2+5b^2\right)\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\frac{1-cx}{cx+1}}+1\right)+3b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(8a^2\left(3c^2x^2+2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x^5,x]
```

```
[Out] (-8*a*(8*a^2 + 3*b^2) - 72*a*b^2*c^2*x^2 + 3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(8*a^2*(2 + 3*c^2*x^2) + b^2*(2 + 15*c^2*x^2)) - 24*b*(8*a^2 + b^2*
(1 + 3*c^2*x^2) - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + 3*c^2*x^2 +
3*c^3*x^3))*ArcSech[c*x] + 24*b^2*(b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x +
 3*c^2*x^2 + 3*c^3*x^3) + a*(-8 + 3*c^4*x^4))*ArcSech[c*x]^2 + 8*b^3*(-8 +
3*c^4*x^4)*ArcSech[c*x]^3 - 9*b*(8*a^2 + 5*b^2)*c^4*x^4*Log[x] + 9*b*(8*a^2
 + 5*b^2)*c^4*x^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1
 + c*x)]])/(256*x^4)
```

fricas [A] time = 0.60, size = 351, normalized size = 1.45

$$-\frac{72\,ab^2c^2x^2 - 8\left(3\,b^3c^4x^4 - 8\,b^3\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 + 64\,a^3 + 24\,ab^2 - 24\left(3\,ab^2c^4x^4 - 8\,ab^2 + \left(3\,b^3c^3x^3 + 2\,b^3c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/256*(72*a*b^2*c^2*x^2 - 8*(3*b^3*c^4*x^4 - 8*b^3)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x))^3 + 64*a^3 + 24*a*b^2 - 24*(3*a*b^2*c^4*x^4 -
 8*a*b^2 + (3*b^3*c^3*x^3 + 2*b^3*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 - 3*(3*(8*a^2*b + 5*b^3)*
c^4*x^4 - 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 + 16*(3*a*b^2*c^3*x^3 + 2*a*b^2
*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + 1)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 + 2*(8*a^2*b + b^3)*c*x)*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx) + a)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^5, x)
```

maple [B] time = 0.48, size = 485, normalized size = 2.00

$$c^4\left(-\frac{a^3}{4c^4x^4} + b^3\left(-\frac{\mathrm{arcsech}\,(cx)^3}{4c^4x^4} + \frac{3\mathrm{arcsech}\,(cx)^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{16c^3x^3} + \frac{9\mathrm{arcsech}\,(cx)^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{32cx} + \frac{3\mathrm{arcs}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^3/x^5,x)
```

```
[Out] c^4*(-1/4*a^3/c^4/x^4+b^3*(-1/4/c^4/x^4*arcsech(c*x)^3+3/16*arcsech(c*x)^2/
c^3/x^3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+9/32*arcsech(c*x)^2/c/x*(-
(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+3/32*arcsech(c*x)^3-3/32/c^4/x^4*arc
sech(c*x)+3/128/c^3/x^3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+45/256/c/x
*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+45/256*arcsech(c*x)-9/32/c^2/x^2*
arcsech(c*x))+3*a*b^2*(-1/4/c^4/x^4*arcsech(c*x)^2+1/8*arcsech(c*x)/c^3/x^3
*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+3/16*arcsech(c*x)/c/x*(-(c*x-1)/c
/x)^(1/2)*((c*x+1)/c/x)^(1/2)+3/32*arcsech(c*x)^2-1/32/c^4/x^4-3/32/c^2/x^2
)+3*a^2*b*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/x^3*((c*
x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*c^2*x^2*(-c^2*x^
2+1)^(1/2)+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{64}a^2b\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) - 3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) - \frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} - 5\,c^6x\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 - 2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c} - \frac{16\,\mathrm{arsech}\,(c}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(
1/(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt
```

```
(1/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) -
1) + 1))/c - 16*arcsech(c*x)/x^4) - 1/4*a^3/x^4 + integrate(b^3*log(sqrt(1/
(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3/x^5 + 3*a*b^2*log(sqrt(1/(c*x) +
1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))^3/x^5,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asech}\,(cx))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**5, x)
```

3.51 $\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{x}{a+b\,\mathrm{sech}^{-1}(cx)},x\right)$$

[Out] Unintegrable(x/(a+b*arcsech(c*x)),x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b*ArcSech[c*x]),x]

[Out] Defer[Int][x/(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

Mathematica [A] time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b*ArcSech[c*x]),x]

[Out] Integrate[x/(a + b*ArcSech[c*x]), x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{b\,\mathrm{arsech}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral(x/(b*arcsech(c*x) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\,\mathrm{arsech}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate(x/(b*arcsech(c*x) + a), x)

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\,\mathrm{arcsech}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsech(c*x)),x)
```

```
[Out] int(x/(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \operatorname{arsech}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsech(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x/(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{asech}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asech(c*x)),x)
```

```
[Out] Integral(x/(a + b*asech(c*x)), x)
```

3.52 $\int \frac{1}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{1}{a+b\,\mathrm{sech}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsech(c*x)),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])^(-1),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcSech[c*x])^(-1), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b\,\mathrm{arsech}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arcsech(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\mathrm{arsech}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arcsech(c*x) + a), x)
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arcsech}\left(cx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsech(c*x)),x)
```

```
[Out] int(1/(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\mathrm{arsech}\left(cx\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsech(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(1/(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{asech}\left(cx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asech(c*x)),x)
```

```
[Out] Integral(1/(a + b*asech(c*x)), x)
```

3.53 $\int \frac{1}{x\left(a+b\mathbf{sech}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsech(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcSech[c*x])),x]
[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcSech[c*x])),x]
[Out] Integrate[1/(x*(a + b*ArcSech[c*x])), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx\,\text{arsech}\,(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arcsech(c*x) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)*x), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\operatorname{arcsech}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsech(c*x)),x)
[Out] int(1/x/(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsech}\left(cx\right)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arcsech(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(1/(c*x)))),x)
[Out] int(1/(x*(a + b*acosh(1/(c*x)))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\operatorname{asech}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asech(c*x)),x)
[Out] Integral(1/(x*(a + b*asech(c*x))), x)
```

3.54 $\int \frac{1}{x^2\left(a+b\textbf{sech}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=46

$$\frac{c\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}$$

```
[Out] -c*cosh(a/b)*Shi(a/b+arcsech(c*x))/b+c*Chi(a/b+arcsech(c*x))*sinh(a/b)/b
```

Rubi [A] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6285, 3303, 3298, 3301}

$$\frac{c\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*ArcSech[c*x])),x]
```

```
[Out] (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/b - (c*Cosh[a/b]*SinhIntegra
l[a/b + ArcSech[c*x]])/b
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\ &= -\left(\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)+\left(c\sinh\left(\frac{a}{b}\right)\right)\text{Subs}\\ &= \frac{c\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.93

$$\frac{c\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])),x]
```

```
[Out] (c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
 + ArcSech[c*x]]))/b
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^2\,\text{arsech}\,(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*arcsech(c*x) + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)*x^2), x)
```

maple [A] time = 0.13, size = 54, normalized size = 1.17

$$c\left(-\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\frac{a}{b}+\text{arcsech}\,(cx)\right)}{2b}+\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsech}\,(cx)-\frac{a}{b}\right)}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsech(c*x)),x)
```

```
[Out] c*(-1/2/b*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/2/b*exp(-a/b)*Ei(1,-arcsech(c*x
)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(1/(c*x)))),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(1/(c*x)))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asech}(cx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asech(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*asech(c*x))), x)
```

3.55 $\int \frac{1}{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=63

$$\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{2b}$$

```
[Out] -1/2*c^2*cosh(2*a/b)*Shi(2*a/b+2*arcsech(c*x))/b+1/2*c^2*Chi(2*a/b+2*arcsec
h(c*x))*sinh(2*a/b)/b
```

Rubi [A] time = 0.14, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6285, 5448, 12, 3303, 3298, 3301}

$$\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*ArcSech[c*x])),x]
```

```
[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*Cos
h[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
```

```
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}\,dx = -\left(c^2\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$
$$= -\left(c^2\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$
$$= -\left(\frac{1}{2}c^2\operatorname{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$
$$= -\left(\frac{1}{2}\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)+\frac{1}{2}\left(c^2\sinh\left(\right.\right.$$
$$= \frac{c^2\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{sech}^{-1}(cx)\right)}{2b}$$

Mathematica [A] time = 0.08, size = 56, normalized size = 0.89

$$\frac{c^2\left(\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{sech}^{-1}(cx)\right)-\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{sech}^{-1}(cx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])),x]
[Out] (c^2*(CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*
SinhIntegral[(2*a)/b + 2*ArcSech[c*x]]))/(2*b)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{bx^3\operatorname{arsech}(cx)+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*arcsech(c*x) + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsech}(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)*x^3), x)
```

maple [A] time = 0.24, size = 60, normalized size = 0.95

$$c^2\left(-\frac{e^{\frac{2a}{b}}\operatorname{Ei}\left(1,\frac{2a}{b}+2\operatorname{arcsech}(cx)\right)}{4b}+\frac{e^{-\frac{2a}{b}}\operatorname{Ei}\left(1,-2\operatorname{arcsech}(cx)-\frac{2a}{b}\right)}{4b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arcsech(c*x)),x)
```

```
[Out] c^2*(-1/4/b*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))+1/4/b*exp(-2*a/b)*Ei(1,-2
*arcsech(c*x)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{arsech}\,(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*acosh(1/(c*x)))),x)
```

```
[Out] int(1/(x^3*(a + b*acosh(1/(c*x)))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(a+b\,\mathrm{asech}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*asech(c*x)),x)
```

```
[Out] Integral(1/(x**3*(a + b*asech(c*x))), x)
```

3.56 $\int \frac{1}{x^4\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=117

$$\frac{c^3\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b}+\frac{c^3\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{sech}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b}-\frac{c^3}{}$$

```
[Out] -1/4*c^3*cosh(a/b)*Shi(a/b+arcsech(c*x))/b-1/4*c^3*cosh(3*a/b)*Shi(3*a/b+3*
arcsech(c*x))/b+1/4*c^3*Chi(a/b+arcsech(c*x))*sinh(a/b)/b+1/4*c^3*Chi(3*a/b
+3*arcsech(c*x))*sinh(3*a/b)/b
```

Rubi [A] time = 0.24, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6285, 5448, 3303, 3298, 3301}

$$\frac{c^3\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b}+\frac{c^3\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{sech}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b}-\frac{c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*ArcSech[c*x])),x]
```

```
[Out] (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(4*b) + (c^3*CoshIntegral[
(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(4*b) - (c^3*Cosh[a/b]*SinhIntegra
l[a/b + ArcSech[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcSech[c*x]])/(4*b)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
```

```
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+b\mathrm{sech}^{-1}(cx)\right)}\,dx &= -\left(c^3\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \mathrm{sech}^{-1}(cx)\right)\right)\\
&= -\left(c^3\,\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \mathrm{sech}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{4}c^3\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \mathrm{sech}^{-1}(cx)\right)\right) - \frac{1}{4}c^3\,\mathrm{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \mathrm{sech}\right.\\
&= -\left(\frac{1}{4}\left(c^3\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \mathrm{sech}^{-1}(cx)\right)\right) - \frac{1}{4}\left(c^3\cosh\left(\frac{3a}{b}\right)\right)\mathrm{S}\\
&= \frac{c^3\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b} + \frac{c^3\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{sech}^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4b} - \frac{c^3\cosh\left(\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 91, normalized size = 0.78

$$-\frac{c^3\left(\sinh\left(\frac{a}{b}\right)\left(-\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)\right)-\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)\right)+\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)+c\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])),x]
```

```
[Out] -1/4*(c^3*(-(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b]) - CoshIntegral[3*(
a/b + ArcSech[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSech[c
*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/b
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{bx^4\,\mathrm{arsech}\,(cx)+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^4*arcsech(c*x) + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{arsech}\,(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)*x^4), x)
```

maple [A] time = 0.40, size = 110, normalized size = 0.94

$$c^3\left(-\frac{e^{\frac{3a}{b}}\,\mathrm{Ei}\left(1, \frac{3a}{b}+3\,\mathrm{arcsech}\,(cx)\right)}{8b} - \frac{e^{\frac{a}{b}}\,\mathrm{Ei}\left(1, \frac{a}{b}+\mathrm{arcsech}\,(cx)\right)}{8b} + \frac{e^{-\frac{a}{b}}\,\mathrm{Ei}\left(1, -\mathrm{arcsech}\,(cx)-\frac{a}{b}\right)}{8b} + \frac{e^{-\frac{3a}{b}}\,\mathrm{Ei}\left(1, -3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arcsech(c*x)),x)
```

```
[Out] c^3*(-1/8/b*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))-1/8/b*exp(a/b)*Ei(1,a/b+a
rcsech(c*x))+1/8/b*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)+1/8/b*exp(-3*a/b)*Ei(1
,-3*arcsech(c*x)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsech}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*acosh(1/(c*x)))),x)
```

```
[Out] int(1/(x^4*(a + b*acosh(1/(c*x)))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+b\operatorname{asech}(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*asech(c*x)),x)
```

```
[Out] Integral(1/(x**4*(a + b*asech(c*x))), x)
```

3.57 $\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsech(c*x))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(a + b*ArcSech[c*x])^2,x]
[Out] Defer[Int][x/(a + b*ArcSech[c*x])^2, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 17.76, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*ArcSech[c*x])^2,x]
[Out] Integrate[x/(a + b*ArcSech[c*x])^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b^2\,\text{arsech}\,(cx)^2+2\,ab\,\text{arsech}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,\text{arsech}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate(x/(b*arcsech(c*x) + a)^2, x)
```

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b\,\mathrm{arcsech}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsech(c*x))^2,x)
```

```
[Out] int(x/(a+b*arcsech(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^3 - x\right)\sqrt{cx+1}\,\sqrt{-cx+1}\,x + \left(c^2x^3 - x\right.}{\left(b^2c^2\log(c) - abc^2\right)x^2 - b^2\log(c) - \left(b^2\log(c) + b^2\log(x) - ab\right)\sqrt{cx+1}\,\sqrt{-cx+1} + ab - \left(b^2c^2x^2 - \sqrt{cx+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + (c^2*x^3 - x)*x)/((b^2*c^2
*log(c) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2*log(x) - a*b)*sqrt(
c*x + 1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)
*b^2 - b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^2*x^2 - b^2)*log
(x)) + integrate((2*(2*c^2*x^2 - 1)*(c*x + 1)*(c*x - 1)*x + (3*c^4*x^4 - 8*
c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + 2*(c^4*x^4 - 2*c^2*x^2 + 1)*x
)/((b^2*c^4*log(c) - a*b*c^4)*x^4 - (b^2*log(c) + b^2*log(x) - a*b)*(c*x +
1)*(c*x - 1) - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*
log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b + (b^2*c^2*x^2 - b^2)*log(x))*sqrt
(c*x + 1)*sqrt(-c*x + 1) - a*b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(
c*x - 1)*b^2 - 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + b^2)*lo
g(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*l
og(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int(x/(a + b*acosh(1/(c*x)))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b\,\mathrm{asech}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(x/(a + b*asech(c*x))**2, x)
```

3.58 $\int \frac{1}{\left(a+b\,\mathbf{sech}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsech(c*x))^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])^(-2),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 9.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])^(-2),x]
[Out] Integrate[(a + b*ArcSech[c*x])^(-2), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\,\text{arsech}\,(cx)^2+2\,ab\,\text{arsech}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arsech}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^(-2), x)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\,\mathrm{arcsech}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsech(c*x))^2,x)
```

```
[Out] int(1/(a+b*arcsech(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^2x^3 + \left(c^2x^3 - x\right)\sqrt{cx+1}\,\sqrt{-cx+1} - x}{\left(b^2c^2\log(c) - abc^2\right)x^2 - b^2\log(c) - \left(b^2\log(c) + b^2\log(x) - ab\right)\sqrt{cx+1}\,\sqrt{-cx+1} + ab - \left(b^2c^2x^2 - \sqrt{cx+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*log(c
) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2*log(x) - a*b)*sqrt(c*x +
1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2 -
 b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^2*x^2 - b^2)*log(x)) +
 integrate((c^4*x^4 - 2*c^2*x^2 + (3*c^2*x^2 - 1)*(c*x + 1)*(c*x - 1) + (2*
c^4*x^4 - 5*c^2*x^2 + 2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/((b^2*c^4*log(c)
 - a*b*c^4)*x^4 - (b^2*log(c) + b^2*log(x) - a*b)*(c*x + 1)*(c*x - 1) - 2*(
b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*log(c) - a*b*c^2)*
x^2 - b^2*log(c) + a*b + (b^2*c^2*x^2 - b^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*
x + 1) - a*b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(c*x - 1)*b^2 - 2*(
b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + b^2)*log(sqrt(c*x + 1)*sq
rt(-c*x + 1) + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int(1/(a + b*acosh(1/(c*x)))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\,\mathrm{asech}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asech(c*x))**2,x)
```

```
[Out] Integral((a + b*asech(c*x))**(-2), x)
```

3.59 $\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsech(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcSech[c*x])^2),x]
[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 5.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcSech[c*x])^2),x]
[Out] Integrate[1/(x*(a + b*ArcSech[c*x])^2), x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b^2x\,\mathrm{arsech}\,(cx)^2+2\,abx\,\mathrm{arsech}\,(cx)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*arcsech(c*x)^2 + 2*a*b*x*arcsech(c*x) + a^2*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{arsech}\,(cx)+a)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arcsech}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsech(c*x))^2,x)
```

```
[Out] int(1/x/(a+b*arcsech(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^2x^3+\left(c^2x^3-x\right)\sqrt{cx+1}\,\sqrt{-cx}}{\left(b^2c^2x^2-b^2\right)x\log(x)-\left(b^2x\log(x)+\left(b^2\log(c)-ab\right)x\right)\sqrt{cx+1}\,\sqrt{-cx+1}+\left(\left(b^2c^2\log(c)-abc^2\right)x^2-b^2\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x*log(x) - (b^2*x*log(x) + (b^2*log(c) - a*b)*x)*sqrt(c*x + 1)*sqrt(-
c*x + 1) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x + (sqrt(c*
x + 1)*sqrt(-c*x + 1)*b^2*x - (b^2*c^2*x^2 - b^2)*x)*log(sqrt(c*x + 1)*sqrt
(-c*x + 1) + 1)) + integrate(-(2*(c*x + 1)*(c*x - 1)*c^2*x^2 + (c^4*x^4 - 2
*c^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1))/((b^2*x*log(x) + (b^2*log(c) - a*b)
*x)*(c*x + 1)*(c*x - 1) - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x*log(x) + 2*
((b^2*c^2*x^2 - b^2)*x*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c
) + a*b)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b^2*c^4*log(c) - a*b*c^4)*x^4
- 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x - ((c*x + 1)*(c*x
- 1)*b^2*x + 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x - (b^2*c^
4*x^4 - 2*b^2*c^2*x^2 + b^2)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(1/(c*x)))^2),x)
```

```
[Out] int(1/(x*(a + b*acosh(1/(c*x)))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{asech}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*asech(c*x))**2), x)
```

3.60 $\int \frac{1}{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=86

$$-\frac{c\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{c\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{bx\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}$$

```
[Out] -c*Chi(a/b+arcsech(c*x))*cosh(a/b)/b^2+c*Shi(a/b+arcsech(c*x))*sinh(a/b)/b^
2+(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*arcsech(c*x))
```

Rubi [A] time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6285, 3297, 3303, 3298, 3301}

$$-\frac{c\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{c\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{bx\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*x*(a + b*ArcSech[c*x])) - (c*Cosh[
a/b]*CoshIntegral[a/b + ArcSech[c*x]])/b^2 + (c*Sinh[a/b]*SinhIntegral[a/b
+ ArcSech[c*x]])/b^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$
$$= \frac{\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b}$$
$$= \frac{\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b} + \frac{(c}{\;}$$
$$= \frac{\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2} + \frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\right.}{b^2}$$

Mathematica [A] time = 0.32, size = 82, normalized size = 0.95

$$\frac{-c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)+c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{x\left(a+b\text{sech}^{-1}(cx)\right)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])) - c*Cosh[
a/b]*CoshIntegral[a/b + ArcSech[c*x]] + c*Sinh[a/b]*SinhIntegral[a/b + ArcS
ech[c*x]])/b^2
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x^2\,\text{arsech}\,(cx)^2+2\,abx^2\,\text{arsech}\,(cx)+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.19, size = 164, normalized size = 1.91

$$c\left(\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx+1}{2bcx\,(a+b\,\text{arcsech}\,(cx))}+\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsech}\,(cx)-\frac{a}{b}\right)}{2b^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx-1}{2bcx\,(a+b\,\text{arcsech}\,(cx))}+\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\frac{a}{b}+\text{arcsec}\right.}{2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsech(c*x))^2,x)
```

```
[Out] c*(1/2/b*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+1)/c/x/(a+b*arcsech(
c*x))+1/2/b^2*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)+1/2*((-(c*x-1)/c/x)^(1/2)*(
(c*x+1)/c/x)^(1/2)*c*x-1)/b/c/x/(a+b*arcsech(c*x))+1/2/b^2*exp(a/b)*Ei(1,a/
b+arcsech(c*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^2x^3 + \left(c^2x^3 - x\right)\sqrt{cx+1}\,\sqrt{-cx}}{\left(b^2c^2x^2 - b^2\right)x^2\log(x) + \left(\left(b^2c^2\log(c) - abc^2\right)x^2 - b^2\log(c) + ab\right)x^2 - \left(b^2x^2\log(x) + \left(b^2\log(c) - ab\right)x^2\right)\sqrt{cx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x^2*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^2
- (b^2*x^2*log(x) + (b^2*log(c) - a*b)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) +
(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^2 - (b^2*c^2*x^2 - b^2)*x^2)*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(-(c^4*x^4 - 2*c^2*x^2 - (c^2*x^2 +
 1)*(c*x + 1)*(c*x - 1) - (c^2*x^2 - 2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(
(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^2*log(x) - (b^2*x^2*log(x) + (b^2*log
(c) - a*b)*x^2)*(c*x + 1)*(c*x - 1) + ((b^2*c^4*log(c) - a*b*c^4)*x^4 - 2*(
b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x^2 - 2*((b^2*c^2*x^2 - b
^2)*x^2*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^2)*s
qrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^2 + 2*(b^2*c^2*x^2
 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b
^2)*x^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(1/(c*x)))^2),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(1/(c*x)))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a + b\,\mathrm{asech}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*asech(c*x))**2), x)
```

3.61 $\int \frac{1}{x^3\left(a+b\,\mathbf{sech}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(2\mathrm{sech}^{-1}(cx)\right)}{2b\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}$$

```
[Out] -c^2*Chi(2*a/b+2*arcsech(c*x))*cosh(2*a/b)/b^2+c^2*Shi(2*a/b+2*arcsech(c*x)
)*sinh(2*a/b)/b^2+1/2*c^2*sinh(2*arcsech(c*x))/b/(a+b*arcsech(c*x))
```

Rubi [A] time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6285, 5448, 12, 3297, 3303, 3298, 3301}

$$-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(2\mathrm{sech}^{-1}(cx)\right)}{2b\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] -((c^2*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]])/b^2) + (c^2*Si
nh[2*ArcSech[c*x]])/(2*b*(a + b*ArcSech[c*x])) + (c^2*Sinh[(2*a)/b]*SinhInt
egral[(2*a)/b + 2*ArcSech[c*x]])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx &= -\left(c^2\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{(a+bx)^2}\,dx,x,\operatorname{sech}^{-1}(cx)\right)\right)\\
&= -\left(c^2\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)^2}\,dx,x,\operatorname{sech}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{2}c^2\operatorname{Subst}\left(\int \frac{\sinh(2x)}{(a+bx)^2}\,dx,x,\operatorname{sech}^{-1}(cx)\right)\right)\\
&= \frac{c^2\sinh\left(2\operatorname{sech}^{-1}(cx)\right)}{2b\left(a+b\operatorname{sech}^{-1}(cx)\right)} - \frac{c^2\operatorname{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{b}\\
&= \frac{c^2\sinh\left(2\operatorname{sech}^{-1}(cx)\right)}{2b\left(a+b\operatorname{sech}^{-1}(cx)\right)} - \frac{\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{b} + \cdots\\
&= -\frac{c^2\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c^2\sinh\left(2\operatorname{sech}^{-1}(cx)\right)}{2b\left(a+b\operatorname{sech}^{-1}(cx)\right)} + \frac{c^2\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.36, size = 92, normalized size = 1.08

$$
\frac{c^2\left(-\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)\right)+c^2\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(2\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)\right)+\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])) - c^2*C
osh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSech[c*x])] + c^2*Sinh[(2*a)/b]*SinhI
ntegral[2*(a/b + ArcSech[c*x])])/b^2
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{1}{b^2x^3\operatorname{arsech}(cx)^2+2\,abx^3\operatorname{arsech}(cx)+a^2x^3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^3*arcsech(c*x)^2 + 2*a*b*x^3*arcsech(c*x) + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arsech}(cx) + a\right)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^3), x)
```

maple [B] time = 0.26, size = 186, normalized size = 2.19

$$c^2 \left(\frac{2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx + c^2x^2 - 2}{4c^2x^2\left(a + b \operatorname{arcsech}(cx)\right) b} + \frac{e^{\frac{2a}{b}} \operatorname{Ei}\left(1, \frac{2a}{b} + 2 \operatorname{arcsech}(cx)\right)}{2b^2} - \frac{c^2x^2 - 2 - 2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx}{4b\,c^2x^2\left(a + b \operatorname{arcsech}(cx)\right)} + \frac{e^{-\frac{2a}{b}} \operatorname{E}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arcsech(c*x))^2,x)
```

```
[Out] c^2*(1/4*(2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+c^2*x^2-2)/c^2/x^2
/(a+b*arcsech(c*x))/b+1/2/b^2*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/4/b*(
c^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x)/c^2/x^2/(a+b*arcs
ech(c*x))+1/2/b^2*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^2x^3 + \left(c^2x^3 - x\right)\sqrt{cx+1}\,\sqrt{-}}{\left(b^2c^2x^2 - b^2\right)x^3 \log(x) + \left(\left(b^2c^2 \log(c) - abc^2\right)x^2 - b^2 \log(c) + ab\right)x^3 - \left(b^2x^3 \log(x) + \left(b^2 \log(c) - ab\right)x^3\right)\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^3
- (b^2*x^3*log(x) + (b^2*log(c) - a*b)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) +
(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^3 - (b^2*c^2*x^2 - b^2)*x^3)*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(-(2*c^4*x^4 - 4*c^2*x^2 - 2*(c*x +
 1)*(c*x - 1) + (c^4*x^4 - 4*c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 2)
/((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^3*log(x) + ((b^2*c^4*log(c) - a*b*c
^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x^3 - (b^2*x
^3*log(x) + (b^2*log(c) - a*b)*x^3)*(c*x + 1)*(c*x - 1) - 2*((b^2*c^2*x^2 -
 b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^3)
*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^3 + 2*(b^2*c^2*x
^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3 - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 +
 b^2)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*acosh(1/(c*x)))^2),x)
```

```
[Out] int(1/(x^3*(a + b*acosh(1/(c*x)))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{asech}(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*asech(c*x))**2), x)
```

3.62 $\int \frac{1}{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=190

$$-\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{4b^2}-\frac{3c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{sech}^{-1}(cx)\right)}{4b^2}+\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{4b^2}+$$

```
[Out] -1/4*c^3*Chi(a/b+arcsech(c*x))*cosh(a/b)/b^2-3/4*c^3*Chi(3*a/b+3*arcsech(c*
x))*cosh(3*a/b)/b^2+1/4*c^3*Shi(a/b+arcsech(c*x))*sinh(a/b)/b^2+3/4*c^3*Shi
(3*a/b+3*arcsech(c*x))*sinh(3*a/b)/b^2+1/4*c^3*sinh(3*arcsech(c*x))/b/(a+b*
arcsech(c*x))+1/4*c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*arcsech(c*x
))
```

Rubi [A] time = 0.29, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5448, 3297, 3303, 3298, 3301}

$$-\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{4b^2}-\frac{3c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{sech}^{-1}(cx)\right)}{4b^2}+\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{4b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*b*x*(a + b*ArcSech[c*x])) - (c
^3*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/(4*b^2) - (3*c^3*Cosh[(3*a)/
b]*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^2) + (c^3*Sinh[3*ArcSech[c*
x]])/(4*b*(a + b*ArcSech[c*x])) + (c^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech
[c*x]])/(4*b^2) + (3*c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x
]])/(4*b^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx = -\left(c^3\operatorname{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{(a+bx)^2}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$

$$= -\left(c^3\operatorname{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)^2}+\frac{\sinh(3x)}{4(a+bx)^2}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$

$$= -\left(\frac{1}{4}c^3\operatorname{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^2}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right) - \frac{1}{4}c^3\operatorname{Subst}\left(\int \frac{\sinh(3x)}{(a+bx)^2}\,dx, x, \operatorname{se}\right.$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\operatorname{sech}^{-1}(cx)\right)} + \frac{c^3\sinh\left(3\operatorname{sech}^{-1}(cx)\right)}{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)} - \frac{c^3\operatorname{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \operatorname{sech}^{-1}\right.}{4b}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\operatorname{sech}^{-1}(cx)\right)} + \frac{c^3\sinh\left(3\operatorname{sech}^{-1}(cx)\right)}{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)} - \frac{\left(c^3\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right.}{a+bx}\right.}{4b}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\operatorname{sech}^{-1}(cx)\right)} - \frac{c^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b^2} - \frac{3c^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\right.}{4b^2}$$

Mathematica [A] time = 0.54, size = 250, normalized size = 1.32

$$\frac{-c^3x^3\cosh\left(\frac{a}{b}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)-3c^3x^3\cosh\left(\frac{3a}{b}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{sech}^{-1}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] (4*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)] - c^3*x^
3*(a + b*ArcSech[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]] - 3*c^3*x
^3*(a + b*ArcSech[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSech[c*x])]
+ a*c^3*x^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + b*c^3*x^3*ArcSech[
c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + 3*a*c^3*x^3*Sinh[(3*a)/b]
*SinhIntegral[3*(a/b + ArcSech[c*x])] + 3*b*c^3*x^3*ArcSech[c*x]*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSech[c*x])])/(4*b^2*x^3*(a + b*ArcSech[c*x]))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{b^2x^4\operatorname{arsech}(cx)^2+2\,abx^4\operatorname{arsech}(cx)+a^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^4*arcsech(c*x)^2 + 2*a*b*x^4*arcsech(c*x) + a^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arsech}(cx) + a\right)^2 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^4), x)
```

maple [B] time = 0.44, size = 420, normalized size = 2.21

$$c^3 \left(-\frac{\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}\,c^3x^3 - 4\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx - 3c^2x^2 + 4}{8c^3x^3b\,(a + b \operatorname{arcsech}(cx))} + \frac{3\,e^{\frac{3a}{b}}\,\operatorname{Ei}\left(1, \frac{3a}{b} + 3\operatorname{arcsech}(cx)\right)}{8b^2} + \frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{}}{8bcx\,(a + b\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arcsech(c*x))^2,x)
```

```
[Out] c^3*(-1/8*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x
)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-3*c^2*x^2+4)/c^3/x^3/b/(a+b*arcsech(c*x))+3
/8/b^2*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))+1/8*((-(c*x-1)/c/x)^(1/2)*((c*
x+1)/c/x)^(1/2)*c*x-1)/b/c/x/(a+b*arcsech(c*x))+1/8/b^2*exp(a/b)*Ei(1,a/b+a
rcsech(c*x))+1/8/b*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+1)/c/x/(a+
b*arcsech(c*x))+1/8/b^2*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)-1/8/b*(((c*x+1)/c
/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)
^(1/2)*c*x+3*c^2*x^2-4)/c^3/x^3/(a+b*arcsech(c*x))+3/8/b^2*exp(-3*a/b)*Ei(1
,-3*arcsech(c*x)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^2x^3 + \left(c^2x^3 - x\right)\sqrt{cx+1}\,\sqrt{-}}{\left(b^2c^2x^2 - b^2\right)x^4\log(x) + \left(\left(b^2c^2\log(c) - abc^2\right)x^2 - b^2\log(c) + ab\right)x^4 - \left(b^2x^4\log(x) + \left(b^2\log(c) - ab\right)x^4\right)\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x^4*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^4
- (b^2*x^4*log(x) + (b^2*log(c) - a*b)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) +
(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^4 - (b^2*c^2*x^2 - b^2)*x^4)*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1)) - integrate((3*c^4*x^4 - 6*c^2*x^2 + (c^2*x^2
- 3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^4 - 7*c^2*x^2 + 6)*sqrt(c*x + 1)*sqrt(-
c*x + 1) + 3)/((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^4*log(x) + ((b^2*c^4*l
og(c) - a*b*c^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)
*x^4 - (b^2*x^4*log(x) + (b^2*log(c) - a*b)*x^4)*(c*x + 1)*(c*x - 1) - 2*((
b^2*c^2*x^2 - b^2)*x^4*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c
) + a*b)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^4 +
 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 - (b^2*c^4*x^4 - 2*
b^2*c^2*x^2 + b^2)*x^4)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*acosh(1/(c*x)))^2),x)
```

```
[Out] int(1/(x^4*(a + b*acosh(1/(c*x)))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + b \operatorname{asech}(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(1/(x**4*(a + b*asech(c*x))**2), x)
```

3.63 $\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{x}{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsech(c*x))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(a + b*ArcSech[c*x])^3,x]
[Out] Defer[Int][x/(a + b*ArcSech[c*x])^3, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Mathematica [A] time = 6.28, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*ArcSech[c*x])^3,x]
[Out] Integrate[x/(a + b*ArcSech[c*x])^3, x]
```

fricas [A] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{b^3\,\mathrm{arsech}\,(cx)^3+3\,ab^2\,\mathrm{arsech}\,(cx)^2+3\,a^2b\,\mathrm{arsech}\,(cx)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral(x/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c
*x) + a^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,\mathrm{arsech}\,(cx)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsech(c*x) + a)^3, x)
```

maple [A] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\,\mathrm{arcsech}\,(cx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsech(c*x))^3,x)
```

```
[Out] int(x/(a+b*arcsech(c*x))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*(2*b*c^4*x^5 - 3*b*c^2*x^3 + b*x)*x*log(x) + (4*(b*c^4*log(c) - a*
c^4)*x^5 - (b*c^2*(6*log(c) + 1) - 6*a*c^2)*x^3 + (b*(2*log(c) + 1) - 2*a)*
x)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (3*(b*c^6*x^7 - 5*b*c^4*x^5 + 6*b*
c^2*x^3 - 2*b*x)*x*log(x) + (3*(b*c^6*log(c) - a*c^6)*x^7 - (b*c^4*(15*log(
c) + 2) - 15*a*c^4)*x^5 + (b*c^2*(18*log(c) + 5) - 18*a*c^2)*x^3 - 3*(b*(2*
log(c) + 1) - 2*a)*x)*x)*(c*x + 1)*(c*x - 1) - 2*(b*c^6*x^7 - 3*b*c^4*x^5 +
 3*b*c^2*x^3 - b*x)*x*log(x) - ((5*b*c^6*x^7 - 17*b*c^4*x^5 + 18*b*c^2*x^3
 - 6*b*x)*x*log(x) + ((b*c^6*(5*log(c) + 1) - 5*a*c^6)*x^7 - (b*c^4*(17*log(
c) + 5) - 17*a*c^4)*x^5 + (b*c^2*(18*log(c) + 7) - 18*a*c^2)*x^3 - 3*(b*(2*
log(c) + 1) - 2*a)*x)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b*c^6*(2*log(c) +
 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*log(c) + 1) - 2*a*c^4)*x^5 + 3*(b*c^2*(2*l
og(c) + 1) - 2*a*c^2)*x^3 - (b*(2*log(c) + 1) - 2*a)*x)*x - (2*(2*b*c^4*x^5
 - 3*b*c^2*x^3 + b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*x + 3*(b*c^6*x^7 - 5
*b*c^4*x^5 + 6*b*c^2*x^3 - 2*b*x)*(c*x + 1)*(c*x - 1)*x - (5*b*c^6*x^7 - 17
*b*c^4*x^5 + 18*b*c^2*x^3 - 6*b*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x - 2*(b*c^
6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1
) + 1))/((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6 - b^4*lo
g(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 + 2*a*
b^3*log(c) - (b^4*log(c)^2 + b^4*log(x)^2 - 2*a*b^3*log(c) + a^2*b^2 + 2*(b
^4*log(c) - a*b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - a^2*b^2 + 3*(
b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*l
og(c) + a^2*b^2*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x)^2 + 2*(b^4*log(c) - a
*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*log(x))*(c*x + 1)*(c*x - 1) + 3*(b
^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^6*x^6 - 3*
b^4*c^4*x^4 + 3*b^4*c^2*x^2 - (c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*b^4 - b^4 -
3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1) - 3*(b^4*c^4*x^4 - 2*b^4*c^2*x^2
+ b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^
2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x)^2 - 3*(b^4*l
og(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 - 2*a*b
^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^
2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x)^2 + 2*((b^4*c^4*log(c)
- a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)
*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6
 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) - (b^4*log(c) + b^4*log(
x) - a*b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + a*b^3 + 3*(b^4*log(c) - a*b^
3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x))*(c*x + 1
)*(c*x - 1) + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2 - 3*((b^4*c^4*log(c) - a*b
^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2 + (b^
4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) + (b^
4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x))*log(sqrt(c*x + 1)*
sqrt(-c*x + 1) + 1) + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(
```

```
c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x
^2)*log(x)) + integrate(-1/2*(4*(6*c^4*x^4 - 6*c^2*x^2 + 1)*(c*x + 1)^2*(c*
x - 1)^2*x - (33*c^6*x^6 - 108*c^4*x^4 + 88*c^2*x^2 - 16)*(c*x + 1)^(3/2)*(
-c*x + 1)^(3/2)*x - 12*(c^8*x^8 - 7*c^6*x^6 + 14*c^4*x^4 - 10*c^2*x^2 + 2)*
(c*x + 1)*(c*x - 1)*x + (15*c^8*x^8 - 67*c^6*x^6 + 108*c^4*x^4 - 72*c^2*x^2
 + 16)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + 4*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4
- 4*c^2*x^2 + 1)*x)/((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) -
 a*b^2*c^6)*x^6 + (b^3*log(c) + b^3*log(x) - a*b^2)*(c*x + 1)^2*(c*x - 1)^2
 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + 4*(b^3*log(c) - a*b^2 - (b^3*c^2*lo
g(c) - a*b^2*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x +
 1)^(3/2) + b^3*log(c) - 6*((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) -
 a*b^2 - 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^4*x^4 - 2*b^3*c^2*x^2
+ b^3)*log(x))*(c*x + 1)*(c*x - 1) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)
*x^2 - 4*((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)
*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^6*x
^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x +
 1) - (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 + (c*x + 1)^2*(c*x - 1)^
2*b^3 - 4*b^3*c^2*x^2 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3
/2) - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1) + b^3 - 4*(
b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x
+ 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6
+ 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*log(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*acosh(1/(c*x)))^3,x)
[Out] int(x/(a + b*acosh(1/(c*x)))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{asech}\,(cx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asech(c*x))**3,x)
[Out] Integral(x/(a + b*asech(c*x))**3, x)
```

3.64 $\int \frac{1}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{1}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsech(c*x))^3,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])^(-3),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])^(-3), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Mathematica [A] time = 3.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])^(-3),x]
```

```
[Out] Integrate[(a + b*ArcSech[c*x])^(-3), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b^3\,\mathrm{arsech}\,(cx)^3+3\,ab^2\,\mathrm{arsech}\,(cx)^2+3\,a^2b\,\mathrm{arsech}\,(cx)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c
*x) + a^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{arsech}\,(cx)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^(-3), x)
```

maple [A] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\,\mathrm{arcsech}\,(cx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsech(c*x))^3,x)
```

```
[Out] int(1/(a+b*arcsech(c*x))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^6*(log(c) + 1) - a*c^6)*x^7 - 3*(b*c^4*(log(c) + 1) - a*c^4)*x^5
- (3*(b*c^4*log(c) - a*c^4)*x^5 - (b*c^2*(4*log(c) + 1) - 4*a*c^2)*x^3 + (b
*(log(c) + 1) - a)*x + (3*b*c^4*x^5 - 4*b*c^2*x^3 + b*x)*log(x))*(c*x + 1)^
(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(log(c) + 1) - a*c^2)*x^3 - (2*(b*c^6*log
(c) - a*c^6)*x^7 - 2*(b*c^4*(5*log(c) + 1) - 5*a*c^4)*x^5 + (b*c^2*(11*log(
c) + 5) - 11*a*c^2)*x^3 - 3*(b*(log(c) + 1) - a)*x + (2*b*c^6*x^7 - 10*b*c^
4*x^5 + 11*b*c^2*x^3 - 3*b*x)*log(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(3*log(
c) + 1) - 3*a*c^6)*x^7 - 5*(b*c^4*(2*log(c) + 1) - 2*a*c^4)*x^5 + (b*c^2*(1
0*log(c) + 7) - 10*a*c^2)*x^3 - 3*(b*(log(c) + 1) - a)*x + (3*b*c^6*x^7 - 1
0*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) -
(b*(log(c) + 1) - a)*x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - (3*b*c^4*
x^5 - 4*b*c^2*x^3 + b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (2*b*c^6*x^7 -
10*b*c^4*x^5 + 11*b*c^2*x^3 - 3*b*x)*(c*x + 1)*(c*x - 1) + (3*b*c^6*x^7 - 1
0*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - b*x)*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3
 - b*x)*log(x))/((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6
- b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^
4 + 2*a*b^3*log(c) - (b^4*log(c)^2 + b^4*log(x)^2 - 2*a*b^3*log(c) + a^2*b^
2 + 2*(b^4*log(c) - a*b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - a^2*b
^2 + 3*(b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b
^3*c^2*log(c) + a^2*b^2*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x)^2 + 2*(b^4*lo
g(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*log(x))*(c*x + 1)*(c*x - 1
) + 3*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^6*
x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - (c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*b^4
- b^4 - 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1) - 3*(b^4*c^4*x^4 - 2*b^4*
c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x +
1) + 1)^2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x)^2 -
3*(b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4
 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^
2*b^2*c^2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x)^2 + 2*((b^4*c^4
*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c
^2)*x^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*((b^4*c^6*log(c) - a*b^3*
c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) - (b^4*log(c) +
b^4*log(x) - a*b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + a*b^3 + 3*(b^4*log(c
) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x))*
(c*x + 1)*(c*x - 1) + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2 - 3*((b^4*c^4*log(
c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x
^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x))*sqrt(c*x + 1)*sqrt(-c*x +
1) + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x))*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1) + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*
```

```
c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^
3*c^2)*x^2)*log(x)) + integrate(-1/2*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4 + (15
*c^4*x^4 - 12*c^2*x^2 + 1)*(c*x + 1)^2*(c*x - 1)^2 - (18*c^6*x^6 - 57*c^4*x
^4 + 40*c^2*x^2 - 4)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 4*c^2*x^2 - 3*(2*c^
8*x^8 - 13*c^6*x^6 + 25*c^4*x^4 - 16*c^2*x^2 + 2)*(c*x + 1)*(c*x - 1) + (6*
c^8*x^8 - 25*c^6*x^6 + 39*c^4*x^4 - 24*c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x
 + 1) + 1)/((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c^
6)*x^6 + (b^3*log(c) + b^3*log(x) - a*b^2)*(c*x + 1)^2*(c*x - 1)^2 + 6*(b^3
*c^4*log(c) - a*b^2*c^4)*x^4 + 4*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*
b^2*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)
 + b^3*log(c) - 6*((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 -
2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*lo
g(x))*(c*x + 1)*(c*x - 1) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 - 4*
((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 - b^
3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^6*x^6 - 3*b^
3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^
3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 + (c*x + 1)^2*(c*x - 1)^2*b^3 - 4
*b^3*c^2*x^2 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 6*(
b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1) + b^3 - 4*(b^3*c^6*x
^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c
^4*x^4 - 4*b^3*c^2*x^2 + b^3)*log(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(1/(c*x)))^3,x)
[Out] int(1/(a + b*acosh(1/(c*x)))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asech}\,(cx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asech(c*x))**3,x)
[Out] Integral((a + b*asech(c*x))**(-3), x)
```

3.65 $\int \frac{1}{x\left(a+b\,\mathbf{sech}^{-1}(cx)\right)^3}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x\left(a+b\,\text{sech}^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsech(c*x))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcSech[c*x])^3),x]
[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])^3), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx$$

Mathematica [A] time = 2.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcSech[c*x])^3),x]
[Out] Integrate[1/(x*(a + b*ArcSech[c*x])^3), x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x\,\text{arsech}\,(cx)^3+3\,ab^2x\,\text{arsech}\,(cx)^2+3\,a^2bx\,\text{arsech}\,(cx)+a^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*x*arcsech(c*x)^3 + 3*a*b^2*x*arcsech(c*x)^2 + 3*a^2*b*x*arc
sech(c*x) + a^3*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

[Out] integrate(1/((b*arcsech(c*x) + a)^3*x), x)

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arcsech}\left(cx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*arcsech(c*x))^3,x)

[Out] int(1/x/(a+b*arcsech(c*x))^3,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")

```
[Out] -1/2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - (2*(b*c^4*log(c) - a*c^4)*x^5
 - (b*c^2*(2*log(c) + 1) - 2*a*c^2)*x^3 + b*x + 2*(b*c^4*x^5 - b*c^2*x^3)*l
og(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - ((b*c^6*log(c) - a*c^6)*x^7 - (b*
c^4*(5*log(c) + 2) - 5*a*c^4)*x^5 + (b*c^2*(4*log(c) + 5) - 4*a*c^2)*x^3 -
3*b*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 4*b*c^2*x^3)*log(x))*(c*x + 1)*(c*x - 1)
 + ((b*c^6*(log(c) + 1) - a*c^6)*x^7 - (b*c^4*(3*log(c) + 5) - 3*a*c^4)*x^5
 + (b*c^2*(2*log(c) + 7) - 2*a*c^2)*x^3 - 3*b*x + (b*c^6*x^7 - 3*b*c^4*x^5
+ 2*b*c^2*x^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - b*x + (2*(b*c^4*x^5 -
 b*c^2*x^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (b*c^6*x^7 - 5*b*c^4*x^5 + 4
*b*c^2*x^3)*(c*x + 1)*(c*x - 1) - (b*c^6*x^7 - 3*b*c^4*x^5 + 2*b*c^2*x^3)*s
qrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1))/((b^4*x
*log(x)^2 + 2*(b^4*log(c) - a*b^3)*x*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c
) + a^2*b^2)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (b^4*c^6*x^6 - 3*b^4*c^4
*x^4 + 3*b^4*c^2*x^2 - b^4)*x*log(x)^2 + 3*((b^4*c^2*x^2 - b^4)*x*log(x)^2
- 2*(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x*log(x) - (b^4
*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(
c) + a^2*b^2*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) + ((c*x + 1)^(3/2)*(-c*x + 1)
^(3/2)*b^4*x + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x + 3*(b^4*c^4*x^4
 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x - (b^4*c^6*x^6 - 3*b
^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^
2 - 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^
4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x*log(x) + 3*(
(b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x*log(x)^2 + 2*((b^4*c^4*log(c) - a*b^3
*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x*log(
x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*
x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) +
 a^2*b^2*c^2)*x^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b^4*c^6*log(c)^2 - 2
*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 -
 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*
c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x - 2*((b^4*x*log(x)
+ (b^4*log(c) - a*b^3)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*((b^4*c^2*x^
2 - b^4)*x*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)
*x)*(c*x + 1)*(c*x - 1) - (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^
4)*x*log(x) + 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x*log(x) + ((b^4*c^4*l
og(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2
)*x^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b^4*c^6*log(c) - a*b^3*c^6)*x^6
- 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(
c) - a*b^3*c^2)*x^2)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(
-1/2*(4*(2*c^4*x^4 - c^2*x^2)*(c*x + 1)^2*(c*x - 1)^2 - (7*c^6*x^6 - 22*c^4
*x^4 + 12*c^2*x^2)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 2*(c^8*x^8 - 5*c^6*x^
```

```
6 + 10*c^4*x^4 - 6*c^2*x^2)*(c*x + 1)*(c*x - 1) + (c^8*x^8 - 3*c^6*x^6 + 6*
c^4*x^4 - 4*c^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1))/((b^3*x*log(x) + (b^3*lo
g(c) - a*b^2)*x)*(c*x + 1)^2*(c*x - 1)^2 - 4*((b^3*c^2*x^2 - b^3)*x*log(x)
- (b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x)*(c*x + 1)^(3/2
)*(-c*x + 1)^(3/2) - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x*log(x) + ((b^
3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3
*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x*log(x) - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4
+ 3*b^3*c^2*x^2 - b^3)*x*log(x) + ((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^
3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((b^3*c^8*log(c) - a*b^2*c^
8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4
)*x^4 + b^3*log(c) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x - ((c*x
+ 1)^2*(c*x - 1)^2*b^3*x - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)
^(3/2)*x - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x - 4*
(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x
 + 1)*x + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^
3)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(1/(c*x)))^3),x)
```

```
[Out] int(1/(x*(a + b*acosh(1/(c*x)))^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{asech}\,(cx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(1/(x*(a + b*asech(c*x))**3), x)
```

3.66 $\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx$

Optimal. Leaf size=114

$$\frac{c\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}+\frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{2bx\left(a+b\text{sech}^{-1}(cx\right.}$$

```
[Out] 1/2/b^2/x/(a+b*arcsech(c*x))-1/2*c*cosh(a/b)*Shi(a/b+arcsech(c*x))/b^3+1/2*
c*Chi(a/b+arcsech(c*x))*sinh(a/b)/b^3+1/2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/
b/x/(a+b*arcsech(c*x))^2
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6285, 3297, 3303, 3298, 3301}

$$\frac{c\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}+\frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{2bx\left(a+b\text{sech}^{-1}(cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(2*b*x*(a + b*ArcSech[c*x])^2) + 1/(2
*b^2*x*(a + b*ArcSech[c*x])) + (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b
])/(2*b^3) - (c*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(2*b^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
```

```
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=-\left(c\,\text{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^3}\,dx,x,\text{sech}^{-1}(cx)\right)\right)$$

$$=\frac{\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{2bx\left(a+b\text{sech}^{-1}(cx)\right)^2}-\frac{c\,\text{Subst}\left(\int \frac{\cosh(x)}{(a+bx)^2}\,dx,x,\text{sech}^{-1}(cx)\right)}{2b}$$

$$=\frac{\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{2bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{c\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\text{sec}\right.}{2b^2}$$

$$=\frac{\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{2bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)}-\frac{\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\text{sinh}}{a\text{-}}\right.}{2b^2}$$

$$=\frac{\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{2bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\text{sin}}{2b^3}$$

Mathematica [A] time = 0.31, size = 103, normalized size = 0.90

$$\frac{\frac{b^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}+c\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)+\frac{b}{ax+bx\text{sech}^{-1}(cx)}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] ((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])^2) + b/(
a*x + b*x*ArcSech[c*x]) + c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - C
osh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]))/(2*b^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x^2\,\text{arsech}\,(cx)^3+3\,ab^2x^2\,\text{arsech}\,(cx)^2+3\,a^2bx^2\,\text{arsech}\,(cx)+a^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^2*arcsech(c*x)^3 + 3*a*b^2*x^2*arcsech(c*x)^2 + 3*a^2*b*x
^2*arcsech(c*x) + a^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arsech}\,(cx)+a)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^2), x)
```

maple [B] time = 0.17, size = 244, normalized size = 2.14

$$c\left(-\frac{\left(\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx-1\right)\left(b\,\mathrm{arcsech}\,(cx)+a-b\right)}{4cx\,b^{2}\left(\mathrm{arcsech}\,(cx)^{2}\,b^{2}+2\,\mathrm{arcsech}\,(cx)\,ab+a^{2}\right)}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\frac{a}{b}+\mathrm{arcsech}\,(cx)\right)}{4b^{3}}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx+1}{4bcx\,(a+b\,\mathrm{arcsech}\,(cx))^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsech(c*x))^3,x)
```

```
[Out] c*(-1/4*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1)*(b*arcsech(c*x)+a-
b)/c/x/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-1/4/b^3*exp(a/b)*Ei(
1,a/b+arcsech(c*x))+1/4/b*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+1)/
c/x/(a+b*arcsech(c*x))^2+1/4/b^2*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*
c*x+1)/c/x/(a+b*arcsech(c*x))+1/4/b^3*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*c^6*(log(c) - 1) - a*c^6)*x^7 - 3*(b*c^4*(log(c) - 1) - a*c^4)*x^5
 - (b*c^2*x^3 - (b*c^4*log(c) - a*c^4)*x^5 + (b*(log(c) - 1) - a)*x - (b*c^
4*x^5 - b*x)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(log(c) -
1) - a*c^2)*x^3 - (2*b*c^4*x^5 + (b*c^2*(3*log(c) - 5) - 3*a*c^2)*x^3 - 3*(
b*(log(c) - 1) - a)*x + 3*(b*c^2*x^3 - b*x)*log(x))*(c*x + 1)*(c*x - 1) + (
(b*c^6*(log(c) - 1) - a*c^6)*x^7 - (b*c^4*(4*log(c) - 5) - 4*a*c^4)*x^5 + (
b*c^2*(6*log(c) - 7) - 6*a*c^2)*x^3 - 3*(b*(log(c) - 1) - a)*x + (b*c^6*x^7
 - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1)
- (b*(log(c) - 1) - a)*x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 + (b*c^4*
x^5 - b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 3*(b*c^2*x^3 - b*x)*(c*x + 1)
*(c*x - 1) + (b*c^6*x^7 - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*sqrt(c*x + 1)*
sqrt(-c*x + 1) - b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b*c^6*x^7 -
3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*
b^4*c^2*x^2 - b^4)*x^2*log(x)^2 - (b^4*x^2*log(x)^2 + 2*(b^4*log(c) - a*b^3
)*x^2*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2)*x^2)*(c*x + 1)^(3/
2)*(-c*x + 1)^(3/2) + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(
c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x
^2)*x^2*log(x) - 3*((b^4*c^2*x^2 - b^4)*x^2*log(x)^2 - 2*(b^4*log(c) - a*b^
3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^2*log(x) - (b^4*log(c)^2 - 2*a*b^3*
log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^
2)*x^2)*(c*x + 1)*(c*x - 1) + ((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a^2
*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a
^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*c^2*log(c)^2 - 2*a*b^3*
c^2*log(c) + a^2*b^2*c^2)*x^2)*x^2 - ((c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*b^4*
x^2 + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^2 + 3*(b^4*c^4*x^4 - 2*b^
4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 - (b^4*c^6*x^6 - 3*b^4*c^
4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 -
 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^2*log(x)^2 + 2*((b^4*c^4*log(c) -
 a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*
x^2*log(x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b
^2*c^4)*x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*
log(c) + a^2*b^2*c^2)*x^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 2*((b^4*x^2*
log(x) + (b^4*log(c) - a*b^3)*x^2)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (b^4*
c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^2*log(x) + 3*((b^4*c^2*x^2
 - b^4)*x^2*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2
)*x^2)*(c*x + 1)*(c*x - 1) - ((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4
```

```
*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c
^2)*x^2)*x^2 + 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^2*log(x) + ((b^4*c^
4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*
c^2)*x^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x +
1) + 1)) + integrate(-1/2*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4 + (3*c^4*x^4 + 1
)*(c*x + 1)^2*(c*x - 1)^2 + (3*c^4*x^4 - 4*c^2*x^2 + 4)*(c*x + 1)^(3/2)*(-c
*x + 1)^(3/2) - 4*c^2*x^2 - 3*(c^6*x^6 + c^4*x^4 - 4*c^2*x^2 + 2)*(c*x + 1)
*(c*x - 1) - (c^6*x^6 - 9*c^4*x^4 + 12*c^2*x^2 - 4)*sqrt(c*x + 1)*sqrt(-c*x
 + 1) + 1)/((b^3*x^2*log(x) + (b^3*log(c) - a*b^2)*x^2)*(c*x + 1)^2*(c*x -
1)^2 - 4*((b^3*c^2*x^2 - b^3)*x^2*log(x) - (b^3*log(c) - a*b^2 - (b^3*c^2*l
og(c) - a*b^2*c^2)*x^2)*x^2)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (b^3*c^8*x^
8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^2*log(x) - 6*((b
^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x^2*log(x) + ((b^3*c^4*log(c) - a*b^2*c^4
)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^2)*(c*x
+ 1)*(c*x - 1) + ((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*
b^2*c^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 4*
(b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^2 - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3
*b^3*c^2*x^2 - b^3)*x^2*log(x) + ((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^3
*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*b
^2*c^2)*x^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1)^2*(c*x - 1)^2*b
^3*x^2 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*x^2 - 6*(b^
3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^2 - 4*(b^3*c^6*x^6 -
 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 + (b
^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^2)*log(
sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(1/(c*x)))^3),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(1/(c*x)))^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\,\mathrm{asech}\,(cx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(1/(x**2*(a + b*asech(c*x))**3), x)
```

3.67 $\int \frac{1}{x^3(a+b\,\mathrm{sech}^{-1}(cx))^3}\,dx$

Optimal. Leaf size=112

$$\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^3}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^3}+\frac{c^2\cosh\left(2\mathrm{sech}^{-1}(cx)\right)}{2b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}+\frac{c^2\sinh\left(2\mathrm{se}\right.}{4b\left(a+b\,\mathrm{sec}\right.}$$

```
[Out] 1/2*c^2*cosh(2*arcsech(c*x))/b^2/(a+b*arcsech(c*x))-c^2*cosh(2*a/b)*Shi(2*a
/b+2*arcsech(c*x))/b^3+c^2*Chi(2*a/b+2*arcsech(c*x))*sinh(2*a/b)/b^3+1/4*c^
2*sinh(2*arcsech(c*x))/b/(a+b*arcsech(c*x))^2
```

Rubi [A] time = 0.21, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6285, 5448, 12, 3297, 3303, 3298, 3301}

$$\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^3}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{sech}^{-1}(cx)\right)}{b^3}+\frac{c^2\cosh\left(2\mathrm{sech}^{-1}(cx)\right)}{2b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}+\frac{c^2\sinh\left(2\mathrm{se}\right.}{4b\left(a+b\,\mathrm{sec}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] (c^2*Cosh[2*ArcSech[c*x]])/(2*b^2*(a + b*ArcSech[c*x])) + (c^2*CoshIntegral
[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/b^3 + (c^2*Sinh[2*ArcSech[c*x]])/
(4*b*(a + b*ArcSech[c*x])^2) - (c^2*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*
ArcSech[c*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = -\left(c^2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$

$$= -\left(c^2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$

$$= -\left(\frac{1}{2}c^2\,\text{Subst}\left(\int \frac{\sinh(2x)}{(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$

$$= \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{c^2\,\text{Subst}\left(\int \frac{\cosh(2x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)}{2b}$$

$$= \frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{c^2\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \text{sec}\right.}{b^2}$$

$$= \frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh}{a}\right.}{b^2}$$

$$= \frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^3} + \frac{c^2\sinh\left(2\text{sech}\right.}{4b\left(a+b\text{sech}\right.}$$

Mathematica [A] time = 0.43, size = 122, normalized size = 1.09

$$\frac{\frac{b^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2} + 2c^2\left(\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right) - \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)\right) + \frac{b\left(2-c^2x^2\right.}{x^2\left(a+b\text{sech}^{-}\right.}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] ((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])^2) + (
b*(2 - c^2*x^2))/(x^2*(a + b*ArcSech[c*x])) + 2*c^2*(CoshIntegral[2*(a/b +
ArcSech[c*x])]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSech[
c*x])]))/(2*b^3)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x^3\,\text{arsech}\,(cx)^3+3\,ab^2x^3\,\text{arsech}\,(cx)^2+3\,a^2bx^3\,\text{arsech}\,(cx)+a^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*x
^3*arcsech(c*x) + a^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\,\text{arsech}\,(cx)+a)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^3), x)
```

maple [B] time = 0.29, size = 277, normalized size = 2.47

$$c^2\left(-\frac{\left(2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx+c^2x^2-2\right)(2b\,\text{arcsech}\,(cx)+2a-b)}{8c^2x^2b^2\left(\text{arcsech}\,(cx)^2\,b^2+2\,\text{arcsech}\,(cx)\,ab+a^2\right)}-\frac{e^{\frac{2a}{b}}\,\text{Ei}\left(1,\frac{2a}{b}+2\,\text{arcsech}\,(cx)\right)}{2b^3}-\frac{c^2x^2-2-2\cdots}{8b\,c^2x^2\,(a+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arcsech(c*x))^3,x)
```

```
[Out] c^2*(-1/8*(2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+c^2*x^2-2)*(2*b*a
rcsech(c*x)+2*a-b)/c^2/x^2/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-
1/2/b^3*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/8/b*(c^2*x^2-2-2*(-(c*x-1)/
c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x)/c^2/x^2/(a+b*arcsech(c*x))^2-1/4/b^2*(c
^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x)/c^2/x^2/(a+b*arcse
ch(c*x))+1/2/b^3*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*c^6*(2*log(c) - 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*log(c) - 1) - 2*a*
c^4)*x^5 + ((b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - (b*(2*log(c) - 1) - 2*a)
*x + 2*(b*c^2*x^3 - b*x)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^
2*(2*log(c) - 1) - 2*a*c^2)*x^3 - ((b*c^6*log(c) - a*c^6)*x^7 - (b*c^4*(5*l
og(c) - 2) - 5*a*c^4)*x^5 + 5*(b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - 3*(b*(
2*log(c) - 1) - 2*a)*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^3 - 6*b*x)*l
og(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 - (b*c^4
*(11*log(c) - 5) - 11*a*c^4)*x^5 + 7*(b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 -
 3*(b*(2*log(c) - 1) - 2*a)*x + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b*c^2*x^3
- 6*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b*(2*log(c) - 1) - 2*a)*x
- (2*b*c^6*x^7 - 6*b*c^4*x^5 + 6*b*c^2*x^3 + 2*(b*c^2*x^3 - b*x)*(c*x + 1)^
(3/2)*(-c*x + 1)^(3/2) - (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^3 - 6*b*x)*(
c*x + 1)*(c*x - 1) + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b*c^2*x^3 - 6*b*x)*sq
rt(c*x + 1)*sqrt(-c*x + 1) - 2*b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) +
```

```
 2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*
b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3*log(x)^2 + 2*((b^4*c^6*log(c) - a*b^
3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b
^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3*log(x) - (b^4*x^3*log(x)^2 + 2*(b^4*log
(c) - a*b^3)*x^3*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2)*x^3)*(c
*x + 1)^(3/2)*(-c*x + 1)^(3/2) + ((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) +
a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c)
+ a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*c^2*log(c)^2 - 2*a*b
^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^3 - 3*((b^4*c^2*x^2 - b^4)*x^3*log(x)^2
 - 2*(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3*log(x) - (
b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*l
og(c) + a^2*b^2*c^2)*x^2)*x^3)*(c*x + 1)*(c*x - 1) - ((c*x + 1)^(3/2)*(-c*x
 + 1)^(3/2)*b^4*x^3 + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^3 + 3*(b^
4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3 - (b^4*c^
6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3)*log(sqrt(c*x + 1)*sqrt(-c
*x + 1) + 1)^2 - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^3*log(x)^2 + 2*((
b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) -
a*b^3*c^2)*x^2)*x^3*log(x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^
4*log(c) + a^2*b^2*c^4)*x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^
2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1
) - 2*((b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3*log(x) - (b^
4*x^3*log(x) + (b^4*log(c) - a*b^3)*x^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) +
 ((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b
^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3 - 3*((b^4*c^2*x
^2 - b^4)*x^3*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x
^2)*x^3)*(c*x + 1)*(c*x - 1) - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^3*l
og(x) + ((b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2
*log(c) - a*b^3*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x +
 1)*sqrt(-c*x + 1) + 1)) + integrate(-1/2*(4*c^8*x^8 - 16*c^6*x^6 + 24*c^4*
x^4 + 4*(c*x + 1)^2*(c*x - 1)^2 + (3*c^6*x^6 - 16*c^2*x^2 + 16)*(c*x + 1)^(
3/2)*(-c*x + 1)^(3/2) - 16*c^2*x^2 - 24*(c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)
*(c*x - 1) + (3*c^8*x^8 - 19*c^6*x^6 + 48*c^4*x^4 - 48*c^2*x^2 + 16)*sqrt(c
*x + 1)*sqrt(-c*x + 1) + 4)/((b^3*x^3*log(x) + (b^3*log(c) - a*b^2)*x^3)*(c
*x + 1)^2*(c*x - 1)^2 + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^
3*c^2*x^2 + b^3)*x^3*log(x) - 4*((b^3*c^2*x^2 - b^3)*x^3*log(x) - (b^3*log(
c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*(c*x + 1)^(3/2)*(-c*x +
 1)^(3/2) + ((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c
^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 4*(b^3*
c^2*log(c) - a*b^2*c^2)*x^2)*x^3 - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x
^3*log(x) + ((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3
*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*(c*x + 1)*(c*x - 1) - 4*((b^3*c^6*x^6 -
3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*x^3*log(x) + ((b^3*c^6*log(c) - a*b^2*
c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3
*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1
)^2*(c*x - 1)^2*b^3*x^3 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^
(3/2)*x^3 - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^3 -
 4*(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-
c*x + 1)*x^3 + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2
 + b^3)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*acosh(1/(c*x)))^3),x)
```

```
[Out] int(1/(x^3*(a + b*acosh(1/(c*x)))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{asech}(cx))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*asech(c*x))**3,x)
[Out] Integral(1/(x**3*(a + b*asech(c*x))**3), x)
```

3.68 $\int \frac{1}{x^4\left(a+b\mathbf{sech}^{-1}(cx)\right)^3}\,dx$

Optimal. Leaf size=240

$$\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{8b^3}+\frac{9c^3\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{sech}^{-1}(cx)\right)}{8b^3}-\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{8b^3}-\cdots$$

```
[Out] 1/8*c^2/b^2/x/(a+b*arcsech(c*x))+3/8*c^3*cosh(3*arcsech(c*x))/b^2/(a+b*arcs
ech(c*x))-1/8*c^3*cosh(a/b)*Shi(a/b+arcsech(c*x))/b^3-9/8*c^3*cosh(3*a/b)*S
hi(3*a/b+3*arcsech(c*x))/b^3+1/8*c^3*Chi(a/b+arcsech(c*x))*sinh(a/b)/b^3+9/
8*c^3*Chi(3*a/b+3*arcsech(c*x))*sinh(3*a/b)/b^3+1/8*c^3*sinh(3*arcsech(c*x)
)/b/(a+b*arcsech(c*x))^2+1/8*c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*
arcsech(c*x))^2
```

Rubi [A] time = 0.37, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6285, 5448, 3297, 3303, 3298, 3301}

$$\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{8b^3}+\frac{9c^3\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{sech}^{-1}(cx)\right)}{8b^3}-\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{8b^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*b*x*(a + b*ArcSech[c*x])^2) +
c^2/(8*b^2*x*(a + b*ArcSech[c*x])) + (3*c^3*Cosh[3*ArcSech[c*x]])/(8*b^2*(a
 + b*ArcSech[c*x])) + (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(8*b
^3) + (9*c^3*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(8*b^3)
+ (c^3*Sinh[3*ArcSech[c*x]])/(8*b*(a + b*ArcSech[c*x])^2) - (c^3*Cosh[a/b]*
SinhIntegral[a/b + ArcSech[c*x]])/(8*b^3) - (9*c^3*Cosh[(3*a)/b]*SinhIntegr
al[(3*a)/b + 3*ArcSech[c*x]])/(8*b^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{(a+bx)^3}\,dx,x,\text{sech}^{-1}(cx)\right)\right)$$

$$= -\left(c^3\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)^3}+\frac{\sinh(3x)}{4(a+bx)^3}\right)dx,x,\text{sech}^{-1}(cx)\right)\right)$$

$$= -\left(\frac{1}{4}c^3\,\text{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^3}\,dx,x,\text{sech}^{-1}(cx)\right)\right)-\frac{1}{4}c^3\,\text{Subst}\left(\int \frac{\sinh(3x)}{(a+bx)^3}\,dx,x,\text{se}\right.$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{c^3\sinh\left(3\text{sech}^{-1}(cx)\right)}{8b\left(a+b\text{sech}^{-1}(cx)\right)^2}-\frac{c^3\,\text{Subst}\left(\int \frac{\cosh(x)}{(a+bx)^2}\,dx,x,\text{sech}^{-1}\right.}{8b}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{c^2}{8b^2x\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{3c^3\cosh\left(3\text{sech}^{-1}(cx)\right)}{8b^2\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^3}{8b}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{c^2}{8b^2x\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{3c^3\cosh\left(3\text{sech}^{-1}(cx)\right)}{8b^2\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^3}{8b}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{c^2}{8b^2x\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{3c^3\cosh\left(3\text{sech}^{-1}(cx)\right)}{8b^2\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^3}{}$$

Mathematica [A] time = 0.63, size = 204, normalized size = 0.85

$$\frac{\frac{4b^2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}-8c^3\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)+9c^3\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] ((4*b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^3*(a + b*ArcSech[c*x])^2) +
 (4*b*(3 - 2*c^2*x^2))/(x^3*(a + b*ArcSech[c*x])) - 8*c^3*(CoshIntegral[a/b
 + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]) +
9*c^3*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + A
```

```
rcSech[c*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] -
Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/(8*b^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b^3x^4\operatorname{arsech}(cx)^3+3\,ab^2x^4\operatorname{arsech}(cx)^2+3\,a^2bx^4\operatorname{arsech}(cx)+a^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^4*arcsech(c*x)^3 + 3*a*b^2*x^4*arcsech(c*x)^2 + 3*a^2*b*x
^4*arcsech(c*x) + a^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsech}(cx)+a)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^4), x)
```

maple [B] time = 0.47, size = 628, normalized size = 2.62

$$c^3\left(\frac{\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\,c^3x^3-4\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx-3c^2x^2+4\right)(3b\operatorname{arcsech}(cx)+3a-b)}{16c^3x^3b^2\left(\operatorname{arcsech}(cx)^2b^2+2\operatorname{arcsech}(cx)ab+a^2\right)}-\frac{9\,e^{\frac{3a}{b}}\operatorname{Ei}\left(1,\frac{3a}{b}+3\operatorname{ar}\right.}{16b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arcsech(c*x))^3,x)
```

```
[Out] c^3*(1/16*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x
)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-3*c^2*x^2+4)*(3*b*arcsech(c*x)+3*a-b)/c^3/x
^3/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-9/16/b^3*exp(3*a/b)*Ei(1
,3*a/b+3*arcsech(c*x))-1/16*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1
)*(b*arcsech(c*x)+a-b)/c/x/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-
1/16/b^3*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/16/b*((-(c*x-1)/c/x)^(1/2)*((c*x
+1)/c/x)^(1/2)*c*x+1)/c/x/(a+b*arcsech(c*x))^2+1/16/b^2*((-(c*x-1)/c/x)^(1/
2)*((c*x+1)/c/x)^(1/2)*c*x+1)/c/x/(a+b*arcsech(c*x))+1/16/b^3*exp(-a/b)*Ei(
1,-arcsech(c*x)-a/b)-1/16/b*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x
^3-4*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+3*c^2*x^2-4)/c^3/x^3/(a+b
*arcsech(c*x))^2-3/16/b^2*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3
-4*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+3*c^2*x^2-4)/c^3/x^3/(a+b*a
rcsech(c*x))+9/16/b^3*exp(-3*a/b)*Ei(1,-3*arcsech(c*x)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 - 3*(b*c^4*(3*log(c) - 1) - 3*a*
c^4)*x^5 - ((b*c^4*log(c) - a*c^4)*x^5 - (b*c^2*(4*log(c) - 1) - 4*a*c^2)*x
^3 + (b*(3*log(c) - 1) - 3*a)*x + (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*log(x))
*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(3*log(c) - 1) - 3*a*c^2)*x^3
```

```
- (2*(b*c^6*log(c) - a*c^6)*x^7 - 2*(b*c^4*(5*log(c) - 1) - 5*a*c^4)*x^5 +
(b*c^2*(17*log(c) - 5) - 17*a*c^2)*x^3 - 3*(b*(3*log(c) - 1) - 3*a)*x + (2*
b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b*x)*log(x))*(c*x + 1)*(c*x - 1
) + ((b*c^6*(5*log(c) - 1) - 5*a*c^6)*x^7 - (b*c^4*(18*log(c) - 5) - 18*a*c
^4)*x^5 + (b*c^2*(22*log(c) - 7) - 22*a*c^2)*x^3 - 3*(b*(3*log(c) - 1) - 3*
a)*x + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b*x)*log(x))*sqrt(c*x
 + 1)*sqrt(-c*x + 1) - (b*(3*log(c) - 1) - 3*a)*x - (3*b*c^6*x^7 - 9*b*c^4*
x^5 + 9*b*c^2*x^3 - (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*(c*x + 1)^(3/2)*(-c*x
 + 1)^(3/2) - (2*b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b*x)*(c*x + 1)
*(c*x - 1) + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b*x)*sqrt(c*x +
 1)*sqrt(-c*x + 1) - 3*b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + 3*(b*c^
6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*b^4*c^4*
x^4 + 3*b^4*c^2*x^2 - b^4)*x^4*log(x)^2 + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x
^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*l
og(c) - a*b^3*c^2)*x^2)*x^4*log(x) + ((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c
) + a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log
(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*c^2*log(c)^2 - 2
*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^4 - (b^4*x^4*log(x)^2 + 2*(b^4*log(
c) - a*b^3)*x^4*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2)*x^4)*(c*
x + 1)^(3/2)*(-c*x + 1)^(3/2) - 3*((b^4*c^2*x^2 - b^4)*x^4*log(x)^2 - 2*(b^
4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^4*log(x) - (b^4*log(
c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) +
a^2*b^2*c^2)*x^2)*x^4)*(c*x + 1)*(c*x - 1) - ((c*x + 1)^(3/2)*(-c*x + 1)^(3
/2)*b^4*x^4 + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^4 + 3*(b^4*c^4*x^
4 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 - (b^4*c^6*x^6 -
3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^4)*log(sqrt(c*x + 1)*sqrt(-c*x + 1)
+ 1)^2 - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^4*log(x)^2 + 2*((b^4*c^4*
log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^
2)*x^2)*x^4*log(x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c)
 + a^2*b^2*c^4)*x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*
b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*((
b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^4*log(x) + ((b^4*c^6*l
og(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) +
a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^4 - (b^4*x^4*log(x) + (b^4*lo
g(c) - a*b^3)*x^4)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 3*((b^4*c^2*x^2 - b^4
)*x^4*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^4)
*(c*x + 1)*(c*x - 1) - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^4*log(x) +
((b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c)
- a*b^3*c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt
(-c*x + 1) + 1)) - integrate(1/2*(9*c^8*x^8 - 36*c^6*x^6 + 54*c^4*x^4 - (c^
4*x^4 + 4*c^2*x^2 - 9)*(c*x + 1)^2*(c*x - 1)^2 + (2*c^6*x^6 + 13*c^4*x^4 -
48*c^2*x^2 + 36)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 36*c^2*x^2 - (2*c^8*x^8
 - 19*c^6*x^6 + 83*c^4*x^4 - 120*c^2*x^2 + 54)*(c*x + 1)*(c*x - 1) + (10*c^
8*x^8 - 57*c^6*x^6 + 123*c^4*x^4 - 112*c^2*x^2 + 36)*sqrt(c*x + 1)*sqrt(-c*
x + 1) + 9)/((b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 +
 b^3)*x^4*log(x) + (b^3*x^4*log(x) + (b^3*log(c) - a*b^2)*x^4)*(c*x + 1)^2*
(c*x - 1)^2 + ((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2
*c^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 4*(b^
3*c^2*log(c) - a*b^2*c^2)*x^2)*x^4 - 4*((b^3*c^2*x^2 - b^3)*x^4*log(x) - (b
^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^4)*(c*x + 1)^(3/2)*
(-c*x + 1)^(3/2) - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x^4*log(x) + ((b^
3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x^4)*(c*x + 1)*(c*x - 1) - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4 +
3*b^3*c^2*x^2 - b^3)*x^4*log(x) + ((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^
3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1)^2*(c*x - 1)^2*
b^3*x^4 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*x^4 - 6*(b
^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^4 - 4*(b^3*c^6*x^6
- 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 + (
```

```
b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^4)*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{cx}\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*acosh(1/(c*x)))^3),x)
```

```
[Out] int(1/(x^4*(a + b*acosh(1/(c*x)))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + b \operatorname{asech}\left(cx\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(1/(x**4*(a + b*asech(c*x))**3), x)
```

3.69 $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcSech[c*x])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcSech[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$$

Mathematica [A] time = 5.61, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3, x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \operatorname{arsech}(cx)^3 + 3\,ab^2 \operatorname{arsech}(cx)^2 + 3\,a^2 b \operatorname{arsech}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x
) + a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsech}(cx) + a)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^3*(d*x)^m, x)
```

maple [A] time = 2.89, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arcsech}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^3/(m + 1) + (d*x)^(m +
1)*a^3/(d*(m + 1)) - integrate(((b^3*c^2*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))
*x^m*log(x)^3 - 3*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (b^3*c^2*d^
m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m*log(x)^2 + 3*((b^3*c^2*d
^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x) + ((b^3*c^2*d^m*(m + 1)*x^2 -
b^3*d^m*(m + 1))*x^m*log(x) - (b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) +
 (a*b^2*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^3*c^2)*x^2)*x^m)*sqr
t(c*x + 1)*sqrt(-c*x + 1) - (b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (
b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m)*log(sqrt(c*x
+ 1)*sqrt(-c*x + 1) + 1)^2 - 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m +
 1)*log(c) + a^2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^
2*d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) + ((b^3*c^2*d
^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x)^3 - 3*(b^3*d^m*(m + 1)*log(c)
- a*b^2*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*
x^2)*x^m*log(x)^2 - 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c
) + a^2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m
+ 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) - (b^3*d^m*(m + 1)*log
(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1)*log(c) - (b^3*c^
2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*c^2*d^m
*(m + 1)*log(c))*x^2)*x^m)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^3*d^m*(m + 1)*
log(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1)*log(c) - (b^3
*c^2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*c^2*
d^m*(m + 1)*log(c))*x^2)*x^m - 3*((b^3*c^2*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1
))*x^m*log(x)^2 - 2*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (b^3*c^2*
d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m*log(x) + ((b^3*c^2*d^m
*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x)^2 - 2*(b^3*d^m*(m + 1)*log(c) -
a*b^2*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^
2)*x^m*log(x) - (b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) + a^
2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)*l
og(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^
3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) + a^2*b*d^m*(m + 1) - (
b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)*log(c) + a^2*b*c^2*d
^m*(m + 1))*x^2)*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1))/(c^2*(m + 1)*x
^2 + (c^2*(m + 1)*x^2 - m - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*acosh(1/(c*x)))^3,x)
```

```
[Out] int((d*x)^m*(a + b*acosh(1/(c*x)))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\,\mathrm{asech}\,(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*asech(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*asech(c*x))**3, x)
```

3.70 $\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcSech[c*x])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcSech[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 3.61, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2, x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \,\text{arsech}\,(cx)^2 + 2\,ab\,\text{arsech}\,(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsech}\,(cx) + a)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^2*(d*x)^m, x)
```

maple [A] time = 2.68, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\,\text{arcsech}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2 d^m x x^m \log\left(\sqrt{cx+1}\sqrt{-cx+1}+1\right)^2}{m+1}+\frac{(dx)^{m+1}a^2}{d(m+1)}-\int -\frac{\left(b^2c^2d^m(m+1)x^2-b^2d^m(m+1)\right)x^m\log(x)^2-2\left(b^2d^m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2/(m + 1) + (d*x)^(m +
1)*a^2/(d*(m + 1)) - integrate(-((b^2*c^2*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1)
)*x^m*log(x)^2 - 2*(b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) - (b^2*c^2*d^m
*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) + ((b^2*c^2*d^m*(m +
 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(x)^2 - 2*(b^2*d^m*(m + 1)*log(c) - a*b*d
^m*(m + 1) - (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m*lo
g(x) - (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c) - (b^2*c^2*d^m*
(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2)*x^m)*sqrt(c*x + 1)*sq
rt(-c*x + 1) - (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c) - (b^2*
c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2)*x^m - 2*((b^2
*c^2*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(x) + ((b^2*c^2*d^m*(m + 1)*
x^2 - b^2*d^m*(m + 1))*x^m*log(x) - (b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m +
1) + (a*b*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^2*c^2)*x^2)*x^m)*s
qrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) - (
b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m)*log(sqrt(c*x +
1)*sqrt(-c*x + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 - m - 1)*sqrt(c
*x + 1)*sqrt(-c*x + 1) - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int((d*x)^m*(a + b*acosh(1/(c*x)))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\mathrm{asech}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*asech(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*asech(c*x))**2, x)
```

3.71 $\int (dx)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=87

$$\frac{(dx)^{m+1}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{d(m+1)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,(dx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)}{d(m+1)^2}$$

```
[Out] (d*x)^(1+m)*(a+b*arcsech(c*x))/d/(1+m)+b*(d*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/(1+m)^2
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6283, 125, 364}

$$\frac{(dx)^{m+1}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{d(m+1)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,(dx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcSech[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcSech[c*x]))/(d*(1 + m)) + (b*(d*x)^(1 + m)*Sqrt[(1
 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^
2*x^2])/(d*(1 + m)^2)
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx &= \frac{(dx)^{1+m}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{d(1+m)}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(dx)^m}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{1+m}\\ &= \frac{(dx)^{1+m}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{d(1+m)}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(dx)^m}{\sqrt{1-c^2x^2}}\,dx}{1+m}\\ &= \frac{(dx)^{1+m}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{d(1+m)}+\frac{b(dx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{d(1+m)^2}\end{aligned}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 1.11

$$\frac{x(dx)^m\left((m+1)(cx-1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\right)}{(m+1)^2(cx-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x]),x]
```

```
[Out] (x*(d*x)^m*((1 + m)*(-1 + c*x)*(a + b*ArcSech[c*x]) - b*Sqrt[(1 - c*x)/(1 +
 c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x
^2]))/((1 + m)^2*(-1 + c*x))
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\,\mathrm{arsech}\,(cx)+a)\,(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{arsech}\,(cx)+a)\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*(d*x)^m, x)
```

maple [F] time = 2.60, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\mathrm{arcsech}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arcsech(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arcsech(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c^2d^m\int\frac{x^2x^m}{c^2(m+1)x^2+\left(c^2(m+1)x^2-m-1\right)\sqrt{cx+1}\,\sqrt{-cx+1}\,-m-1}\,dx+\frac{d^mxx^m\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right.}{m+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] (c^2*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 - m - 1)*sqr
t(c*x + 1)*sqrt(-c*x + 1) - m - 1), x) + (d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(
-c*x + 1) + 1) - d^m*x*x^m*log(x))/(m + 1) - integrate((c^2*d^m*(m + 1)*x^2
*log(c) - d^m*(m + 1)*log(c) + d^m)*x^m/(c^2*(m + 1)*x^2 - m - 1), x))*b +
(d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*acosh(1/(c*x))),x)
[Out] int((d*x)^m*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\,\text{asech}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*asech(c*x)),x)
[Out] Integral((d*x)**m*(a + b*asech(c*x)), x)
```

3.72 $\int \frac{(dx)^m}{a+b\,\mathbf{sech}^{-1}(cx)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{a+b\,\text{sech}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arcsech(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\,\text{sech}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcSech[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\,\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\text{sech}^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\text{sech}^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcSech[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\,\text{arsech}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arcsech(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\text{arsech}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arcsech(c*x) + a), x)
```

maple [A] time = 1.85, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{arcsech}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arcsech(c*x)),x)
[Out] int((d*x)^m/(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\mathrm{arsech}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arcsech(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*acosh(1/(c*x))),x)
[Out] int((d*x)^m/(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{asech}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*asech(c*x)),x)
[Out] Integral((d*x)**m/(a + b*asech(c*x)), x)
```

3.73 $\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arcsech(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcSech[c*x])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcSech[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\,\mathrm{arsech}\,(cx)^2+2\,ab\,\mathrm{arsech}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b\,\mathrm{arsech}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arcsech(c*x) + a)^2, x)
```

maple [A] time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\,\mathrm{arcsech}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arcsech(c*x))^2,x)
[Out] int((d*x)^m/(a+b*arcsech(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2 d^m x^3 - d^m x\right)\sqrt{cx+1}\,\sqrt{-cx+1}\,x^m + \left(c^2 d^m x^3\right.}{\left(b^2c^2\log(c) - abc^2\right)x^2 - b^2\log(c) - \left(b^2\log(c) + b^2\log(x) - ab\right)\sqrt{cx+1}\,\sqrt{-cx+1} + ab - \left(b^2c^2x^2 - \sqrt{cx+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*d^m*x^3 - d^m*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m + (c^2*d^m*x^3 - d
^m*x)*x^m)/((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2
*log(x) - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x
 + 1)*sqrt(-c*x + 1)*b^2 - b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^
2*c^2*x^2 - b^2)*log(x)) + integrate(((c^2*d^m*(m + 3)*x^2 - d^m*(m + 1))*(
c*x + 1)*(c*x - 1)*x^m + (c^4*d^m*(m + 2)*x^4 - c^2*d^m*(3*m + 5)*x^2 + 2*d
^m*(m + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m + (c^4*d^m*(m + 1)*x^4 - 2*c^2
*d^m*(m + 1)*x^2 + d^m*(m + 1))*x^m)/((b^2*c^4*log(c) - a*b*c^4)*x^4 - (b^2
*log(c) + b^2*log(x) - a*b)*(c*x + 1)*(c*x - 1) - 2*(b^2*c^2*log(c) - a*b*c
^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b
 + (b^2*c^2*x^2 - b^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - a*b - (b^2*c^
4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(c*x - 1)*b^2 - 2*(b^2*c^2*x^2 - b^2)*sqr
t(c*x + 1)*sqrt(-c*x + 1) + b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b
^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*acosh(1/(c*x)))^2,x)
[Out] int((d*x)^m/(a + b*acosh(1/(c*x)))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\,\mathrm{asech}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*asech(c*x))**2,x)
[Out] Integral((d*x)**m/(a + b*asech(c*x))**2, x)
```

3.74 $\int (d+ex)^3 \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=264

$$\frac{(d+ex)^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e}-\frac{bd^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4e}-\frac{bde^2x\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2c^2}-\frac{be^3x^2\sqrt{\frac{1}{c}}}{}$$

```
[Out] 1/4*(e*x+d)^4*(a+b*arcsech(c*x))/e+1/2*b*d*(2*c^2*d^2+e^2)*arcsin(c*x)*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/4*b*d^4*arctanh((-c^2*x^2+1)^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*e*(9*c^2*d^2+e^2)*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/2*b*d*e^2*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1
/2)*(-c^2*x^2+1)^(1/2)/c^2-1/12*b*e^3*x^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] time = 0.36, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6288, 1809, 844, 216, 266, 63, 208}

$$\frac{(d+ex)^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e}-\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(9c^2d^2+e^2\right)}{6c^4}+\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(2c^2d^2+e^2\right)\sin^{-1}(cx)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*e*(9*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(6*c^4) - (b*d*e^2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(2*c^2) - (b*e^3*x^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(12*c^2) + ((d + e*x)^4*(a + b*ArcSech[c*x]))/(4*e) + (b*d*(2*c^2*d^2 +
e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(2*c^3) - (b*d^4*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx = \frac{(d+ex)^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^4}{x\sqrt{1-c^2x^2}}\,dx}{4e}$$

$$= -\frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e} - \frac{\left(b\sqrt{\frac{1}{1+c\ldots}}\right.}{}$$

$$= -\frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d\ldots}{}$$

$$= -\frac{be\left(9c^2d^2+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c\ldots}}{2c^2}$$

$$= -\frac{be\left(9c^2d^2+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c\ldots}}{2c^2}$$

$$= -\frac{be\left(9c^2d^2+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c\ldots}}{2c^2}$$

$$= -\frac{be\left(9c^2d^2+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c\ldots}}{2c^2}$$

$$= -\frac{be\left(9c^2d^2+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c\ldots}}{2c^2}$$

Mathematica [C] time = 0.42, size = 190, normalized size = 0.72

$$\frac{1}{4}\left(4ad^3x+6ad^2ex^2+4ade^2x^3+ae^3x^4-\frac{be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2\left(18d^2+6dex+e^2x^2\right)+2e^2\right)}{3c^4}+\frac{2ibd\left(2c^2d^2+e^2\ldots\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] (4*a*d^3*x + 6*a*d^2*e*x^2 + 4*a*d*e^2*x^3 + a*e^3*x^4 - (b*e*Sqrt[(1 - c*x
)/(1 + c*x)]*(1 + c*x)*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)))/(3*c^4)
+ b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcSech[c*x] + ((2*I)*b*d
*(2*c^2*d^2 + e^2)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])
/c^3)/4
```

fricas [B] time = 0.76, size = 358, normalized size = 1.36

$$\frac{3\,ac^3e^3x^4 + 12\,ac^3de^2x^3 + 18\,ac^3d^2ex^2 + 12\,ac^3d^3x - 12\left(2\,bc^2d^3 + bde^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - 3\left(4\,bc^3d^3 + 6\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e^3*x^4 + 12*a*c^3*d*e^2*x^3 + 18*a*c^3*d^2*e*x^2 + 12*a*c^3*
d^3*x - 12*(2*b*c^2*d^3 + b*d*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) - 1)/(c*x)) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e + 4*b*c^3*d*e^2 + b*c^3*e^3
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 3*(b*c^3*e^3*x^4 + 4*b*
c^3*d*e^2*x^3 + 6*b*c^3*d^2*e*x^2 + 4*b*c^3*d^3*x - 4*b*c^3*d^3 - 6*b*c^3*d
^2*e - 4*b*c^3*d*e^2 - b*c^3*e^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
 1)/(c*x)) - (b*c^2*e^3*x^3 + 6*b*c^2*d*e^2*x^2 + 2*(9*b*c^2*d^2*e + b*e^3)
*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^3 (b\,\mathrm{arsech}\,(cx) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arcsech(c*x) + a), x)
```

maple [A] time = 0.07, size = 283, normalized size = 1.07

$$\frac{\frac{(cxe+cd)^4 a}{4c^3e} + \frac{b\left(\frac{e^3\mathrm{arcsech}(cx)c^4x^4}{4} + e^2\mathrm{arcsech}(cx)c^4x^3d + \frac{3e\,\mathrm{arcsech}(cx)c^4x^2d^2}{2} + \mathrm{arcsech}(cx)c^4x\,d^3 + \frac{\mathrm{arcsech}(cx)c^4d^4}{4e} + \frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(-3c^4d^4\,\mathrm{arctanh}\left(\frac{}{\sqrt{-c^2}}\right.\right.}{}\right.}{c^3}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+b/c^3*(1/4*e^3*arcsech(c*x)*c^4*x^4+e^2*arcs
ech(c*x)*c^4*x^3*d+3/2*e*arcsech(c*x)*c^4*x^2*d^2+arcsech(c*x)*c^4*x*d^3+1/
4/e*arcsech(c*x)*c^4*d^4+1/12/e*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2
)*(-3*c^4*d^4*arctanh(1/(-c^2*x^2+1)^(1/2))+12*c^3*d^3*e*arcsin(c*x)-c^2*x^
2*e^4*(-c^2*x^2+1)^(1/2)-6*c^2*d*e^3*x*(-c^2*x^2+1)^(1/2)-18*c^2*d^2*e^2*(-
c^2*x^2+1)^(1/2)+6*c*d*e^3*arcsin(c*x)-2*e^4*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+
1)^(1/2)))
```

maxima [A] time = 0.42, size = 221, normalized size = 0.84

$$\frac{1}{4}\,ae^3x^4 + ade^2x^3 + \frac{3}{2}\,ad^2ex^2 + \frac{3}{2}\left(x^2\,\mathrm{arsech}\,(cx) - \frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd^2e + \frac{1}{2}\left(2\,x^3\,\mathrm{arsech}\,(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{}{c^2}}\right.}{c^2}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arcsech(c*x) - x*s
qrt(1/(c^2*x^2) - 1)/c)*b*d^2*e + 1/2*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^
2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/
c)*b*d*e^2 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*e^3 + a*d^3*x + (c*x*arcsech(c*x) - arcta
n(sqrt(1/(c^2*x^2) - 1)))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{asech}{\left(cx\right)}\right)\left(d+ex\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x)**3, x)
```

3.75 $\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=201

$$\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}-\frac{bd^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{3e}-\frac{bde\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{c^2}-\frac{be^2x\sqrt{\frac{1}{cx+1}}}{}$$

```
[Out] 1/3*(e*x+d)^3*(a+b*arcsech(c*x))/e+1/6*b*(6*c^2*d^2+e^2)*arcsin(c*x)*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/3*b*d^3*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)/e-b*d*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)/c^2-1/6*b*e^2*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
/c^2
```

Rubi [A] time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6288, 1809, 844, 216, 266, 63, 208}

$$\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(6c^2d^2+e^2\right)\sin^{-1}(cx)}{6c^3}-\frac{bd^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{3e}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*d*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/c^2) - (b*e^
2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) + ((d + e
*x)^3*(a + b*ArcSech[c*x]))/(3*e) + (b*(6*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3) - (b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(3*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)dx = \frac{(d+ex)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^3}{x\sqrt{1-c^2x^2}}\,dx}{3e}$$

$$= -\frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\right.}{}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex}{}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex}{}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex}{}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex}{}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex}{}$$

Mathematica [C] time = 0.25, size = 147, normalized size = 0.73

$$\frac{2ac^3x\left(3d^2+3dex+e^2x^2\right)+2bc^3x\operatorname{sech}^{-1}(cx)\left(3d^2+3dex+e^2x^2\right)+ib\left(6c^2d^2+e^2\right)\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2ic\right.}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-(b*c*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(6*d + e*x)) + 2*a*c^3*x*(3*d^
2 + 3*d*e*x + e^2*x^2) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcSech[c*x]
 + I*b*(6*c^2*d^2 + e^2)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
c*x)])/(6*c^3)
```

fricas [B] time = 0.70, size = 280, normalized size = 1.39

$$\frac{2\,ac^3e^2x^3+6\,ac^3dex^2+6\,ac^3d^2x-2\left(6\,bc^2d^2+be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-2\left(3\,bc^3d^2+3\,bc^3de+bc^3e^2\right)\log\left(\frac{cx}{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x - 2*(6*b*c^2*d^2 + b
*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 2*(3*b*c^3*d
^2 + 3*b*c^3*d*e + b*c^3*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/
x) + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b
*c^3*d*e - b*c^3*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) -
 (b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2(b\,\mathrm{arsech}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arcsech(c*x) + a), x)
```

maple [A] time = 0.07, size = 215, normalized size = 1.07

$$\frac{\frac{(cxe+cd)^3a}{3c^2e}+\frac{b\left(\frac{e^2\mathrm{arcsech}(cx)c^3x^3}{3}+e\,\mathrm{arcsech}(cx)c^3x^2d+\mathrm{arcsech}(cx)c^3x\,d^2+\frac{\mathrm{arcsech}(cx)c^3d^3}{3e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(-2c^3d^3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)+6c^2d^2e\,\mathrm{arcsin}(c\cdots\right.}{6e\sqrt{-c^2x^2+1}}\right.}{c^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+b/c^2*(1/3*e^2*arcsech(c*x)*c^3*x^3+e*arcsec
h(c*x)*c^3*x^2*d+arcsech(c*x)*c^3*x*d^2+1/3/e*arcsech(c*x)*c^3*d^3+1/6/e*(-
(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(-2*c^3*d^3*arctanh(1/(-c^2*x^2+
1)^(1/2))+6*c^2*d^2*e*arcsin(c*x)-e^3*c*x*(-c^2*x^2+1)^(1/2)-6*c*d*e^2*(-c^
2*x^2+1)^(1/2)+e^3*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.42, size = 152, normalized size = 0.76

$$\frac{1}{3}\,ae^2x^3+adex^2+\left(x^2\,\mathrm{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bde+\frac{1}{6}\left(2\,x^3\,\mathrm{arsech}\,(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be^2+ad^2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*
b*d*e + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2)
```

```
- 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e^2 + a*d^2*x + (c*x*
arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{asech}{\left(c x \right)}\right) \left(d + e x\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x)**2, x)
```

3.76 $\int (d+ex)\left(a+b\,\mathbf{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=142

$$\frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e}-\frac{bd^2\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2e}-\frac{be\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{2c^2}+\frac{bd\sqrt{\frac{1}{cx+1}}\,\sqrt{c}}{}$$

```
[Out] 1/2*(e*x+d)^2*(a+b*arcsech(c*x))/e+b*d*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1
)^(1/2)/c-1/2*b*d^2*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(
1/2)/e-1/2*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6288, 1809, 844, 216, 266, 63, 208}

$$\frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e}-\frac{bd^2\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2e}-\frac{be\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{2c^2}+\frac{bd\sqrt{\frac{1}{cx+1}}\,\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(2*c^2) + ((d +
 e*x)^2*(a + b*ArcSech[c*x]))/(2*e) + (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*
x]*ArcSin[c*x])/c - (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[
1 - c^2*x^2]])/(2*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)dx &= \frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^2}{x\sqrt{1-c^2x^2}}dx}{2e}\\
&= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1\right.}}{}\\
&= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\left(bd\sqrt{\frac{1}{1+c}}\right.\\
&= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{}}{}\\
&= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{}}{}\\
&= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{}}{}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 122, normalized size = 0.86

$$adx+\frac{1}{2}aex^2-\frac{bd\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sin^{-1}(cx)}{c(cx-1)}+be\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{cx+1}}+bdx\text{sech}^{-1}(cx)+\frac{1}{2}bex^2\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcSech[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + b*e*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)]
+ b*d*x*ArcSech[c*x] + (b*e*x^2*ArcSech[c*x])/2 - (b*d*Sqrt[(1 - c*x)/(1 +
c*x)]*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*(-1 + c*x))
```

fricas [B] time = 0.68, size = 177, normalized size = 1.25

$$\frac{acex^2+2\,acdx-bex\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-4\,bd\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-(2\,bcd+bce)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)+\left(bcex^2+2\,bc\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c*e*x^2 + 2*a*c*d*x - b*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*b*d*a
rctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - (2*b*c*d + b*c*e)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 + 2*b*c*d*x - 2
*b*c*d - b*c*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)(b \operatorname{arsech}(cx) + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arcsech(c*x) + a), x)
```

maple [A] time = 0.06, size = 125, normalized size = 0.88

$$\frac{\frac{a\left(\frac{1}{2}c^2x^2e+c^2dx\right)}{c}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^2x^2e}{2}+\operatorname{arcsech}(cx)c^2xd+\frac{\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}\left(2cd\arcsin(cx)-e\sqrt{-c^2x^2+1}\right)}{2\sqrt{-c^2x^2+1}}\right)}{c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c*(a/c*(1/2*c^2*x^2*e+c^2*d*x)+b/c*(1/2*arcsech(c*x)*c^2*x^2*e+arcsech(c*
x)*c^2*x*d+1/2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(2*c*d*arcsin(c
*x)-e*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.32, size = 70, normalized size = 0.49

$$\frac{1}{2}\, aex^2 + \frac{1}{2}\left(x^2 \operatorname{arsech}(cx) - \frac{x\sqrt{\frac{1}{c^2x^2} - 1}}{c}\right)be + adx + \frac{\left(cx \operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2} - 1}\right)\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b*e + a*d*
x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d/c
```

mupad [B] time = 1.53, size = 99, normalized size = 0.70

$$\frac{a\,x\,\left(2\,d+e\,x\right)}{2}+\frac{b\,d\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}+\frac{b\,e\,x^2\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{2}+b\,d\,x\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)-\frac{b\,e\,x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 + (b*d*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))
))/c + (b*e*x^2*acosh(1/(c*x)))/2 + b*d*x*acosh(1/(c*x)) - (b*e*x*(1/(c*x)
- 1)^(1/2)*(1/(c*x) + 1)^(1/2))/(2*c)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\text{asech}\,(cx))\,(d + ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x), x)
```

3.77 $\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=40

$$ax+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\sin^{-1}(cx)}{c}+bx\text{sech}^{-1}(cx)$$

```
[Out] a*x+b*x*arcsech(c*x)+b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6277, 216}

$$ax+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\sin^{-1}(cx)}{c}+bx\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSech[c*x],x]
```

```
[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])
/c
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6277

```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx &= ax+b\int \text{sech}^{-1}(cx)\,dx\\
&= ax+bx\text{sech}^{-1}(cx)+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx\\
&= ax+bx\text{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\sin^{-1}(cx)}{c}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 60, normalized size = 1.50

$$ax-\frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\,\sin^{-1}(cx)}{c(cx-1)}+bx\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSech[c*x],x]
```

```
[Out] a*x + b*x*ArcSech[c*x] - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Arc
Sin[c*x])/(c*(-1 + c*x))
```

fricas [B] time = 0.68, size = 119, normalized size = 2.98

$$\frac{acx - bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx - bc)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x - b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*log((c*x*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int b\,\mathrm{arsech}\,(cx) + a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arcsech(c*x) + a, x)
```

maple [A] time = 0.04, size = 42, normalized size = 1.05

$$ax + bx\,\mathrm{arcsech}\,(cx) - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsech(c*x),x)
```

```
[Out] a*x+b*x*arcsech(c*x)-b/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
```

maxima [A] time = 0.31, size = 31, normalized size = 0.78

$$ax + \frac{\left(cx\,\mathrm{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b/c
```

mupad [B] time = 1.39, size = 44, normalized size = 1.10

$$a\,x + b\,x\,\mathrm{acosh}\left(\frac{1}{c\,x}\right) + \frac{b\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(1/(c*x)),x)
```

```
[Out] a*x + b*x*acosh(1/(c*x)) + (b*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/
2))))/c
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asech}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asech(c*x),x)
```

```
[Out] Integral(a + b*asech(c*x), x)
```

3.78 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=229

$$\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e}+\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e}-\frac{\log\left(e^{-2s\cdots}\right.}{\cdots}$$

```
[Out] -(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e+(a
+b*arcsech(c*x))*ln(1+(e-(-c^2*d^2+e^2)^(1/2))/c/d/(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2)))/e+(a+b*arcsech(c*x))*ln(1+(e+(-c^2*d^2+e^2)^(1/2))/c/d/(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e-b*polylog(2,(-e+(-c^2*d^2+e^2)^(1/2))/c/d/(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e-b*polylog(2,(-e-(-c^2*d^2+e^2)^(
1/2))/c/d/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e
```

Rubi [A] time = 0.93, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6287, 2518}

$$-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{2e}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e) + ((a + b*ArcSech[
c*x])*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e + ((a +
 b*ArcSech[c*x])*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])]
)/e + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e) - (b*PolyLog[2, -((e - Sqr
t[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]))])/e - (b*PolyLog[2, -((e + Sqrt[
-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]))])/e
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 6287

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :>
Simp[((a + b*ArcSech[c*x])*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcS
ech[c*x])])/e, x] + (Dist[b/e, Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + (e -
Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x], x] + Dist
[b/e, Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(
c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x], x] - Dist[b/e, Int[(Sqrt[(1 - c*x)
/(1 + c*x)]*Log[1 + 1/E^(2*ArcSech[c*x])])/(x*(1 - c*x)), x], x] + Simp[((a
 + b*ArcSech[c*x])*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]
)])/e, x] - Simp[((a + b*ArcSech[c*x])*Log[1 + 1/E^(2*ArcSech[c*x])])/e, x]
) /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex}\,dx = -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\operatorname{sec}}}{cd}\right.}{e}$$

$$= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\operatorname{sec}}}{cd}\right.}{e}$$

Mathematica [C] time = 0.58, size = 393, normalized size = 1.72

$$\frac{a\log(d+ex)}{e} + \frac{b\left(\operatorname{Li}_2\left(-e^{-2\operatorname{sech}^{-1}(cx)}\right) - 2\left(\operatorname{Li}_2\left(\frac{\left(\sqrt{e^2-c^2d^2}-e\right)e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right) + \operatorname{Li}_2\left(-\frac{\left(e+\sqrt{e^2-c^2d^2}\right)e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right) - \operatorname{sech}^{-1}(cx)\,\mathrm{l}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSi
n[Sqrt[1 + e/(c*d)]/Sqrt[2]]*ArcTanh[((-(c*d) + e)*Tanh[ArcSech[c*x]/2])/Sq
rt[-(c^2*d^2) + e^2]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech
[c*x]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + (2*I)*Ar
cSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E
^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^
ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*Log[1 + (e + Sqrt[
-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, (-e + Sqrt[-(c^2*d^2)
 + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, -((e + Sqrt[-(c^2*d^2) + e^2])/
(c*d*E^ArcSech[c*x]))])))/(2*e)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\operatorname{arsech}(cx)+a}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arsech}(cx)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d), x)
```

maple [C] time = 0.68, size = 514, normalized size = 2.24

$$\frac{a\ln(cxe+cd)}{e} + \frac{b\operatorname{arcsech}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e} + \frac{b\operatorname{arcsech}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-}}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e+b/e*arcsech(c*x)*ln((-c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*arcsech(c*
x)*ln((c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)
/(e+(-c^2*d^2+e^2)^(1/2)))+b/e*dilog((c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e^2)^(1/2)))+b/e*dilog((-c*d*(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d
^2+e^2)^(1/2)))-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2)))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/
e*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog(1-I*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{ex+d}\,dx+\frac{a\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x + d), x
) + a*log(e*x + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\left(cx\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x), x)
```

3.79 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{de}$$

```
[Out] (-a-b*arcsech(c*x))/e/(e*x+d)+b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/
2)*(c*x+1)^(1/2)/d/e+b*arctan((c^2*d*x+e)/(c^2*d^2-e^2)^(1/2)/(-c^2*x^2+1)^
(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/(c^2*d^2-e^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6288, 961, 266, 63, 208, 725, 204}

$$-\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSech[c*x])/(e*(d + e*x))) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*
x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(d*Sqrt[c
^2*d^2 - e^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2
*x^2]])/(d*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x(d+ex)\sqrt{1-c^2x^2}}\,dx}{e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(\frac{1}{dx\sqrt{1-c^2x^2}} - \frac{e}{d(d+ex)\sqrt{1-c^2x^2}}\right)dx}{e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)\sqrt{1-c^2x^2}}\,dx}{d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{}{x\sqrt{}}}{de}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-c^2d^2+e^2-x^2}\,dx, x, \frac{e+c^2dx}{\sqrt{1-c^2x^2}}\right)}{d} - \frac{\left(b\sqrt{\frac{1}{1+}}\right.}{}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan^{-1}\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{S}}{}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan^{-1}\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan}{de}$$

Mathematica [A] time = 0.24, size = 222, normalized size = 1.51

$$-\frac{a}{e(d+ex)}+\frac{b\log(d+ex)}{d\sqrt{e^2-c^2d^2}}-\frac{b\log\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{e^2-c^2d^2}+\sqrt{\frac{1-cx}{cx+1}}\sqrt{e^2-c^2d^2}+c^2dx+e\right)}{d\sqrt{e^2-c^2d^2}}+\frac{b\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{}\right.}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcSech[c*x])/(e*(d + e*x)) - (b*Log[x])/(d*e) + (b
*Log[d + e*x])/(d*Sqrt[-(c^2*d^2) + e^2]) + (b*Log[1 + Sqrt[(1 - c*x)/(1 +
c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(d*e) - (b*Log[e + c^2*d*x + Sqrt[-
(c^2*d^2) + e^2]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[-(c^2*d^2) + e^2]*x*Sqr
t[(1 - c*x)/(1 + c*x)]])/(d*Sqrt[-(c^2*d^2) + e^2])
```

fricas [B] time = 1.08, size = 578, normalized size = 3.93

$$\left[-\frac{ac^2d^3 - ade^2 + \sqrt{-c^2d^2 + e^2}\left(be^2x + bde\right)\log\left(\frac{c^2dex - \left(c^3d^2 - ce^2\right)x\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + e^2 - \sqrt{-c^2d^2+e^2}\left(c^2dx + cex\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + e\right)}{ex+d}\right) + \left(bc^2d\right.}{c^2d^4e - d^2e^3 + \left(c^2d^3e^2 - de^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 + sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*log((c^2*d*
e*x - (c^3*d^2 - c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2 - sqrt(-c^2*
d^2 + e^2)*(c^2*d*x + c*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e))/(e*x + d))
 + (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^3 - b*d*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x),
-(a*c^2*d^3 - a*d*e^2 - 2*sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*arctan(-(sq
rt(c^2*d^2 - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(c^2*d^2 - e^2
)*(e*x + d))/((c^2*d^2 - e^2)*x)) + (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b
*e^3)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^3 - b*d
*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e - d^2
*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx) + a}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^2, x)
```

maple [A] time = 0.12, size = 231, normalized size = 1.57

$$-\frac{ca}{(cxe + cd)\,e} - \frac{cb\,\mathrm{arcsech}\,(cx)}{(cxe + cd)\,e} + \frac{cb\sqrt{-\frac{cx-1}{cx}}\;x\sqrt{\frac{cx+1}{cx}}\;\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{ed\sqrt{-c^2x^2+1}} - \frac{cb\sqrt{-\frac{cx-1}{cx}}\;x\sqrt{\frac{cx+1}{cx}}\;\ln\left(\frac{2\sqrt{-c^2x^2+1}\;\sqrt{-\frac{c^2d^2}{e}}}{cxe+c}\right.}{e\sqrt{-\frac{c^2d^2-e^2}{e^2}}\;d\sqrt{-c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arcsech(c*x)+c*b/e*(-(c*x-1)/c/x)^(1/2
)*x*((c*x+1)/c/x)^(1/2)/d/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))-
c*b/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-(c^2*d^2-e^2)/e^2)^(1/2)
/d/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*(-(c^2*d^2-e^2)/e^2)^(1/2)*e
+c^2*d*x+e)/(c*e*x+c*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c^2\int \frac{x^2}{c^2d^2x^2 + \left(c^2d^2x^2 - d^2 + \left(c^2dex^2 - de\right)x\right)\sqrt{cx+1}\,\sqrt{-cx+1} - d^2 + \left(c^2dex^2 - de\right)x}\,dx + \frac{x\log\left(\sqrt{cx+1}\,\sqrt{-c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] (c^2*integrate(x^2/(c^2*d^2*x^2 + (c^2*d^2*x^2 - d^2 + (c^2*d*e*x^2 - d*e)*
x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - d^2 + (c^2*d*e*x^2 - d*e)*x), x) + (x*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - x*log(c) - x*log(x))/(d*e*x + d^2) - i
ntegrate(1/(c^2*d^2*x^2 - d^2 + (c^2*d*e*x^2 - d*e)*x), x))*b - a/(e^2*x +
d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\,(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x)**2, x)
```

3.80 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=306

$$-\frac{a+b\,\text{sech}^{-1}(cx)}{2e(d+ex)^2}+\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{2\left(c^2d^2-e^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{ta}}{2d^2\sqrt{}}$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/e/(e*x+d)^2+1/2*b*c^2*arctan((c^2*d*x+e)/(c^2*d^2-e
^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)
^(3/2)+1/2*b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^
2/e+1/2*b*arctan((c^2*d*x+e)/(c^2*d^2-e^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*d^2-e^2)^(1/2)+1/2*b*e*(1/(c*x+1))^(1/2)
*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A] time = 0.19, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6288, 961, 266, 63, 208, 731, 725, 204}

$$-\frac{a+b\,\text{sech}^{-1}(cx)}{2e(d+ex)^2}+\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{2\left(c^2d^2-e^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{ta}}{2d^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^3,x]
```

```
[Out] (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(2*d*(c^2*d^2 -
e^2)*(d + e*x)) - (a + b*ArcSech[c*x])/(2*e*(d + e*x)^2) + (b*c^2*Sqrt[(1 +
 c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1
- c^2*x^2])])/(2*(c^2*d^2 - e^2)^(3/2)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(2*d^2*
Sqrt[c^2*d^2 - e^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1
 - c^2*x^2]])/(2*d^2*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e(d+ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x(d+ex)^2\sqrt{1-c^2x^2}}\,dx}{2e}$$

$$= -\frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e(d+ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\left(\frac{1}{d^2x\sqrt{1-c^2x^2}} - \frac{e}{d(d+ex)^2\sqrt{1-c^2x^2}} - \frac{e}{d^2(d+ex)\sqrt{1-c\cdots}}\right.}{2e}$$

$$= -\frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)\sqrt{1-c^2x^2}}\,dx}{2d^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\cdots}{(d\cdots}}{2d}$$

$$= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e(d+ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{1}{-c^2d^2+\cdots}\right.}{2d^2}$$

$$= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan^{-1}\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1\cdots}}\right.}{2d^2\sqrt{c^2d^2-e^2}}$$

$$= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)} - \frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan^{-1}\left(\frac{e+c^2\cdots}{\sqrt{c^2d^2-e^2}\cdots}\right.}{2\left(c^2d^2-e^2\right)^{3/2}}$$

Mathematica [C] time = 0.70, size = 342, normalized size = 1.12

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{ib\left(2c^2d^2-e^2\right)\log\left(\frac{4d^2e\sqrt{c^2d^2-e^2}\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d^2-e^2}+\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d^2-e^2}+ic^2dx+ie\right)}{b\left(2c^2d^2-e^2\right)(d+ex)}\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}+\frac{b\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\right.}{d^2e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(e + c*e*x))/(d*(c*d -
 e)*(c*d + e)*(d + e*x)) - (b*ArcSech[c*x])/(e*(d + e*x)^2) - (b*Log[x])/(d
^2*e) + (b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)
]])/(d^2*e) - (I*b*(2*c^2*d^2 - e^2)*Log[(4*d^2*e*Sqrt[c^2*d^2 - e^2]*(I*e
+ I*c^2*d*x + Sqrt[c^2*d^2 - e^2]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d^
2 - e^2]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*(2*c^2*d^2 - e^2)*(d + e*x))])/(d
^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

fricas [B] time = 0.88, size = 1212, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a + b)*d^2*e^4 - (b*c^2*d^2*e^4
 - b*e^6)*x^2 + (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2
+ 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(-c^2*d^2 + e^2)*log((c^2*d*e*x - (c
^3*d^2 - c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2 - sqrt(-c^2*d^2 + e^
2)*(c^2*d*x + c*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e))/(e*x + d)) - 2*(b*
c^2*d^3*e^3 - b*d*e^5)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^
4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3
 + b*d*e^5)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^4*d^6
 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1
)/(c*x)) - ((b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 - b*c*d^2*e^4)
*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 +
(c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^
4 + d^3*e^6)*x), -1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a + b)*d^2*e^4
- (b*c^2*d^2*e^4 - b*e^6)*x^2 - 2*(2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2
*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^2)*ar
ctan(-(sqrt(c^2*d^2 - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(c^2*
d^2 - e^2)*(e*x + d))/((c^2*d^2 - e^2)*x)) - 2*(b*c^2*d^3*e^3 - b*d*e^5)*x
+ (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e
^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log((c*x*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^
2*e^4)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - ((b*c^3*d^3*e^
3 - b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 - b*c*d^2*e^4)*x)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^4*
e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^3, x)
```

maple [B] time = 0.09, size = 1090, normalized size = 3.56

$$-\frac{c^2 a}{2\left(cxe+cd\right)^2 e}-\frac{c^2 b\,\mathrm{arcsech}\left(cx\right)}{2\left(cxe+cd\right)^2 e}+\frac{c^4 b\sqrt{-\frac{cx-1}{cx}}\,x^2\sqrt{\frac{cx+1}{cx}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2\sqrt{-c^2x^2+1}\,\left(cd+e\right)\left(cd-e\right)\left(cxe+cd\right)}+\frac{c^4 b\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,d\,\operatorname{arctanh}}{2e\sqrt{-c^2x^2+1}\,\left(cd+e\right)\left(cd-e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arcsech(c*x)+1/2*c^4*b
*(-(c*x-1)/c/x)^(1/2)*x^2*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/(c*d+e)/(c
*d-e)/(c*e*x+c*d)*arctanh(1/(-c^2*x^2+1)^(1/2))+1/2*c^4*b/e*(-(c*x-1)/c/x)^
(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)*d/(c*d+e)/(c*d-e)/(c*e*x+c*d
)*arctanh(1/(-c^2*x^2+1)^(1/2))-c^4*b*(-(c*x-1)/c/x)^(1/2)*x^2*((c*x+1)/c/x
)^(1/2)/(-c^2*x^2+1)^(1/2)/(c*d+e)/(c*d-e)/(-(c^2*d^2-e^2)/e^2)^(1/2)/(c*e*
x+c*d)*ln(2*((-c^2*x^2+1)^(1/2)*(-(c^2*d^2-e^2)/e^2)^(1/2)*e+c^2*d*x+e)/(c*
e*x+c*d))-c^4*b/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(
1/2)*d/(c*d+e)/(c*d-e)/(-(c^2*d^2-e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln(2*((-c^2*x
^2+1)^(1/2)*(-(c^2*d^2-e^2)/e^2)^(1/2)*e+c^2*d*x+e)/(c*e*x+c*d))+1/2*c^2*b*
e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/d/(c*d+e)/(c*d-e)/(c*e*x+c*d)-
1/2*c^2*b*e^2*(-(c*x-1)/c/x)^(1/2)*x^2*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/
2)/d^2/(c*d+e)/(c*d-e)/(c*e*x+c*d)*arctanh(1/(-c^2*x^2+1)^(1/2))-1/2*c^2*b*
e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/d/(c*d+e)/(
c*d-e)/(c*e*x+c*d)*arctanh(1/(-c^2*x^2+1)^(1/2))+1/2*c^2*b*e^2*(-(c*x-1)/c/
x)^(1/2)*x^2*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/d^2/(c*d+e)/(c*d-e)/(-(
c^2*d^2-e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln(2*((-c^2*x^2+1)^(1/2)*(-(c^2*d^2-e^2
)/e^2)^(1/2)*e+c^2*d*x+e)/(c*e*x+c*d))+1/2*c^2*b*e*(-(c*x-1)/c/x)^(1/2)*x*(
(c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/d/(c*d+e)/(c*d-e)/(-(c^2*d^2-e^2)/e^2
)^(1/2)/(c*e*x+c*d)*ln(2*((-c^2*x^2+1)^(1/2)*(-(c^2*d^2-e^2)/e^2)^(1/2)*e+c
^2*d*x+e)/(c*e*x+c*d))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\left(cx\right)}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x)**3, x)
```

3.81 $\int (d+ex)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx$

Optimal. Leaf size=343

$$\frac{2(d+ex)^{5/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5e}-\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex}}{15c^2}-\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}{15c^3\sqrt{d+ex}}$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arcsech(c*x))/e-28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/
2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x+
d)^(1/2)/c/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(
-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^
(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/c^3/(e*x+d)^(1/2)-4/5*b*d^3*EllipticPi(1/2*
(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/e/(e*x+d)^(1/2)-4/15*b*e*(1/(c*x+1))^(1/
2)*(c*x+1)^(1/2)*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] time = 0.62, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6288, 958, 719, 419, 932, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2(d+ex)^{5/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5e}-\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^3\sqrt{d+ex}}-\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2])
/(15*c^2) + (2*(d + e*x)^(5/2)*(a + b*ArcSech[c*x]))/(5*e) - (28*b*d*Sqrt[(
1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)])/(15*c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c
^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d +
e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^3*Sqrt
[d + e*x]) - (4*b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))
/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/
(5*e*Sqrt[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)dx = \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex)^{5/2}}{x\sqrt{1-c^2x^2}}dx}{5e}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \right.}{}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} + \frac{1}{5}\left(6bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{}{\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$

Mathematica [C] time = 10.78, size = 2653, normalized size = 7.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x]*((-4*b*e)/(15*c^2) - (4*b*e*x)/(15*
c)) + Sqrt[d + e*x]*((2*a*d^2)/(5*e) + (4*a*d*x)/5 + (2*a*e*x^2)/5) + (2*b*
(d + e*x)^(5/2)*ArcSech[c*x])/(5*e) - (4*b*(7*c*d*e*Sqrt[(1 - c*x)/(1 + c*x
)]*(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x))) + ((7*I)*c
^2*d^2*e*(c*d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1
 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*(EllipticE[I*ArcSinh[Sq
rt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh[Sqrt[(
1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)]))/(c*d - e) - ((7*I)*c*d*e^2*(c*
d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*
d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)
/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1
+ c*x)]], (c*d - e)/(c*d + e)]))/(c*d - e) + (3*I)*c^3*d^3*Sqrt[1 + (1 - c*
x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c
*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/
(c*d + e)] - (2*I)*c^2*d^2*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1
- c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*A
rcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - I*e^3*Sqrt[1 + (1
 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(
1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d
- e)/(c*d + e)] + ((3 + 3*I)*c^3*d^3*(-I + Sqrt[(1 - c*x)/(1 + c*x)])*(I +
Sqrt[(1 - c*x)/(1 + c*x)])*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d
*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqr
t[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[
((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*S
qrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*
(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e
] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] +
I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I
*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (1 - I)*Ellipti
cPi[(I*Sqrt[-(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e
]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(
1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 +
 c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sq
rt[c*d - e])^2]))/Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 -
 c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*
x)/(1 + c*x)]))] + ((3 + 3*I)*c^3*d^3*(1 + I*Sqrt[(1 - c*x)/(1 + c*x)])*(I
+ Sqrt[(1 - c*x)/(1 + c*x)])*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c
*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + S
qrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqr
t[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e
*Sqrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e
)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) -
 e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e]
+ I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] +
 I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (1 + I)*Ellip
ticPi[((-I)*Sqrt[-(c*d) - e] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*
d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c
*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)
/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] -
 I*Sqrt[c*d - e])^2]))/Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt
[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1
 - c*x)/(1 + c*x)]))]))/(15*c^3*e*(1 + (1 - c*x)/(1 + c*x))*Sqrt[(c*d + e +
 (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1
 + c*x))])
```

fricas [F] time = 173.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex + ad + (bex + bd)\,\text{arsech}\,(cx)\right)\sqrt{ex + d}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x + a*d + (b*e*x + b*d)*arcsech(c*x))*sqrt(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*arcsech(c*x) + a), x)
```

maple [B] time = 0.23, size = 830, normalized size = 2.42

$$\frac{\frac{2(ex+d)^{\frac{5}{2}}a}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsech}(cx)}{5}-\frac{2e^2\sqrt{-\frac{(ex+d)c-cd-e}{cxe}}\,x\sqrt{\frac{(ex+d)c-cd+e}{cxe}}\left(\sqrt{\frac{c}{cd+e}}\,(ex+d)^{\frac{5}{2}}c^2+9\sqrt{-\frac{(ex+d)c-cd-e}{cd+e}}\sqrt{-\frac{(ex+d)c-cd+e}{cd-e}}\operatorname{EllipticF}\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

```
[Out] 2/e*(1/5*(e*x+d)^(5/2)*a+b*(1/5*(e*x+d)^(5/2)*arcsech(c*x)-2/15/c*e^2*(-((e
*x+d)*c-c*d-e)/c/x/e)^(1/2)*x*(((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*((c/(c*d+e))^
(1/2)*(e*x+d)^(5/2)*c^2+9*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c
*d+e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*
d-e))^(1/2))*c^2*d^2-7*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+
e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e
))^(1/2))*c^2*d^2-3*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/
(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/
(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*c^2*d^2-2*(c/(c*d+e))^(1/2)*(e*x+d)^(3/2)
*c^2*d-7*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d*
e+7*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*E
llipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d*e+(c/
(c*d+e))^(1/2)*(e*x+d)^(1/2)*c^2*d^2+(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-(
(e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),(
(c*d+e)/(c*d-e))^(1/2))*e^2-(c/(c*d+e))^(1/2)*(e*x+d)^(1/2)*e^2)/(c/(c*d+e)
)^(1/2)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asech}(cx)) (d + ex)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x)**(3/2), x)
```

3.82 $\int \sqrt{d+ex}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx$

Optimal. Leaf size=279

$$\frac{2(d+ex)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e}-\frac{4bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e\sqrt{d+ex}}-\frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{}}{3}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arcsech(c*x))/e-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x+d)^(
1/2)/c/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/
2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c
*d+e))^(1/2)/c/(e*x+d)^(1/2)-4/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2
),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(
c*d+e))^(1/2)/e/(e*x+d)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6288, 958, 719, 419, 932, 168, 538, 537, 844, 424}

$$\frac{2(d+ex)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e}-\frac{4bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e\sqrt{d+ex}}-\frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) - (4*b*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/(3*c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e*Sqrt[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)dx = \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{3e}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(\frac{2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \ldots\right.}{3e}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} + \frac{1}{3}\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\ldots}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} + \frac{1}{3}\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}d\ldots$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{8bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1\ldots}}{\sqrt{\ldots}}\right.\right.}{3c\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\ldots}}{\sqrt{2}}\right.\right.}{3c\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\ldots}}{\sqrt{2}}\right.\right.}{3c\sqrt{\frac{c(d+ex)}{cd+e}}}$$

Mathematica [C] time = 13.56, size = 2938, normalized size = 10.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcSech[c*x]),x]
```

```
[Out] ((2*a*d)/(3*e) + (2*a*x)/3)*Sqrt[d + e*x] + (2*b*(d + e*x)^(3/2)*ArcSech[c*
x])/(3*e) + (4*b*(-((e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[c*(1 + (1 - c*x)/(1 +
 c*x))]*Sqrt[c + (c*(1 - c*x))/(1 + c*x)]*Sqrt[(c*d + e + (c*d*(1 - c*x))/(
1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))])/(c*(1 +
 (1 - c*x)/(1 + c*x)))) + (Sqrt[c*(1 + (1 - c*x)/(1 + c*x))]*Sqrt[c + (c*(1
 - c*x))/(1 + c*x)]*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c
*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))]*Sqrt[(c*d + e + (c*d*(1 - c*x))/
(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))]*((I*c*d
*(-(c*d) - e)*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x)
)/((-(c*d) - e)*(1 + c*x))]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]]
, -((c*d - e)/(-(c*d) - e))] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)
]], -((c*d - e)/(-(c*d) - e))]))/((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x)
)*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*(-(c*d) - e)*e^2*Sqrt[
1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 +
c*x))]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d)
 - e))] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*
d) - e))]))/((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d -
e)*(1 - c*x))/(1 + c*x))]) - (I*c^2*d^2*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[
```

```
1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*EllipticF[I*ArcSinh[Sqr
t[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))])/Sqrt[c*(1 + (1 - c*x)/
(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))] + (I*c*d*e*Sqrt[1 +
 (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x
))]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e
))])/Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 +
 c*x))] + (I*c^2*d^2*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c
*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 -
 c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*
x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x
)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(
1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 +
c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x
)]))]*((1 + I)*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*
(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I
+ Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sq
rt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (2*I)*EllipticPi[((-I)*(I + Sqrt[-(c
*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt
[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((S
qrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sq
rt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]
))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(
c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*c^2*d^2*(I + Sqrt[-(c*d)
- e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) -
 e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e]
+ I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d
) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/
Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]
/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*
d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((-1 + I)*EllipticF[ArcSin[Sqrt[
((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sq
rt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqr
t[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]
- (2*I)*EllipticPi[(I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*
d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(
I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I +
 Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqr
t[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])
*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x
))])))/(c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) -
(e*(1 - c*x))/(1 + c*x)))))/(3*c*e)
```

fricas [F] time = 9.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex+d}\,(b\,\text{arsech}\,(cx)+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\,(b\,\text{arsech}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arcsech(c*x) + a), x)
```

maple [A] time = 0.10, size = 415, normalized size = 1.49

$$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsech}(cx)}{3}-\frac{2e^{2}\sqrt{-\frac{(ex+d)c-cd-e}{cxe}}\,x\sqrt{\frac{(ex+d)c-cd+e}{cxe}}\left(2\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cd-\operatorname{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}}\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arcsech(c*x)-2/3*e^2*(-((e*x+
d)*c-c*d-e)/c/x/e)^(1/2)*x*(((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*(2*EllipticF((e*
x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d-EllipticE((e*x+d)
^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d-EllipticPi((e*x+d)^(1
/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*c*
d-EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*e+Elli
pticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*e)*(-((e*x+d
)*c-c*d+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)/(c/(c*d+e))^(1
/2)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{asech}(cx)\right)\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x), x)
```

3.83 $\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=187

$$\frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e}-\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(\right.}{e\sqrt{d+ex}}$$

```
[Out] 2*(a+b*arcsech(c*x))*(e*x+d)^(1/2)/e-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/
2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c
*d+e))^(1/2)/c/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,
2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+
e))^(1/2)/e/(e*x+d)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6288, 944, 719, 419, 932, 168, 538, 537}

$$\frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e}-\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(\right.}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcSech[c*x]))/e - (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (4*b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}\,dx}{e}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} + \left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx + \frac{\left(2bd\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} + \frac{\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{e} - \frac{\left(4b\sqrt{\frac{1}{1+}}\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{\left(4b\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{\left(4b\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{4bd}{}$$

Mathematica [C] time = 11.75, size = 1707, normalized size = 9.13

$$\frac{2\sqrt{d+ex}\,a}{e} + \frac{2b\sqrt{d+ex}\,\text{sech}^{-1}(cx)}{e} - \frac{4ib\sqrt{\frac{cd+\frac{c(1-cx)d}{cx+1}+e-\frac{e(1-cx)}{cx+1}}{\frac{(1-cx)c}{cx+1}+c}}\left(2cd\sqrt{-\frac{i\left(c\sqrt{\frac{1-cx}{cx+1}}\,d+\sqrt{-cd-e}\sqrt{cd-e}-e\sqrt{\frac{1-cx}{cx+1}}\right)}{\left(-icd+ie+\sqrt{-cd-e}\sqrt{cd-e}\right)\left(\sqrt{\frac{1-cx}{cx+1}}-i\right)}}\sqrt{-\frac{i\left(-c\sqrt{\frac{1-cx}{cx+1}}\right.}{\left(icd-ie+\right.}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*a*Sqrt[d + e*x])/e + (2*b*Sqrt[d + e*x]*ArcSech[c*x])/e - ((4*I)*b*Sqrt[
(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1
- c*x))/(1 + c*x))]*(2*c*d*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d
*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqr
t[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[
((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*S
qrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*
(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*EllipticF[ArcS
in[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)
]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))
]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d -
 e])^2] + (c*d - e)*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1
 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 -
c*x)/(1 + c*x)]))]*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1
 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqr
t[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] + (2*I)*c*d*Sqrt[((-I)*(Sqrt[
-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x
```

```
)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqr
t[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*
Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(
c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 -
 c*x)/(1 + c*x))*(EllipticPi[(I*Sqrt[-(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c
*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d -
e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*
(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2
/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - EllipticPi[((-I)*Sqrt[-(c*d) - e
] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt
[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c
*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*
d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2])))/(e*
Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))
/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(
e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/sqrt(e*x + d), x)
```

maple [A] time = 0.10, size = 288, normalized size = 1.54

$$\frac{2a\sqrt{ex+d} + 2b\left(\sqrt{ex+d}\,\operatorname{arcsech}(cx) - \frac{2c\,e^2\sqrt{-\frac{(ex+d)c-cd-e}{cxe}}\,x\sqrt{\frac{(ex+d)c-cd+e}{cxe}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)-\operatorname{EllipticPi}\left(\sqrt{ex+\cdots}\right.\right.}{\sqrt{\frac{c}{cd+e}}\left((ex+d)^2c^2-2(ex+d)c^2d+c^2d^2\cdots\right.}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e*(a*(e*x+d)^(1/2)+b*((e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*(-((e*x+d)*c-c*d
-e)/c/x/e)^(1/2)*x*(((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*(EllipticF((e*x+d)^(1/2)
*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c*
d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2)))*(-((e*x+d)*
c-c*d+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)/(c/(c*d+e))^(1/2
)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\,(cx)}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/sqrt(d + e*x), x)
```

3.84 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=105

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arcsech(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(
1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d
)/(c*d+e))^(1/2)/e/(e*x+d)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6288, 932, 168, 538, 537}

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSech[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{e}$$

$$= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{e}$$

$$= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-c\right.}{e}$$

$$= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,\right.}{e\sqrt{d+ex}}$$

$$= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

Mathematica [C] time = 11.34, size = 1675, normalized size = 15.95

$$-\frac{2a}{e\sqrt{d+ex}} - \frac{2b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex}} + \frac{4ib\left(2\sqrt{-\frac{i\left(c\sqrt{\frac{1-cx}{cx+1}}d+\sqrt{-cd-e}\sqrt{cd-e}-e\sqrt{\frac{1-cx}{cx+1}}\right)}{\left(-icd+ie+\sqrt{-cd-e}\sqrt{cd-e}\right)\left(\sqrt{\frac{1-cx}{cx+1}}-i\right)}}\sqrt{-\frac{i\left(-c\sqrt{\frac{1-cx}{cx+1}}d+\sqrt{-cd-e}\sqrt{cd-e}+e\sqrt{\frac{1-cx}{cx+1}}\right)}{\left(icd-ie+\sqrt{-cd-e}\sqrt{cd-e}\right)\left(\sqrt{\frac{1-cx}{cx+1}}-i\right)}}\left(\frac{1-cx}{cx+1}+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*a)/(e*Sqrt[d + e*x]) - (2*b*ArcSech[c*x])/(e*Sqrt[d + e*x]) + ((4*I)*b*
(2*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x
)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d -
 e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(Sqrt[-(c*d) - e]*
Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)]
))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1
+ c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) -
e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] +
 I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] +
I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] + Sqrt[((Sqrt[-(
c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d)
 - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[1 + (1 - c
*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 +
c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)
/(c*d + e)] + (2*I)*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(
```

```
1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d
) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(
Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1
- c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + S
qrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*(EllipticPi[(I*Sqrt[-
(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sq
rt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/(
(Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (
Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^
2] - EllipticPi[((-I)*Sqrt[-(c*d) - e] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] -
 I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + S
qrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt
[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c
*d) - e] - I*Sqrt[c*d - e])^2])))/(e*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d -
 e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])
*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*Sqrt[(c*d + e
 + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/
(1 + c*x))])
```

fricas [F] time = 7.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arsech}\,(cx)+a)}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(3/2), x)
```

maple [B] time = 0.09, size = 253, normalized size = 2.41

$$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\text{arcsech}(cx)}{\sqrt{ex+d}}-\frac{2c\,e^2\sqrt{-\frac{(ex+d)c-cd-e}{cxe}}\;x\sqrt{\frac{(ex+d)c-cd+e}{cxe}}\;\text{EllipticPi}\left(\sqrt{ex+d}\;\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)\sqrt{-\frac{(ex+d)c-cd+e}{cd-e}}\;\sqrt{-\frac{(ex+d)c-cd-}{cd+e}}}{d\sqrt{\frac{c}{cd+e}}\,\left((ex+d)^2c^2-2(ex+d)c^2d+c^2d^2-e^2\right)}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(3/2),x)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*(-((e*x+d)*c
-c*d-e)/c/x/e)^(1/2)*x*(((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*EllipticPi((e*x+d)^(
1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*(
-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)/d/(c/(
c*d+e))^(1/2)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asech}\,(cx)}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x)**(3/2), x)
```

3.85 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=278

$$-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b}{}$$

```
[Out] -2/3*(a+b*arcsech(c*x))/e/(e*x+d)^(3/2)-4/3*b*c*EllipticE(1/2*(-c*x+1)^(1/2
)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x+d
)^(1/2)/d/(c^2*d^2-e^2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*EllipticPi(1/2*(-c*
x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(
1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d/e/(e*x+d)^(1/2)+4/3*b*e*(1/(c*x+1))^(1/2)*
(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6288, 958, 745, 21, 719, 424, 932, 168, 538, 537}

$$-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d^2
- e^2)*Sqrt[d + e*x]) - (2*(a + b*ArcSech[c*x]))/(3*e*(d + e*x)^(3/2)) - (4
*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[(c*(d + e*x
))/(c*d + e)]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))
/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/
(3*d*e*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3e}$$

$$= -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)}{3e}$$

$$= -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3d} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+c\right.}{}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{}{x\sqrt{1-cx}}}{3de}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)}{}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\,E}{3d\left(c^2d^2-e^2\right)\sqrt{}}$$

Mathematica [C] time = 13.74, size = 4527, normalized size = 16.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-2*a)/(3*e*(d + e*x)^(3/2)) + Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x]*((4*
b*c)/(3*d*(c^2*d^2 - e^2)) - (4*b)/(3*d*(c*d + e)*(d + e*x))) - (2*b*ArcSec
h[c*x])/(3*e*(d + e*x)^(3/2)) - (4*b*((e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[c*(
1 + (1 - c*x)/(1 + c*x))]*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*
x))/(1 + c*x)))/((1 + (1 - c*x)/(1 + c*x))*Sqrt[c + (c*(1 - c*x))/(1 + c*x)
]*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c +
 (c*(1 - c*x))/(1 + c*x))]) - ((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))]*
Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*
(1 - c*x))/(1 + c*x))]*((I*(-(c*d) - e)*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqr
t[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*(EllipticE[I*ArcSinh[
Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))] - EllipticF[I*ArcSin
h[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))]))/((c*d - e)*Sqrt[
c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) +
 (I*c*d*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*
d) - e)*(1 + c*x))]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d
- e)/(-(c*d) - e))])/Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)
*(1 - c*x))/(1 + c*x))] + (I*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d
 - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*EllipticF[I*ArcSinh[Sqrt[(1 - c*
x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))])/Sqrt[c*(1 + (1 - c*x)/(1 + c*x)
)*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))] - (I*c*d*(I + Sqrt[-(c*d) -
```

```
e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) - e
] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] +
I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d)
- e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/Sq
rt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]/S
qrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d
- e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((1 + I)*EllipticF[ArcSin[Sqrt[((S
qrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[
-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-
(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (
2*I)*EllipticPi[((-I)*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*
d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(
I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I +
 Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqr
t[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])
*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x
))]) - (I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*e*(-I + Sqrt[(1 - c*x)/(1 +
c*x)])^2*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1
+ c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c
*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*
x)]))/((I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]
))]*Sqrt[(I*(Sqrt[-(c*d) - e]/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/(
(I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((1
 + I)*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt
[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1
 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d)
 - e] - I*Sqrt[c*d - e])^2] - (2*I)*EllipticPi[((-I)*(I + Sqrt[-(c*d) - e]/
Sqrt[c*d - e]))/(-I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-
(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d
) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d)
 - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I -
Sqrt[-(c*d) - e]/Sqrt[c*d - e])*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e +
 ((c*d - e)*(1 - c*x))/(1 + c*x))]) + (I*c*d*(I + Sqrt[-(c*d) - e]/Sqrt[c*d
 - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[
c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d
- e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d) - e]/Sqrt[c
*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]
)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]/Sqrt[c*d - e
] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I +
 Sqrt[(1 - c*x)/(1 + c*x)]))]*((-1 + I)*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d)
 - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e
] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e]
 + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (2*I)*Ellip
ticPi[(I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*d) - e]/Sqrt[
c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 -
 c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*
x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e]
 - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*Sqrt[c*(1 +
(1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) + (I*(I
+ Sqrt[-(c*d) - e]/Sqrt[c*d - e])*e*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt
[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((S
qrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[
(I*(-(Sqrt[-(c*d) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + S
qrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(
Sqrt[-(c*d) - e]/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c
*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((-1 + I)*Ellipt
icF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(
1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 +
 c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sq
```

```
rt[c*d - e])^2] - (2*I)*EllipticPi[(I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))
/(-I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*
Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt
[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[
c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e
]/Sqrt[c*d - e])*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1
- c*x))/(1 + c*x))]])))/((1 + (1 - c*x)/(1 + c*x))*Sqrt[c + (c*(1 - c*x))/(1
 + c*x)]*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x
))/(c + (c*(1 - c*x))/(1 + c*x))]])))/(3*d*e*(c^2*d^2 - e^2))
```

fricas [F] time = 5.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{e^{3}x^{3}+3\,de^{2}x^{2}+3\,d^{2}ex+d^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*
e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(5/2), x)
```

maple [B] time = 0.12, size = 902, normalized size = 3.24

$$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\mathrm{arcsech}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2c\,e^{2}\sqrt{-\frac{(ex+d)c-cd-e}{cxe}}\;x\sqrt{\frac{(ex+d)c-cd+e}{cxe}}\left(\sqrt{\frac{c}{cd+e}}\,(ex+d)^{2}c^{2}d-\sqrt{-\frac{(ex+d)c-cd-e}{cd+e}}\,\sqrt{-\frac{(ex+d)c-cd+e}{cd-e}}\,\mathrm{EllipticF}\Big(\sqrt{ex\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e*(-1/3/(e*x+d)^(3/2)*a+b*(-1/3/(e*x+d)^(3/2)*arcsech(c*x)+2/3*c*e^2*(-((
e*x+d)*c-c*d-e)/c/x/e)^(1/2)*x*(((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*((c/(c*d+e))
^(1/2)*(e*x+d)^2*c^2*d-(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+
e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e
))^(1/2))*(e*x+d)^(1/2)*c^2*d^2+c^2*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((
e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((
c*d+e)/(c*d-e))^(1/2))*d^2*(e*x+d)^(1/2)-(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)
*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1
/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^
2-2*(c/(c*d+e))^(1/2)*(e*x+d)*c^2*d^2+(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-
((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),
((c*d+e)/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c*d*e-(-((e*x+d)*c-c*d-e)/(c*d+e))^(
1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))
^(1/2),((c*d+e)/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(c/(c*d+e))^(1/2)*c^2*d
^3+(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*El
lipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c
```

```
/(c*d+e))^(1/2))*(e*x+d)^(1/2)*e^2-(c/(c*d+e))^(1/2)*d*e^2)/(e*x+d)^(1/2)/(
c*d-e)/(c*d+e)/(c/(c*d+e))^(1/2)/d^2/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2
-e^2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asech}{\left(c x \right)}}{\left(d + e x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x)**(5/2), x)
```

3.86 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=609

$$-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}+\frac{16bc^2e\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{}}{15d\left(c^2d^2-e^2\right)(d+}$$

```
[Out] -2/5*(a+b*arcsech(c*x))/e/(e*x+d)^(5/2)-16/15*b*c^3*EllipticE(1/2*(-c*x+1)^
(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e
*x+d)^(1/2)/(c^2*d^2-e^2)^2/(c*(e*x+d)/(c*d+e))^(1/2)-4/5*b*c*EllipticE(1/2
*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1
)^(1/2)*(e*x+d)^(1/2)/d^2/(c^2*d^2-e^2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/15*b*c*
EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)^(1/2
)+4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*
(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d^2/e/(e*x+d)^(1/
2)+4/15*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e
^2)/(e*x+d)^(3/2)+16/15*b*c^2*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1
)^(1/2)/(c^2*d^2-e^2)^2/(e*x+d)^(1/2)+4/5*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*d^2-e^2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6288, 958, 745, 835, 844, 719, 424, 419, 21, 932, 168, 538, 537}

$$-\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}+\frac{16bc^2e\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{}}{15d\left(c^2d^2-e^2\right)(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(15*d*(c^2*d^2
 - e^2)*(d + e*x)^(3/2)) + (16*b*c^2*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*S
qrt[1 - c^2*x^2])/(15*(c^2*d^2 - e^2)^2*Sqrt[d + e*x]) + (4*b*e*Sqrt[(1 + c
*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[d +
e*x]) - (2*(a + b*ArcSech[c*x]))/(5*e*(d + e*x)^(5/2)) - (16*b*c^3*Sqrt[(1
+ c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[(c*(d + e*x))/(c*d + e)
]) - (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[(c
*(d + e*x))/(c*d + e)]) + (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c
*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d
+ e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*e*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x(d+ex)^{5/2}\sqrt{1-c^2x^2}}\,dx}{5e}$$

$$= -\frac{2\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{1-c^2x^2}} - \frac{e}{d^2(d+ex)^{3/2}\sqrt{1-c^2x^2}} + \cdots\right.}{5e}$$

$$= -\frac{2\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{5d^2} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\cdots}{5\cdots}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \cdots$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\cdots}{5d^2\left(c^2d^2-e^2\right)\cdots}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\cdots}{5d^2\left(c^2d^2-e^2\right)\cdots}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\cdots}{5d^2\left(c^2d^2-e^2\right)\cdots}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\cdots}{5d^2\left(c^2d^2-e^2\right)\cdots}$$

Mathematica [C] time = 14.96, size = 8675, normalized size = 14.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(7/2),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(7/2), x)
```

maple [B] time = 0.13, size = 1632, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(7/2),x)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arcsech(c*x)+2/15*c*e^2*(-(
(e*x+d)*c-c*d-e)/c/x/e)^(1/2)*x*(((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*(7*(c/(c*d+
e))^(1/2)*(e*x+d)^3*c^4*d^3-6*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)
*c-c*d+e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)
/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c^4*d^4+7*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)
*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/
2),((c*d+e)/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c^4*d^4-3*(-((e*x+d)*c-c*d-e)/(c*
d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/
(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*(e*x+d)^(
3/2)*c^4*d^4-13*(c/(c*d+e))^(1/2)*(e*x+d)^2*c^4*d^4+7*(-((e*x+d)*c-c*d-e)/(
c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c^3*d^3*e-7*(-((e*x+
d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticE((e*
x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c^3*d^3
*e-3*(c/(c*d+e))^(1/2)*(e*x+d)^3*c^2*d*e^2+5*(c/(c*d+e))^(1/2)*(e*x+d)*c^4*
d^5+2*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)
*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*(e*x+d)
^(3/2)*c^2*d^2*e^2-3*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)
/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))
^(1/2))*(e*x+d)^(3/2)*c^2*d^2*e^2+6*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((
e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1
/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^2*d^2*e^2
+5*(c/(c*d+e))^(1/2)*(e*x+d)^2*c^2*d^2*e^2+(c/(c*d+e))^(1/2)*c^4*d^6-3*(-((
e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c*d
*e^3+3*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2
)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*(e*x+d
)^(3/2)*c*d*e^3-8*(c/(c*d+e))^(1/2)*(e*x+d)*c^2*d^3*e^2-3*(-((e*x+d)*c-c*d-
e)/(c*d+e))^(1/2)*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/
2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*(e*
x+d)^(3/2)*e^4-2*(c/(c*d+e))^(1/2)*c^2*d^4*e^2+3*(c/(c*d+e))^(1/2)*(e*x+d)*
d*e^4+(c/(c*d+e))^(1/2)*d^2*e^4)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2
)/d^3/(c/(c*d+e))^(1/2)/(c*d+e)/(c*d-e)/(e*x+d)^(3/2)/(c^2*d^2-e^2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(7/2),x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(7/2),x)
[Out] Timed out
```

3.87 $\int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=87

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\mathrm{Int}\left(\frac{(d+ex)^{m+1}}{x\sqrt{1-c^2x^2}},x\right)}{e(m+1)}+\frac{(d+ex)^{m+1}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e(m+1)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsech(c*x))/e/(1+m)+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
Unintegrable((e*x+d)^(1+m)/x/(-c^2*x^2+1)^(1/2),x)/e/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcSech[c*x]),x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcSech[c*x]))/(e*(1 + m)) + (b*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Defer[Int][(d + e*x)^(1 + m)/(x*Sqrt[1 - c^2*x^2]), x])/(
e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx=\frac{(d+ex)^{1+m}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e(1+m)}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex)^{1+m}}{x\sqrt{1-c^2x^2}}\,dx}{e(1+m)}$$

Mathematica [A] time = 1.96, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSech[c*x]),x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 1.19, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\,\mathrm{arsech}\,(cx)+a)(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)*(e*x + d)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{arsech}\,(cx)+a)(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*(e*x + d)^m, x)
```

maple [A] time = 2.10, size = 0, normalized size = 0.00

$$\int (ex+d)^m (a+b\,\mathrm{arcsech}\,(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arcsech(c*x)),x)
```

```
[Out] int((e*x+d)^m*(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{(ex+d)(ex+d)^m \log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)-(ex+d)(ex+d)^m \log(x)}{e(m+1)} - \int \frac{\left(c^2e(m+1)x^3\log(c)-\left(e(m+1\right.\right.}{c^2e(m+1)x^3-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] b*(((e*x + d)*(e*x + d)^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - (e*x + d)
*(e*x + d)^m*log(x))/(e*(m + 1)) - integrate((c^2*e*(m + 1)*x^3*log(c) - (e
*(m + 1)*log(c) - e)*x + d)*(e*x + d)^m/(c^2*e*(m + 1)*x^3 - e*(m + 1)*x),
x) + integrate((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 + (c^2*
e*(m + 1)*x^2 - e*(m + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1) - e*(m + 1)), x)) +
 (e*x + d)^(m + 1)*a/(e*(m + 1))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^m,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{asech}\,(cx))\,(d+ex)^m\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x)**m, x)
```

3.88 $\int x^4 \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=229

$$\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bex^5\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{42c^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(42c^2d+2\right.}{560c^7}$$

```
[Out] 1/5*d*x^5*(a+b*arcsech(c*x))+1/7*e*x^7*(a+b*arcsech(c*x))+1/560*b*(42*c^2*d
+25*e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7-1/560*b*(42*c^2*d+25
*e)*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6-1/840*b*(42*c^
2*d+25*e)*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/42*b
*e*x^5*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] time = 0.13, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 6301, 12, 459, 321, 216}

$$\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bx^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(42c^2d+25e\right)}{840c^4}-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(42*c^2*d + 25*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(560*c^6) - (b*(42*c^2*d + 25*e)*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/(840*c^4) - (b*e*x^5*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/(42*c^2) + (d*x^5*(a + b*ArcSech[c*x]))/5 + (e*x^7*(a +
b*ArcSech[c*x]))/7 + (b*(42*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x
]*ArcSin[c*x])/(560*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^4\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)dx=\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right.$$

$$=\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{35}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$=-\frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2}+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+bs\right.$$

$$=-\frac{b\left(42c^2d+25e\right)x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4}-\frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-}}{42c^2}$$

$$=-\frac{b\left(42c^2d+25e\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{560c^6}-\frac{b\left(42c^2d+25e\right)x^3\sqrt{\frac{1}{1+cx}}}{840c^4}$$

$$=-\frac{b\left(42c^2d+25e\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{560c^6}-\frac{b\left(42c^2d+25e\right)x^3\sqrt{\frac{1}{1+cx}}}{840c^4}$$

Mathematica [C] time = 0.34, size = 162, normalized size = 0.71

$$\frac{48ac^7x^5\left(7d+5ex^2\right)+48bc^7x^5\text{sech}^{-1}(cx)\left(7d+5ex^2\right)+3ib\left(42c^2d+25e\right)\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2icx\right)-bcx\sqrt{\frac{1}{c}}}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(
75*e + 2*c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*
(7*d + 5*e*x^2)*ArcSech[c*x] + (3*I)*b*(42*c^2*d + 25*e)*Log[(-2*I)*c*x + 2
*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(1680*c^7)
```

fricas [A] time = 1.01, size = 259, normalized size = 1.13

$$240\,ac^7ex^7+336\,ac^7dx^5-6\left(42\,bc^2d+25\,be\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-48\left(7\,bc^7d+5\,bc^7e\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 - 6*(42*b*c^2*d + 25*b*e)*arctan(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 48*(7*b*c^7*d + 5*b*c^7*e
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 48*(5*b*c^7*e*x^7 + 7*b
*c^7*d*x^5 - 7*b*c^7*d - 5*b*c^7*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
 + 1)/(c*x)) - (40*b*c^6*e*x^6 + 2*(42*b*c^6*d + 25*b*c^4*e)*x^4 + 3*(42*b*
c^4*d + 25*b*c^2*e)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)(b \operatorname{arsech}(cx) + a)x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^4, x)
```

maple [A] time = 0.07, size = 224, normalized size = 0.98

$$\frac{\frac{a\left(\frac{1}{7}e\,c^7x^7+\frac{1}{5}c^7x^5d\right)}{c^2}+\frac{b\left(\frac{\operatorname{arcsech}(cx)e\,c^7x^7}{7}+\frac{\operatorname{arcsech}(cx)c^7x^5d}{5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\,\sqrt{\frac{cx+1}{cx}}\left(-40c^5x^5e\sqrt{-c^2x^2+1}-84c^5x^3d\sqrt{-c^2x^2+1}-50e\,c^3x^3\sqrt{-c^2x^2+1}-126\sqrt{-c^2x^2+1}\right.}{1680\sqrt{-c^2x^2+1}}\right.}{c^2}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^5*(a/c^2*(1/7*e*c^7*x^7+1/5*c^7*x^5*d)+b/c^2*(1/7*arcsech(c*x)*e*c^7*x^
7+1/5*arcsech(c*x)*c^7*x^5*d+1/1680*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^
(1/2)*(-40*c^5*x^5*e*(-c^2*x^2+1)^(1/2)-84*c^5*x^3*d*(-c^2*x^2+1)^(1/2)-50*
e*c^3*x^3*(-c^2*x^2+1)^(1/2)-126*(-c^2*x^2+1)^(1/2)*c^3*x*d+126*arcsin(c*x)
*c^2*d-75*e*c*x*(-c^2*x^2+1)^(1/2)+75*e*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.42, size = 244, normalized size = 1.07

$$\frac{1}{7}aex^7+\frac{1}{5}adx^5+\frac{1}{40}\left(8x^5\operatorname{arsech}(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)bd+\frac{1}{336}\left(48x^7\operatorname{arsech}(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1
)^(3/2) + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2
*x^2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*d + 1/336*(48
*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*(1/(c^2*x^2) - 1)^(3/
2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2
) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(sqrt(1/(c^2*x^2) - 1)
)/c^6)/c)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^4*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{asech}(cx)) \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**4*(a + b*asech(c*x))*(d + e*x**2), x)
```

3.89 $\int x^2 \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=174

$$\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bex^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{20c^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(20c^2d+9\right.}{120c^5}$$

[Out] 1/3*d*x^3*(a+b*arcsech(c*x))+1/5*e*x^5*(a+b*arcsech(c*x))+1/120*b*(20*c^2*d +9*e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5-1/120*b*(20*c^2*d+9*e)*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/20*b*e*x^3*(1/ (c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2

Rubi [A] time = 0.10, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6301, 12, 459, 321, 216}

$$\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(20c^2d+9e\right)}{120c^4}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(20*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]
)/(120*c^4) - (b*e*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]
)/(20*c^2) + (d*x^3*(a + b*ArcSech[c*x]))/3 + (e*x^5*(a + b*ArcSech[c*x]))/
5 + (b*(20*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(12
0*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right.$$

$$= \frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{15}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$= -\frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2}+\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+bs\right.$$

$$= -\frac{b\left(20c^2d+9e\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2}}{20c^2}$$

$$= -\frac{b\left(20c^2d+9e\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2}}{20c^2}$$

Mathematica [C] time = 0.22, size = 144, normalized size = 0.83

$$\frac{8ac^5x^3\left(5d+3ex^2\right)+8bc^5x^3\text{sech}^{-1}(cx)\left(5d+3ex^2\right)-bcx\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2\left(20d+6ex^2\right)+9e\right)+ib\left(20c^2d+9e\right)}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (8*a*c^5*x^3*(5*d + 3*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(9
*e + c^2*(20*d + 6*e*x^2)) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcSech[c*x] + I*b
*(20*c^2*d + 9*e)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/
(120*c^5)
```

fricas [B] time = 1.02, size = 238, normalized size = 1.37

$$\frac{24\,ac^5ex^5+40\,ac^5dx^3-2\left(20\,bc^2d+9\,be\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-8\left(5\,bc^5d+3\,bc^5e\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)+8\left(3\,b\right.}{120\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 - 2*(20*b*c^2*d + 9*b*e)*arctan((c*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 8*(5*b*c^5*d + 3*b*c^5*e)*log
```

```
((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d
*x^3 - 5*b*c^5*d - 3*b*c^5*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x)) - (6*b*c^4*e*x^4 + (20*b*c^4*d + 9*b*c^2*e)*x^2)*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)))/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)(b\operatorname{arsech}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

maple [A] time = 0.06, size = 182, normalized size = 1.05

$$\frac{\frac{a\left(\frac{1}{5}c^5x^5e+\frac{1}{3}c^5x^3d\right)}{c^2}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^5x^5e}{5}+\frac{\operatorname{arcsech}(cx)c^5x^3d}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\,\sqrt{\frac{cx+1}{cx}}\left(-6ec^3x^3\sqrt{-c^2x^2+1}-20\sqrt{-c^2x^2+1}\,c^3xd+20\arcsin(cx)c^2d-9ecx\sqrt{-c^2x^2+1}+9e\right.}{120\sqrt{-c^2x^2+1}}\right.}{c^2}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^3*(a/c^2*(1/5*c^5*x^5*e+1/3*c^5*x^3*d)+b/c^2*(1/5*arcsech(c*x)*c^5*x^5*
e+1/3*arcsech(c*x)*c^5*x^3*d+1/120*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(
1/2)*(-6*e*c^3*x^3*(-c^2*x^2+1)^(1/2)-20*(-c^2*x^2+1)^(1/2)*c^3*x*d+20*arcs
in(c*x)*c^2*d-9*e*c*x*(-c^2*x^2+1)^(1/2)+9*e*arcsin(c*x))/(-c^2*x^2+1)^(1/2
)))
```

maxima [A] time = 0.42, size = 182, normalized size = 1.05

$$\frac{1}{5}aex^5+\frac{1}{3}adx^3+\frac{1}{6}\left(2x^3\operatorname{arsech}(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bd+\frac{1}{40}\left(8x^5\operatorname{arsech}(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\cdots}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1
)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*d
 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt(1/(c^2*x
^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) + 3*arc
tan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asech}(cx)) \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2), x)
```

3.90 $\int \left(d+ex^2\right)\left(a+b\,\mathbf{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=112

$$dx\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\,\text{sech}^{-1}(cx)\right)-\frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{6c^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(6c^2d+e\right)\sin$$

[Out] d*x*(a+b*arcsech(c*x))+1/3*e*x^3*(a+b*arcsech(c*x))+1/6*b*(6*c^2*d+e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/6*b*e*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2

Rubi [A] time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6291, 12, 388, 216}

$$dx\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(6c^2d+e\right)\sin^{-1}(cx)}{6c^3}-\frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*e*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) + d*x
*(a + b*ArcSech[c*x]) + (e*x^3*(a + b*ArcSech[c*x]))/3 + (b*(6*c^2*d + e)*S
qrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 6291

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)dx=dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}(cx)\right)+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{}{3}$$

$$=dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int$$

$$=-\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2}+dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}\right.$$

$$=-\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2}+dx\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{sech}^{-1}\right.$$

Mathematica [C] time = 0.37, size = 169, normalized size = 1.51

$$adx+\frac{1}{3}aex^3+\frac{ibe\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2icx\right)}{6c^3}-\frac{bd\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sin^{-1}(cx)}{c(cx-1)}+be\sqrt{\frac{1-cx}{cx+1}}\left(-\frac{x}{6c^2}-\frac{x^2}{6c}\right)+bdx\text{sec}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*x/c^2 - x^2/(6*c)
) + b*d*x*ArcSech[c*x] + (b*e*x^3*ArcSech[c*x])/3 - (b*d*Sqrt[(1 - c*x)/(1
+ c*x)]*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*(-1 + c*x)) + ((I/6)*b*e*Log[(-2*
I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^3
```

fricas [B] time = 0.98, size = 209, normalized size = 1.87

$$\frac{2\,ac^3ex^3-bc^2ex^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+6\,ac^3dx-2\left(6\,bc^2d+be\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-2\left(3\,bc^3d+bc^3e\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-}{x}\right.}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 - b*c^2*e*x^2*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 6*a*c^3*d
*x - 2*(6*b*c^2*d + b*e)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c
*x)) - 2*(3*b*c^3*d + b*c^3*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)
/x) + 2*(b*c^3*e*x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*log((c*x*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex^2+d\right)\left(b\,\text{arsech}\,(cx)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

maple [A] time = 0.06, size = 135, normalized size = 1.21

$$\frac{\frac{a\left(\frac{1}{3}e\,c^3x^3+x\,c^3d\right)}{c^2}+\frac{b\left(\frac{\text{arcsech}(cx)e\,c^3x^3}{3}+\text{arcsech}(cx)c^3dx+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(6\arcsin(cx)c^2d-ecx\sqrt{-c^2x^2+1}+e\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c*(a/c^2*(1/3*e*c^3*x^3+x*c^3*d)+b/c^2*(1/3*arcsech(c*x)*e*c^3*x^3+arcsec
h(c*x)*c^3*d*x+1/6*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(6*arcsin(c
*x)*c^2*d-e*c*x*(-c^2*x^2+1)^(1/2)+e*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.41, size = 107, normalized size = 0.96

$$\frac{1}{3}aex^3+\frac{1}{6}\left(2x^3\operatorname{arsech}(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be+adx+\frac{\left(cx\operatorname{arsech}(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2
*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e + a*d*x + (c*
x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{asech}{\left(c x \right)}\right) \left(d + e x^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2), x)
```

3.91 $\int \frac{(d+ex^2)(a+b\,\mathbf{sech}^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=96

$$-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{x}+ex\left(a+b\text{sech}^{-1}(cx)\right)+\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x}+\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c}$$

```
[Out] -d*(a+b*arcsech(c*x))/x+e*x*(a+b*arcsech(c*x))+b*e*arcsin(c*x)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)/c+b*d*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2
)/x
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {14, 6301, 451, 216}

$$-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{x}+ex\left(a+b\text{sech}^{-1}(cx)\right)+\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x}+\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (d*(a + b*Ar
cSech[c*x]))/x + e*x*(a + b*ArcSech[c*x]) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*ArcSin[c*x])/c
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\text{sech}^{-1}(cx))}{x^2}\,dx = -\frac{d(a+b\text{sech}^{-1}(cx))}{x} + ex(a+b\text{sech}^{-1}(cx)) + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int$$
$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d(a+b\text{sech}^{-1}(cx))}{x} + ex(a+b\text{sech}^{-1}(cx$$
$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d(a+b\text{sech}^{-1}(cx))}{x} + ex(a+b\text{sech}^{-1}(cx$$

Mathematica [A] time = 0.26, size = 107, normalized size = 1.11

$$-\frac{ad}{x}+aex-\frac{be\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sin^{-1}(cx)}{c(cx-1)}+bd\left(c+\frac{1}{x}\right)\sqrt{\frac{1-cx}{cx+1}}-\frac{bd\text{sech}^{-1}(cx)}{x}+bex\text{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x + b*d*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*d*ArcS
ech[c*x])/x + b*e*x*ArcSech[c*x] - (b*e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 -
c^2*x^2]*ArcSin[c*x])/(c*(-1 + c*x))
```

fricas [B] time = 0.53, size = 182, normalized size = 1.90

$$\frac{bc^2dx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + acex^2 - 2\,bex\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - acd + (bcd-bce)x\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) + \left(bcex^2 - bcd\right.}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (b*c^2*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + a*c*e*x^2 - 2*b*e*x*arctan((c*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - a*c*d + (b*c*d - b*c*e)*x*log
((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 - b*c*d + (b*c*d
- b*c*e)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)(b\,\text{arsech}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

maple [A] time = 0.07, size = 114, normalized size = 1.19

$$c\left(\frac{a\left(cxe-\frac{cd}{x}\right)}{c^2}+\frac{b\left(\text{arcsech}(cx)\,cxe-\frac{\text{arcsech}(cx)cd}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(\sqrt{-c^2x^2+1}\,c^2d+\arcsin(cx)cxe\right)}{\sqrt{-c^2x^2+1}}\right)}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x)
```

```
[Out] c*(a/c^2*(c*x*e-c*d/x)+b/c^2*(arcsech(c*x)*c*x*e-arcsech(c*x)*c*d/x+(-(c*x-
1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*((-c^2*x^2+1)^(1/2)*c^2*d+arcsin(c*x)*c*x
*e)/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.32, size = 66, normalized size = 0.69

$$\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\operatorname{arsech}(cx)}{x}\right)bd+aex+\frac{\left(cx\operatorname{arsech}(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)be}{c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d + a*e*x + (c*x*arcsech(c*x)
- arctan(sqrt(1/(c^2*x^2) - 1)))*b*e/c - a*d/x
```

mupad [B] time = 1.81, size = 98, normalized size = 1.02

$$aex-\frac{ad}{x}+bcd\left(\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}-\frac{\operatorname{acosh}\left(\frac{1}{cx}\right)}{cx}\right)+\frac{be\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}\right)}{c}+bex\operatorname{acosh}\left(\frac{1}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^2,x)
```

```
[Out] a*e*x - (a*d)/x + b*c*d*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - acosh(1/
(c*x))/(c*x)) + (b*e*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))))/c +
 b*e*x*acosh(1/(c*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{asech}(cx))\left(d+ex^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**2, x)
```

3.92 $\int \frac{(d+ex^2)(a+b\,\mathbf{sech}^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=126

$$-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(2c^2d+9e\right)}{9x}+\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c}}{9x^3}$$

```
[Out] -1/3*d*(a+b*arcsech(c*x))/x^3-e*(a+b*arcsech(c*x))/x+1/9*b*d*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+1/9*b*(2*c^2*d+9*e)*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6301, 12, 453, 264}

$$-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(2c^2d+9e\right)}{9x}+\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (b*(2*
c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x) -
(d*(a + b*ArcSech[c*x]))/(3*x^3) - (e*(a + b*ArcSech[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
```

```
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-}{3x^4}$$

$$= -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x} + \frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-}{x^4}$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} - \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x} +$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} + \frac{b\left(2c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x} - \frac{d(}{}$$

Mathematica [A] time = 0.12, size = 76, normalized size = 0.60

$$\frac{-3a\left(d+3ex^2\right)+b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(2c^2dx^2+d+9ex^2\right)-3b\text{sech}^{-1}(cx)\left(d+3ex^2\right)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (-3*a*(d + 3*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x^
2 + 9*e*x^2) - 3*b*(d + 3*e*x^2)*ArcSech[c*x])/(9*x^3)
```

fricas [A] time = 0.61, size = 106, normalized size = 0.84

$$-\frac{9\,aex^2+3\,ad+3\left(3\,bex^2+bd\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(bcdx+\left(2\,bc^3d+9\,bce\right)x^3\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log((c*x*sqrt(-(c^2*x^2 - 1)/
(c^2*x^2)) + 1)/(c*x)) - (b*c*d*x + (2*b*c^3*d + 9*b*c*e)*x^3)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(b\,\text{arsech}\,(cx)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)
```

maple [A] time = 0.07, size = 123, normalized size = 0.98

$$c^{3}\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^{3}}\right)}{c^{2}}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)e}{cx}-\frac{\mathrm{arcsech}(cx)d}{3c\,x^{3}}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(2c^{4}d\,x^{2}+9c^{2}x^{2}e+c^{2}d\right)}{9c^{2}x^{2}}\right)}{c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x)
```

```
[Out] c^3*(a/c^2*(-e/c/x-1/3/c*d/x^3)+b/c^2*(-arcsech(c*x)*e/c/x-1/3*arcsech(c*x)
/c*d/x^3+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2)*(2*c^4*d*x^2+
9*c^2*e*x^2+c^2*d)))
```

maxima [A] time = 0.31, size = 91, normalized size = 0.72

$$\left(c\sqrt{\frac{1}{c^{2}x^{2}}-1}-\frac{\mathrm{arsech}\,(cx)}{x}\right)be+\frac{1}{9}\,bd\left(\frac{c^{4}\left(\frac{1}{c^{2}x^{2}}-1\right)^{\frac{3}{2}}+3\,c^{4}\sqrt{\frac{1}{c^{2}x^{2}}-1}}{c}-\frac{3\,\mathrm{arsech}\,(cx)}{x^{3}}\right)-\frac{ae}{x}-\frac{ad}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2)
 - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - a*e/x
- 1/3*a*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^{2}+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{asech}\,(cx)\right)\left(d+ex^{2}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**4, x)
```

3.93 $\int \frac{(d+ex^2)(a+b\,\mathbf{sech}^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=183

$$-\frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3}+\frac{2bc^2\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}\,\left(12c^2d+25e\right)}{225x}+\frac{b\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{1-\cdots}}{225\cdots}$$

```
[Out] -1/5*d*(a+b*arcsech(c*x))/x^5-1/3*e*(a+b*arcsech(c*x))/x^3+1/25*b*d*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+1/225*b*(12*c^2*d+25*e)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/225*b*c^2*(12*c^2*d+2
5*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = $\{14, 6301, 12, 453, 271, 264\}$

$$-\frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3}+\frac{2bc^2\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}\,\left(12c^2d+25e\right)}{225x}+\frac{b\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{1-\cdots}}{225\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (b*(1
2*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*
x^3) + (2*b*c^2*(12*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
 - c^2*x^2])/(225*x) - (d*(a + b*ArcSech[c*x]))/(5*x^5) - (e*(a + b*ArcSech
[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -$$

$$= -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} + \frac{1}{15}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} - \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{b\left(12c^2d+25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3}$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{b\left(12c^2d+25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3}$$

Mathematica [A] time = 0.18, size = 101, normalized size = 0.55

$$\frac{-15a\left(3d+5ex^2\right)+b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(25ex^2\left(2c^2x^2+1\right)+3d\left(8c^4x^4+4c^2x^2+3\right)\right)-15b\text{sech}^{-1}(cx)\left(3d+5ex^2\right.}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d + 5*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(25*e*x^2*(1
 + 2*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*Arc
Sech[c*x])/(225*x^5)
```

fricas [A] time = 0.63, size = 128, normalized size = 0.70

$$-\frac{75\,aex^2+45\,ad+15\left(5\,bex^2+3\,bd\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(2\left(12\,bc^5d+25\,bc^3e\right)x^5+9\,bcdx+\left(12\,bc^3d+25\,b\right.\right.}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*(12*b*c^5*d + 25*b*c^3*e)*x^5 + 9*b*c*d*
x + (12*b*c^3*d + 25*b*c*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

maple [A] time = 0.07, size = 142, normalized size = 0.78

$$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)e}{3c^3x^3}-\frac{\mathrm{arcsech}(cx)d}{5c^3x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2x^2e+9c^2d\right)}{225c^4x^4}\right)}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x)
```

```
[Out] c^5*(a/c^2*(-1/3*e/c^3/x^3-1/5/c^3*d/x^5)+b/c^2*(-1/3*arcsech(c*x)*e/c^3/x^
3-1/5*arcsech(c*x)/c^3*d/x^5+1/225*(-(c*x-1)/c/x)^(1/2)/c^4/x^4*((c*x+1)/c/
x)^(1/2)*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+25*c^2*e*x^2+9*c^2*d)))
```

maxima [A] time = 0.32, size = 132, normalized size = 0.72

$$\frac{1}{75}\,bd\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\,\mathrm{arsech}\,(cx)}{x^5}\right)+\frac{1}{9}\,be\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*d*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) +
 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) + 1/9*b*e*((c^4*(1/
(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3)
 - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asech}\,(cx))\left(d+ex^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**6,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**6, x)
```

3.94 $\int \frac{(d+ex^2)(a+b\,\mathrm{sech}^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=238

$$-\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5x^5}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(30c^2d+49e\right)}{1225x^5}+\frac{4bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{}}{367}$$

```
[Out] -1/7*d*(a+b*arcsech(c*x))/x^7-1/5*e*(a+b*arcsech(c*x))/x^5+1/49*b*d*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^7+1/1225*b*(30*c^2*d+49*e)*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+4/3675*b*c^2*(30*c^2*d
+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+8/3675*b*c^4*
(30*c^2*d+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.12, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 6301, 12, 453, 271, 264}

$$-\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5x^5}+\frac{8bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(30c^2d+49e\right)}{3675x}+\frac{4bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(49*x^7) + (b*(3
0*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(1225
*x^5) + (4*b*c^2*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[
1 - c^2*x^2])/(3675*x^3) + (8*b*c^4*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3675*x) - (d*(a + b*ArcSech[c*x]))/(7*x^7
) - (e*(a + b*ArcSech[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\,\text{sech}^{-1}(cx))}{x^8}\,dx = -\frac{d(a+b\,\text{sech}^{-1}(cx))}{7x^7}-\frac{e(a+b\,\text{sech}^{-1}(cx))}{5x^5}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-5}{35x^8}$$

$$= -\frac{d(a+b\,\text{sech}^{-1}(cx))}{7x^7}-\frac{e(a+b\,\text{sech}^{-1}(cx))}{5x^5}+\frac{1}{35}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7}-\frac{d(a+b\,\text{sech}^{-1}(cx))}{7x^7}-\frac{e(a+b\,\text{sech}^{-1}(cx))}{5x^5}+$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7}+\frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5}-$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7}+\frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5}+$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7}+\frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5}+$$

Mathematica [A] time = 0.22, size = 117, normalized size = 0.49

$$\frac{-105a(5d+7ex^2)+b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(49ex^2(8c^4x^4+4c^2x^2+3)+15d(16c^6x^6+8c^4x^4+6c^2x^2+5)\right)-105b\,\text{sec}}{3675x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (-105*a*(5*d + 7*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(49*e*x^2*(
3 + 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6))
 - 105*b*(5*d + 7*e*x^2)*ArcSech[c*x])/(3675*x^7)
```

fricas [A] time = 0.58, size = 149, normalized size = 0.63

$$-\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(8\left(30\,bc^7d+49\,bc^5e\right)x^7+4\left(30\,bc^5d+49\,bc^3e\right)x^5\right.}{3675\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*log((c*x*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*(30*b*c^7*d + 49*b*c^5*e)*x^7 + 4*(3
0*b*c^5*d + 49*b*c^3*e)*x^5 + 75*b*c*d*x + 3*(30*b*c^3*d + 49*b*c*e)*x^3)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^8, x)
```

maple [A] time = 0.08, size = 160, normalized size = 0.67

$$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)d}{7c^5x^7}-\frac{\mathrm{arcsech}(cx)e}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x^2+1\right.}{3675c^6x^6}\right.}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x)
```

```
[Out] c^7*(a/c^2*(-1/7/c^5*d/x^7-1/5*e/c^5/x^5)+b/c^2*(-1/7*arcsech(c*x)/c^5*d/x^
7-1/5*arcsech(c*x)*e/c^5/x^5+1/3675*(-(c*x-1)/c/x)^(1/2)/c^6/x^6*((c*x+1)/c
/x)^(1/2)*(240*c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d
*x^2+147*c^2*e*x^2+75*c^2*d)))
```

maxima [A] time = 0.32, size = 165, normalized size = 0.69

$$\frac{1}{245}\,bd\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}+21\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+35\,c^8\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{35\,\mathrm{arsech}\,(cx)}{x^7}\right)+\frac{1}{75}\,be\left(\frac{3\,c^6(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/2)
+ 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35*arc
sech(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*
x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) -
1/5*a*e/x^5 - 1/7*a*d/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{asech}\,(cx))\left(d + ex^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**8,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**8, x)
```

3.95 $\int x^5\left(d+ex^2\right)\left(a+b\,\mathbf{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=232

$$\frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}\left(4c^2d+9e\right)}{120c^8}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{}$$

```
[Out] 1/6*d*x^6*(a+b*arcsech(c*x))+1/8*e*x^8*(a+b*arcsech(c*x))+1/72*b*(8*c^2*d+9
*e)*(-c^2*x^2+1)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/120*b*(4*c^2*d
+9*e)*(-c^2*x^2+1)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8+1/56*b*e*(-c^2
*x^2+1)^(7/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/24*b*(4*c^2*d+3*e)*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^8
```

Rubi [A] time = 0.16, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6301, 12, 446, 77}

$$\frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}\left(4c^2d+9e\right)}{120c^8}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(4*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(
24*c^8) + (b*(8*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(72*c^8) - (b*(4*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
(1 - c^2*x^2)^(5/2))/(120*c^8) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1
 - c^2*x^2)^(7/2))/(56*c^8) + (d*x^6*(a + b*ArcSech[c*x]))/6 + (e*x^8*(a +
b*ArcSech[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5 \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)dx = \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right.$$

$$= \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{24}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$= \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{48}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$= \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{48}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$= -\frac{b\left(4c^2d+3e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8}+\frac{b\left(8c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{72c^8}$$

Mathematica [A] time = 0.22, size = 126, normalized size = 0.54

$$\frac{1}{24}ax^6\left(4d+3ex^2\right)-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^6\left(84dx^4+45ex^6\right)+2c^4\left(56dx^2+27ex^4\right)+8c^2\left(28d+9ex^2\right)+144e\right)}{2520c^8}+\frac{1}{24}b$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^6*(4*d + 3*e*x^2))/24 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(144*e
+ 8*c^2*(28*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45
*e*x^6)))/(2520*c^8) + (b*x^6*(4*d + 3*e*x^2)*ArcSech[c*x])/24
```

fricas [A] time = 0.66, size = 168, normalized size = 0.72

$$\frac{315\,ac^7ex^8+420\,ac^7dx^6+105\left(3\,bc^7ex^8+4\,bc^7dx^6\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(45\,bc^6ex^7+6\left(14\,bc^6d+9\,bc^4e\right)x^5+8\right.}{2520\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e*x^8 + 420*a*c^7*d*x^6 + 105*(3*b*c^7*e*x^8 + 4*b*c^7*d*
x^6)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (45*b*c^6*e*x^7
+ 6*(14*b*c^6*d + 9*b*c^4*e)*x^5 + 8*(14*b*c^4*d + 9*b*c^2*e)*x^3 + 16*(14*
b*c^2*d + 9*b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex^2+d\right)\left(b\,\text{arsech}\left(cx\right)+a\right)x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

maple [A] time = 0.07, size = 150, normalized size = 0.65

$$\frac{\frac{a\left(\frac{1}{8}e\,c^8x^8+\frac{1}{6}c^8d\,x^6\right)}{c^2}+\frac{b\left(\frac{\operatorname{arcsech}(cx)e\,c^8x^8}{8}+\frac{\operatorname{arcsech}(cx)c^8x^6d}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\,\sqrt{\frac{cx+1}{cx}}\,\left(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2x^2e+224c^2d+144e\right)}{2520}\right)}{c^2}}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^6*(a/c^2*(1/8*e*c^8*x^8+1/6*c^8*d*x^6)+b/c^2*(1/8*arcsech(c*x)*e*c^8*x^
8+1/6*arcsech(c*x)*c^8*x^6*d-1/2520*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^
(1/2)*(45*c^6*e*x^6+84*c^6*d*x^4+54*c^4*e*x^4+112*c^4*d*x^2+72*c^2*e*x^2+22
4*c^2*d+144*e)))
```

maxima [A] time = 0.32, size = 177, normalized size = 0.76

$$\frac{1}{8}\,aex^8+\frac{1}{6}\,adx^6+\frac{1}{90}\left(15\,x^6\operatorname{arsech}(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)bd+\frac{1}{280}\left(35\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*
x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2
) - 1))/c^5)*b*d + 1/280*(35*x^8*arcsech(c*x) + (5*c^6*x^7*(1/(c^2*x^2) - 1
)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) - 1)
^(3/2) - 35*x*sqrt(1/(c^2*x^2) - 1))/c^7)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 15.66, size = 228, normalized size = 0.98

$$\begin{cases}\frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{asech}(cx)}{6}+\frac{bex^8\operatorname{asech}(cx)}{8}-\frac{bdx^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{bex^6\sqrt{-c^2x^2+1}}{56c^2}-\frac{2bdx^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{3bex^4\sqrt{-c^2x^2+1}}{140c^4}-\frac{4bd\sqrt{-}}{4} \\ (a+\infty b)\left(\frac{dx^6}{6}+\frac{ex^8}{8}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*asech(c*x)/6 + b*e*x**8*asech
(c*x)/8 - b*d*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*e*x**6*sqrt(-c**2*x**
2 + 1)/(56*c**2) - 2*b*d*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e*x**4*s
qrt(-c**2*x**2 + 1)/(140*c**4) - 4*b*d*sqrt(-c**2*x**2 + 1)/(45*c**6) - b*e
*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e*sqrt(-c**2*x**2 + 1)/(35*c**8)
, Ne(c, 0)), ((a + oo*b)*(d*x**6/6 + e*x**8/8), True))
```

3.96 $\int x^3\left(d+ex^2\right)\left(a+b\,\mathbf{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=180

$$\frac{1}{4}dx^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}\left(3c^2d+4e\right)}{36c^6}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{}$$

```
[Out] 1/4*d*x^4*(a+b*arcsech(c*x))+1/6*e*x^6*(a+b*arcsech(c*x))+1/36*b*(3*c^2*d+4
*e)*(-c^2*x^2+1)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/30*b*e*(-c^2*x
^2+1)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/12*b*(3*c^2*d+2*e)*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6
```

Rubi [A] time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6301, 12, 446, 77}

$$\frac{1}{4}dx^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}\left(3c^2d+4e\right)}{36c^6}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(3*c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(
12*c^6) + (b*(3*c^2*d + 4*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(36*c^6) - (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^
(5/2))/(30*c^6) + (d*x^4*(a + b*ArcSech[c*x]))/4 + (e*x^6*(a + b*ArcSech[c*
x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)dx = \frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right.$$

$$= \frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{12}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$= \frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{24}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$= \frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{24}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.$$

$$= -\frac{b\left(3c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^6}+\frac{b\left(3c^2d+4e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{36c^6}$$

Mathematica [A] time = 0.19, size = 106, normalized size = 0.59

$$\frac{1}{180}\left(15ax^4\left(3d+2ex^2\right)-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(3c^4\left(5dx^2+2ex^4\right)+c^2\left(30d+8ex^2\right)+16e\right)}{c^6}+15bx^4\text{sech}^{-1}(cx)\left(3d+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (15*a*x^4*(3*d + 2*e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(16*e +
c^2*(30*d + 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^6 + 15*b*x^4*(3*d + 2*
e*x^2)*ArcSech[c*x])/180
```

fricas [A] time = 0.50, size = 147, normalized size = 0.82

$$\frac{30\,ac^5ex^6+45\,ac^5dx^4+15\left(2\,bc^5ex^6+3\,bc^5dx^4\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(6\,bc^4ex^5+\left(15\,bc^4d+8\,bc^2e\right)x^3+2\left(15\,bc\right.\right.}{180\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^5*e*x^6 + 45*a*c^5*d*x^4 + 15*(2*b*c^5*e*x^6 + 3*b*c^5*d*x^4)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (6*b*c^4*e*x^5 + (15
*b*c^4*d + 8*b*c^2*e)*x^3 + 2*(15*b*c^2*d + 8*b*e)*x)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)))/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex^2+d\right)\left(b\,\text{arsech}\,(cx)+a\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)
```

maple [A] time = 0.06, size = 132, normalized size = 0.73

$$\frac{\frac{a\left(\frac{1}{6}c^6ex^6+\frac{1}{4}c^6dx^4\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsech}(cx)c^6x^6e}{6}+\frac{\mathrm{arcsech}(cx)c^6x^4d}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(6c^4ex^4+15c^4dx^2+8c^2x^2e+30c^2d+16e\right)}{180}\right)}{c^2}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^4*(a/c^2*(1/6*c^6*e*x^6+1/4*c^6*d*x^4)+b/c^2*(1/6*arcsech(c*x)*c^6*x^6*
e+1/4*arcsech(c*x)*c^6*x^4*d-1/180*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(
1/2)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2*e*x^2+30*c^2*d+16*e)))
```

maxima [A] time = 0.32, size = 138, normalized size = 0.77

$$\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\,\mathrm{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bd+\frac{1}{90}\left(15x^6\,\mathrm{arsech}\,(cx)-\frac{3c^4x^5\left(\frac{1}{c^2x}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2
) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d + 1/90*(15*x^6*arcsech(c
*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/
2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 5.81, size = 177, normalized size = 0.98

$$\begin{cases} \frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\,\mathrm{asech}\,(cx)}{4}+\frac{bex^6\,\mathrm{asech}\,(cx)}{6}-\frac{bdx^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{bex^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{bd\sqrt{-c^2x^2+1}}{6c^4}-\frac{2bex^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{4be\sqrt{-c^2x}}{45c^6} \\ (a+\infty b)\left(\frac{dx^4}{4}+\frac{ex^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asech(c*x)/4 + b*e*x**6*asech
(c*x)/6 - b*d*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*e*x**4*sqrt(-c**2*x**
2 + 1)/(30*c**2) - b*d*sqrt(-c**2*x**2 + 1)/(6*c**4) - 2*b*e*x**2*sqrt(-c**
2*x**2 + 1)/(45*c**4) - 4*b*e*sqrt(-c**2*x**2 + 1)/(45*c**6), Ne(c, 0)), ((
a + oo*b)*(d*x**4/4 + e*x**6/6), True))
```

3.97 $\int x\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=164

$$\frac{\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}-\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(2c^2d+e\right)}{4c^4}+\frac{b}{}$$

[Out] 1/4*(e*x^2+d)^2*(a+b*arcsech(c*x))/e+1/12*b*e*(-c^2*x^2+1)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^4-1/4*b*d^2*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/4*b*(2*c^2*d+e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4

Rubi [A] time = 0.19, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6299, 517, 446, 88, 63, 208}

$$\frac{\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}-\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(2c^2d+e\right)}{4c^4}+\frac{b}{}$$

Antiderivative was successfully verified.

[In] Int[x*(d + e*x^2)*(a + b*ArcSech[c*x]),x]

[Out] -(b*(2*c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(4*c^4) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(12*c^4) + ((d + e*x^2)^2*(a + b*ArcSech[c*x]))/(4*e) - (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*e)

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
```

```
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(d+ex^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)dx &= \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^2}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{4e}\\
&= \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^2}{x\sqrt{1-c^2x^2}}\,dx}{4e}\\
&= \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{(d+ex)^2}{x\sqrt{1-c^2x}}\,dx\right.}{8e}\\
&= \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\left(\frac{e\left(2c^2d+e\right)}{c^2\sqrt{1-c^2x}}\right.\right.}{8e}\\
&= -\frac{b\left(2c^2d+e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4}+\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^3}{12c^4}\\
&= -\frac{b\left(2c^2d+e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4}+\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^3}{12c^4}\\
&= -\frac{b\left(2c^2d+e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4}+\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^3}{12c^4}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 85, normalized size = 0.52

$$\frac{1}{12}\left(3ax^2\left(2d+ex^2\right)-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2\left(6d+ex^2\right)+2e\right)}{c^4}+3bx^2\operatorname{sech}^{-1}(cx)\left(2d+ex^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (3*a*x^2*(2*d + e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2*e + c^2*
(6*d + e*x^2)))/c^4 + 3*b*x^2*(2*d + e*x^2)*ArcSech[c*x])/12
```

fricas [A] time = 0.69, size = 125, normalized size = 0.76

$$\frac{3\,ac^3ex^4+6\,ac^3dx^2+3\left(bc^3ex^4+2\,bc^3dx^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(bc^2ex^3+2\left(3\,bc^2d+be\right)x\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*log((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*e*x^3 + 2*(3*b*c^2*
d + b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)(b\,\mathrm{arsech}\,(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

maple [A] time = 0.07, size = 113, normalized size = 0.69

$$\frac{\frac{a\left(\frac{1}{4}c^4 e\,x^4+\frac{1}{2}c^4 d\,x^2\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsech}(cx)c^4x^4e}{4}+\frac{\mathrm{arcsech}(cx)c^4x^2d}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\,\sqrt{\frac{cx+1}{cx}}\,\left(c^2x^2e+6c^2d+2e\right)}{12}\right)}{c^2}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^2*(a/c^2*(1/4*c^4*e*x^4+1/2*c^4*d*x^2)+b/c^2*(1/4*arcsech(c*x)*c^4*x^4*
e+1/2*arcsech(c*x)*c^4*x^2*d-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1
/2)*(c^2*e*x^2+6*c^2*d+2*e)))
```

maxima [A] time = 0.32, size = 96, normalized size = 0.59

$$\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\,\mathrm{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd+\frac{1}{12}\left(3\,x^4\,\mathrm{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)
/c)*b*d + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x
*sqrt(1/(c^2*x^2) - 1))/c^3)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 2.19, size = 126, normalized size = 0.77

$$\begin{cases} \frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\,\mathrm{asech}\,(cx)}{2}+\frac{bex^4\,\mathrm{asech}\,(cx)}{4}-\frac{bd\sqrt{-c^2x^2+1}}{2c^2}-\frac{bex^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{be\sqrt{-c^2x^2+1}}{6c^4} & \text{for}\ c\neq 0 \\ (a+\infty b)\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asech(c*x)/2 + b*e*x**4*asech
(c*x)/4 - b*d*sqrt(-c**2*x**2 + 1)/(2*c**2) - b*e*x**2*sqrt(-c**2*x**2 + 1)
/(12*c**2) - b*e*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), ((a + oo*b)*(d*x
**2/2 + e*x**4/4), True))
```

3.98 $\int \frac{(d+ex^2)(a+b\,\mathbf{sech}^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=296

$$-d\log\left(\frac{1}{x}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\,\mathrm{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\,\csc^{-1}(cx)^2}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-$$

```
[Out] 1/2*e*x^2*(a+b*arcsech(c*x))-d*(a+b*arcsech(c*x))*ln(1/x)+1/2*I*b*d*arccsc(
c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-b*d*arccsc(c*x)
*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(
1+1/c/x)^(1/2)+b*d*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)
/(1+1/c/x)^(1/2)+1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^
2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/2*b*e*x*(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2)/c
```

Rubi [A] time = 0.88, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {6303, 14, 5790, 6742, 95, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-d\log\left(\frac{1}{x}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\,\csc^{-1}(}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] -(b*e*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c) + ((I/2)*b*d*Sqrt[1 - 1
/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (e*x^2*
(a + b*ArcSech[c*x]))/2 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^
((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*d*Sqrt[1
- 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x
)]) - d*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/2)*b*d*Sqrt[1 - 1/(c^2*x^2)]
*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\text{sech}^{-1}(cx))}{x}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx)) - d(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int \frac{-\frac{e}{2x^2}+d\,l}{\sqrt{-1+\frac{x}{c}}\ldots}\right.}{c}$$

$$= \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx)) - d(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int\left(-\frac{}{2x^2\sqrt{\ldots}}\right.\right.}{}$$

$$= \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx)) - d(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{(bd)\,\text{Subst}\left(\int \frac{l}{\sqrt{-1+\ldots}}\right.}{c}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx)) - d(a+b\text{sech}^{-1}(cx))\log\ldots$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx)) + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\ldots}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\ldots}}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx)) + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\ldots}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\ldots}}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx\ldots$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx\ldots$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx\ldots$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2(a+b\text{sech}^{-1}(cx\ldots$$

Mathematica [A] time = 0.31, size = 98, normalized size = 0.33

$$\frac{1}{2}\left(2ad\log(x) + aex^2 - \frac{be\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^2} + bd\text{Li}_2\left(-e^{-2\text{sech}^{-1}(cx)}\right) - bd\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx) + 2\log\left(e^{-2\text{sech}^{-1}(cx)}\ldots\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x,x]
```

[Out] (a*e*x^2 - (b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/c^2 + b*e*x^2*ArcSech[c*x] - b*d*ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])]) + 2*a*d*Log[x] + b*d*PolyLog[2, -E^(-2*ArcSech[c*x])])/2

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^2 + ad + \left(bex^2 + bd\right)\text{arsech}\,(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)(b\,\text{arsech}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)

maple [A] time = 1.49, size = 166, normalized size = 0.56

$$\frac{a\,x^2e}{2} + \ln(cx)\,ad + \frac{b\text{arcsech}\,(cx)^2\,d}{2} - \frac{b\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,xe}{2c} + \frac{b\,\text{arcsech}\,(cx)\,x^2e}{2} + \frac{be}{2c^2} - bd\,\text{arcsech}\,(cx)\ln\left(1 + \left(\frac{1}{cx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x,x)

[Out] 1/2*a*x^2*e+ln(c*x)*a*d+1/2*b*arcsech(c*x)^2*d-1/2*b/c*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x*e+1/2*b*arcsech(c*x)*x^2*e+1/2*b/c^2*e-b*d*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*b*d*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,aex^2 + ad\log(x) + \int bex\log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right) + \frac{bd\log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")

[Out] 1/2*a*e*x^2 + a*d*log(x) + integrate(b*e*x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x)) + b*d*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2 + d\right)\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asech}(cx))\left(d + ex^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x, x)
```

3.99 $\int \frac{(d+ex^2)\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=309

$$-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx)+\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\operatorname{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{ibe\sqrt{1-}}{2\sqrt{\frac{1}{cx}}}$$

```
[Out] 1/4*b*c^2*d*arcsech(c*x)-1/2*d*(a+b*arcsech(c*x))/x^2-e*(a+b*arcsech(c*x))*
ln(1/x)+1/2*I*b*e*arccsc(c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)
^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+b*e*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^
2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*I*b*e*polylog(2,(I/c/x+(1-1/c
^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/4*
b*c*d*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/x
```

Rubi [A] time = 0.78, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {6303, 14, 5790, 12, 6742, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx)+\frac{ib}{-}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) + ((I/2)*b*e*Sqrt[1 - 1/
(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d
*ArcSech[c*x])/4 - (d*(a + b*ArcSech[c*x]))/(2*x^2) - (b*e*Sqrt[1 - 1/(c^2*
x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[
1 + 1/(c*x)]) + (b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-
1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - e*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/
2)*b*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 +
1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
```

```
+ p, 0]))
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} - e(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int \frac{dx^2+2e\log}{2\sqrt{-1+\frac{x}{c}}\sqrt{}}\right.}{c}$$

$$= -\frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} - e(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int \frac{dx^2+2e\log(}{\sqrt{-1+\frac{x}{c}}\sqrt{1}}\right.}{2c}$$

$$= -\frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} - e(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int\left(\frac{dx^2}{\sqrt{-1+\frac{x}{c}}\sqrt{}}\right.\right.}{}$$

$$= -\frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} - e(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{(bd)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+\frac{x}{c}}}\right.}{2c}$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} - e(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right)$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{1}{4}bc^2d\text{sech}^{-1}(cx) - \frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} + \frac{be\sqrt{1-}}{\sqrt{}}$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{1}{4}bc^2d\text{sech}^{-1}(cx) - \frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} + \frac{be\sqrt{1-}}{\sqrt{}}$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d\text{sech}^{-1}(cx) - \frac{d(a}{}$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d\text{sech}^{-1}(cx) - \frac{d(a}{}$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d\text{sech}^{-1}(cx) - \frac{d(a}{}$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d\text{sech}^{-1}(cx) - \frac{d(a}{}$$

Mathematica [A] time = 0.61, size = 149, normalized size = 0.48

$$\frac{1}{4}\left(-\frac{2ad}{x^2} + 4ae\log(x) - \frac{bd\sqrt{\frac{1-cx}{cx+1}}\left(-c^2x^2+c^2x^2\sqrt{1-c^2x^2}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)+1\right)}{x^2(cx-1)} - \frac{2bd\text{sech}^{-1}(cx)}{x^2} + 2be\text{Li}_2\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d)/x^2 - (2*b*d*ArcSech[c*x])/x^2 - (b*d*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 - c^2*x^2 + c^2*x^2*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 - c^2*x^2]]))/(x^2*(
-1 + c*x)) - 2*b*e*ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x
])]) + 4*a*e*Log[x] + 2*b*e*PolyLog[2, -E^(-2*ArcSech[c*x])])/4
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^2 + ad + \left(bex^2 + bd\right)\operatorname{arsech}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)(b \operatorname{arsech}(cx) + a)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

maple [A] time = 0.79, size = 170, normalized size = 0.55

$$ae\ln(cx) - \frac{da}{2x^2} + \frac{b\operatorname{arcsech}(cx)^2 e}{2} + \frac{bc^2 d\operatorname{arcsech}(cx)}{4} + \frac{cbd\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4x} - \frac{b\operatorname{arcsech}(cx)\, d}{2x^2} - be\operatorname{arcsech}(cx)\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x)
```

```
[Out] a*e*ln(c*x)-1/2/x^2*d*a+1/2*b*arcsech(c*x)^2*e+1/4*b*c^2*d*arcsech(c*x)+1/4
*c*b*d/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-1/2*b*arcsech(c*x)*d/x^2-
b*e*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*b*e*p
olylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}bd\left(\frac{\frac{2c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)}{c} + \frac{4\operatorname{arsech}(cx)}{x^2}\right) + be\int \frac{\log\left(\sqrt{\frac{1}{cx}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1)
- 1))/c + 4*arcsech(c*x)/x^2) + b*e*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/
(c*x) - 1) + 1/(c*x))/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asech}\,(cx))\left(d + ex^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**3,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**3, x)
```

3.100 $\int x^2 \left(d+ex^2\right)^2 \left(a+b\,\mathbf{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=275

$$\frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}^{-1}(cx)\right)-\frac{be^2x^5\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{42c^2}$$

```
[Out] 1/3*d^2*x^3*(a+b*arcsech(c*x))+2/5*d*e*x^5*(a+b*arcsech(c*x))+1/7*e^2*x^7*(
a+b*arcsech(c*x))+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*arcsin(c*x)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*
x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6-1/840*b*e*(84*c^2*
d+25*e)*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/42*b*e
^2*x^5*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] time = 0.23, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 6301, 12, 1267, 459, 321, 216}

$$\frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}^{-1}(cx)\right)-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\,(28}{1680c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*
x]*Sqrt[1 - c^2*x^2])/(1680*c^6) - (b*e*(84*c^2*d + 25*e)*x^3*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(840*c^4) - (b*e^2*x^5*Sqrt[(1 + c
*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(42*c^2) + (d^2*x^3*(a + b*ArcSe
ch[c*x]))/3 + (2*d*e*x^5*(a + b*ArcSech[c*x]))/5 + (e^2*x^7*(a + b*ArcSech[
c*x]))/7 + (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*ArcSin[c*x])/(1680*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)dx = \frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}\right.$$

$$= -\frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2}+\frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a\right.$$

$$= -\frac{be\left(84c^2d+25e\right)x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4}-\frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{}}{42c^2}$$

$$= -\frac{b\left(280c^4d^2+252c^2de+75e^2\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1680c^6}-\frac{be\left(84c^2d\right.}{}$$

$$= -\frac{b\left(280c^4d^2+252c^2de+75e^2\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1680c^6}-\frac{be\left(84c^2d\right.}{}$$

Mathematica [C] time = 0.50, size = 207, normalized size = 0.75

$$\frac{16ac^7x^3\left(35d^2+42dex^2+15e^2x^4\right)+16bc^7x^3\text{sech}^{-1}(cx)\left(35d^2+42dex^2+15e^2x^4\right)-bcx\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(8c^4\left(35d\right.\right.}{168}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (16*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - b*c*x*Sqrt[(1 - c*x)/(1
+ c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21
```

```
*d*e*x^2 + 5*e^2*x^4)) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*Ar
cSech[c*x] + I*b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(-2*I)*c*x + 2*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(1680*c^7)
```

fricas [A] time = 1.82, size = 341, normalized size = 1.24

$$\frac{240\,ac^7e^2x^7 + 672\,ac^7dex^5 + 560\,ac^7d^2x^3 - 2\left(280\,bc^4d^2 + 252\,bc^2de + 75\,be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - 16\,(35}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 - 2*(280*
b*c^4*d^2 + 252*b*c^2*d*e + 75*b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)) - 1)/(c*x)) - 16*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2)*log((c*
x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 16*(15*b*c^7*e^2*x^7 + 42*b*c^7*
d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c^7*d*e - 15*b*c^7*e^2)*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (40*b*c^6*e^2*x^6 + 2*(
84*b*c^6*d*e + 25*b*c^4*e^2)*x^4 + (280*b*c^6*d^2 + 252*b*c^4*d*e + 75*b*c^
2*e^2)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)^2 (b\,\text{arsech}\,(cx) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^2, x)
```

maple [A] time = 0.07, size = 300, normalized size = 1.09

$$\frac{\frac{a\left(\frac{1}{7}e^2c^7x^7+\frac{2}{5}c^7dex^5+\frac{1}{3}x^3c^7d^2\right)}{c^4} + \frac{b\left(\frac{\text{arcsech}(cx)e^2c^7x^7}{7}+\frac{2\,\text{arcsech}(cx)c^7dex^5}{5}+\frac{\text{arcsech}(cx)c^7x^3d^2}{3}+\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(-40c^5x^5e^2\sqrt{-c^2x^2+1}-168c^5x^3de\sqrt{-c^2}\right.\right.}{}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^3*(a/c^4*(1/7*e^2*c^7*x^7+2/5*c^7*d*e*x^5+1/3*x^3*c^7*d^2)+b/c^4*(1/7*a
rcsech(c*x)*e^2*c^7*x^7+2/5*arcsech(c*x)*c^7*d*e*x^5+1/3*arcsech(c*x)*c^7*x
^3*d^2+1/1680*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(-40*c^5*x^5*e^2
*(-c^2*x^2+1)^(1/2)-168*c^5*x^3*d*e*(-c^2*x^2+1)^(1/2)-280*d^2*c^5*x*(-c^2*
x^2+1)^(1/2)+280*d^2*c^4*arcsin(c*x)-50*e^2*c^3*x^3*(-c^2*x^2+1)^(1/2)-252*
c^3*d*e*x*(-c^2*x^2+1)^(1/2)+252*c^2*d*e*arcsin(c*x)-75*e^2*c*x*(-c^2*x^2+1
)^(1/2)+75*e^2*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.42, size = 328, normalized size = 1.19

$$\frac{1}{7}\,ae^2x^7+\frac{2}{5}\,adex^5+\frac{1}{3}\,ad^2x^3+\frac{1}{6}\left(2\,x^3\,\text{arsech}\,(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bd^2+\frac{1}{20}\left(8\,x^5\,\text{arsech}\,(cx)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arcsech(c*x) - (
sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^
2) - 1))/c^2)/c)*b*d^2 + 1/20*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(
3/2) + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^
2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*d*e + 1/336*(48*
x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*(1/(c^2*x^2) - 1)^(3/2
) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2)
 - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(sqrt(1/(c^2*x^2) - 1))
/c^6)/c)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a + b \operatorname{asech}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2)**2, x)
```

3.101 $\int \left(d+ex^2\right)^2 \left(a+b\,\mathbf{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=204

$$d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)-\frac{be^2x^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{20c^2}$$

```
[Out] d^2*x*(a+b*arcsech(c*x))+2/3*d*e*x^3*(a+b*arcsech(c*x))+1/5*e^2*x^5*(a+b*ar
csech(c*x))+1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*arcsin(c*x)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)/c^5-1/120*b*e*(40*c^2*d+9*e)*x*(1/(c*x+1))^(1/2)*(c*x+1
)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/20*b*e^2*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {194, 6291, 12, 1159, 388, 216}

$$d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(120c^4d^2+40c^2\right.}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*e*(40*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^
2])/(120*c^4) - (b*e^2*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*
x^2])/(20*c^2) + d^2*x*(a + b*ArcSech[c*x]) + (2*d*e*x^3*(a + b*ArcSech[c*x
]))/3 + (e^2*x^5*(a + b*ArcSech[c*x]))/5 + (b*(120*c^4*d^2 + 40*c^2*d*e + 9
*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(120*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 6291

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx = d^2x\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{sech}^{-1}(cx\right.$$

$$= d^2x\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{sech}^{-1}(cx\right.$$

$$= -\frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2}+d^2x\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{se}\right.$$

$$= -\frac{be\left(40c^2d+9e\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c}}{20c^2}$$

$$= -\frac{be\left(40c^2d+9e\right)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c}}{20c^2}$$

Mathematica [C] time = 0.35, size = 174, normalized size = 0.85

$$\frac{8ac^5x\left(15d^2+10dex^2+3e^2x^4\right)+8bc^5x\text{sech}^{-1}(cx)\left(15d^2+10dex^2+3e^2x^4\right)-bcex\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2\left(40d+6ex^2\right.\right.}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (8*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*c*e*x*Sqrt[(1 - c*x)/(1 +
c*x)]*(1 + c*x)*(9*e + c^2*(40*d + 6*e*x^2)) + 8*b*c^5*x*(15*d^2 + 10*d*e*x
^2 + 3*e^2*x^4)*ArcSech[c*x] + I*b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(
-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(120*c^5)
```

fricas [B] time = 0.81, size = 305, normalized size = 1.50

$$24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x-2\left(120\,bc^4d^2+40\,bc^2de+9\,be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-8\left(15\,bc^5d^2+10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x - 2*(120*b*c^4
*d^2 + 40*b*c^2*d*e + 9*b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) -
 1)/(c*x)) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log((c*x*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 1
5*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*log((c*x*sqrt(-(
```

```
c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (6*b*c^4*e^2*x^4 + (40*b*c^4*d*e + 9*
b*c^2*e^2)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^2 (b \operatorname{arsech}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a), x)
```

maple [A] time = 0.07, size = 228, normalized size = 1.12

$$\frac{\frac{a\left(\frac{1}{5}e^2c^5x^5+\frac{2}{3}c^5dex^3+xc^5d^2\right)}{c^4}+\frac{b\left(\frac{\operatorname{arcsech}(cx)e^2c^5x^5}{5}+\frac{2\operatorname{arcsech}(cx)c^5dex^3}{3}+\operatorname{arcsech}(cx)c^5xd^2+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(120d^2c^4\arcsin(cx)-6e^2c^3x^3\sqrt{-c^2x^2+1}-\cdots\right)}{12\cdots}\right)}{c^4}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*arcsech(c
*x)*e^2*c^5*x^5+2/3*arcsech(c*x)*c^5*d*e*x^3+arcsech(c*x)*c^5*x*d^2+1/120*(
-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(120*d^2*c^4*arcsin(c*x)-6*e^2*
c^3*x^3*(-c^2*x^2+1)^(1/2)-40*c^3*d*e*x*(-c^2*x^2+1)^(1/2)+40*c^2*d*e*arcsi
n(c*x)-9*e^2*c*x*(-c^2*x^2+1)^(1/2)+9*e^2*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.43, size = 224, normalized size = 1.10

$$\frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{1}{3}\left(2x^3\operatorname{arsech}(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bde+\frac{1}{40}\left(8x^5\operatorname{arsech}(cx)-\frac{3\left(\frac{1}{c^2x^2}-1\right)\cdots}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/3*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2)
 - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)
*b*d*e + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt(1/
(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) +
 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*e^2 + a*d^2*x + (c*x*arcsech(c*x
) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\text{asech}\,(cx))\left(d + ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2, x)
```

3.102 $\int \frac{(d+ex^2)^2 (a+b\,\mathbf{sech}^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=177

$$-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+2dex\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x}-\frac{be^2}{}$$

```
[Out] -d^2*(a+b*arcsech(c*x))/x+2*d*e*x*(a+b*arcsech(c*x))+1/3*e^2*x^3*(a+b*arcse
ch(c*x))+1/6*b*e*(12*c^2*d+e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
^3+b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x-1/6*b*e^2*x*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 6301, 12, 1265, 388, 216}

$$-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+2dex\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x}+\frac{be}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (b*e^2*x*S
qrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) - (d^2*(a + b*
ArcSech[c*x]))/x + 2*d*e*x*(a + b*ArcSech[c*x]) + (e^2*x^3*(a + b*ArcSech[c
*x]))/3 + (b*e*(12*c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x
])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
```

```
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\text{sech}^{-1}(cx)\right)$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\text{sech}^{-1}(cx)\right)$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{sech}^{-1}(\right.$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} - \frac{d^2\left(a+b\right.}{}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} - \frac{d^2\left(a+b\right.}{}$$

Mathematica [C] time = 0.32, size = 158, normalized size = 0.89

$$\frac{2ac^3\left(-3d^2+6dex^2+e^2x^4\right)+2bc^3\text{sech}^{-1}(cx)\left(-3d^2+6dex^2+e^2x^4\right)-bc\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(e^2x^2-6c^2d^2\right)+ibex\left(12\right.}{6c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] (-(b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(-6*c^2*d^2 + e^2*x^2)) + 2*a*c^
3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*A
rcSech[c*x] + I*b*e*(12*c^2*d + e)*x*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 +
 c*x)]*(1 + c*x)])/(6*c^3*x)
```

fricas [B] time = 1.84, size = 287, normalized size = 1.62

$$\frac{2\,ac^3e^2x^4+12\,ac^3dex^2-6\,ac^3d^2-2\left(12\,bc^2de+be^2\right)x\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)+2\left(3\,bc^3d^2-6\,bc^3de-bc^3e^2\right)x\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 2*(12*b*c^2*d*e + b
*e^2)*x*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 2*(3*b*c^3
*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) -
 1)/x) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 -
6*b*c^3*d*e - b*c^3*e^2)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c
*x)) + (6*b*c^4*d^2*x - b*c^2*e^2*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^3
*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2 (b\,\text{arsech}\,(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^2, x)
```

maple [A] time = 0.08, size = 197, normalized size = 1.11

$$c\left(\frac{a\left(\frac{c^3x^3e^2}{3}+2c^3dex-\frac{d^2c^3}{x}\right)}{c^4}+\frac{b\left(\frac{e^2\text{arcsech}(cx)c^3x^3}{3}+2\,\text{arcsech}\,(cx)\,c^3dex-\frac{\text{arcsech}(cx)d^2c^3}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(6\sqrt{-c^2x^2\cdots}\right.}{\cdots}\right.}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x)
```

```
[Out] c*(a/c^4*(1/3*c^3*x^3*e^2+2*c^3*d*e*x-d^2*c^3/x)+b/c^4*(1/3*e^2*arcsech(c*x
)*c^3*x^3+2*arcsech(c*x)*c^3*d*e*x-arcsech(c*x)*d^2*c^3/x+1/6*(-(c*x-1)/c/x
)^(1/2)*((c*x+1)/c/x)^(1/2)*(6*(-c^2*x^2+1)^(1/2)*c^4*d^2+12*arcsin(c*x)*c^
3*x*d*e-c^2*x^2*e^2*(-c^2*x^2+1)^(1/2)+arcsin(c*x)*c*x*e^2)/(-c^2*x^2+1)^(1
/2)))
```

maxima [A] time = 0.41, size = 152, normalized size = 0.86

$$\frac{1}{3}\,ae^2x^3+\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\text{arsech}\,(cx)}{x}\right)bd^2+\frac{1}{6}\left(2\,x^3\,\text{arsech}\,(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be^2+2\,adex+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d^2 + 1/6*(2*x
^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + ar
ctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e^2 + 2*a*d*e*x + 2*(c*x*arcsech(c*x)
 - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d*e/c - a*d^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asech}\,(cx))\left(d + ex^2\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**2, x)
```

3.103 $\int \frac{\left(d+ex^2\right)^2\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}+e^2x\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{9x^3}+\frac{2bd\sqrt{\frac{1}{c\ldots}}}{}$$

```
[Out] -1/3*d^2*(a+b*arcsech(c*x))/x^3-2*d*e*(a+b*arcsech(c*x))/x+e^2*x*(a+b*arcse
ch(c*x))+b*e^2*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c+1/9*b*d^2*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/9*b*d*(c^2*d+9*e)*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 6301, 12, 1265, 451, 216}

$$-\frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}+e^2x\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{9x^3}+\frac{2bd\sqrt{\frac{1}{c\ldots}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (2*b
*d*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x
) - (d^2*(a + b*ArcSech[c*x]))/(3*x^3) - (2*d*e*(a + b*ArcSech[c*x]))/x + e
^2*x*(a + b*ArcSech[c*x]) + (b*e^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSi
n[c*x])/c
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
```

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}+e^2x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+$$

$$= -\frac{d^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}+e^2x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3}-\frac{d^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\,\mathrm{sech}^{-1}(c\right.}{x}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3}+\frac{2bd\left(c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x}-$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3}+\frac{2bd\left(c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x}-$$

Mathematica [C] time = 0.31, size = 149, normalized size = 0.85

$$\frac{-3ac\left(d^2+6dex^2-3e^2x^4\right)+bcd\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(2c^2dx^2+d+18ex^2\right)-3bc\,\mathrm{sech}^{-1}(cx)\left(d^2+6dex^2-3e^2x^4\right)+9ib}{9cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x^2 + 18*e*x^2) - 3
*a*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4) - 3*b*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*Ar
cSech[c*x] + (9*I)*b*e^2*x^3*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)])/(9*c*x^3)
```

fricas [B] time = 0.70, size = 267, normalized size = 1.52

$$\frac{9\,ace^2x^4-18\,be^2x^3\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-18\,acdex^2+3\left(bcd^2+6\,bcde-3\,bce^2\right)x^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-3\,acd^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*c*e^2*x^4 - 18*b*e^2*x^3*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) - 1)/(c*x)) - 18*a*c*d*e*x^2 + 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 3*a*c*d^2 + 3*(3*b*c*e^2*x^
4 - 6*b*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log((c
*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*d^2*x + 2*(b*c^4*d^2
 + 9*b*c^2*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2 (b \operatorname{arsech}(cx)+a)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^4, x)
```

maple [A] time = 0.07, size = 205, normalized size = 1.16

$$c^3 \left(\frac{a \left(c x e^2 - \frac{2cde}{x} - \frac{d^2 c}{3x^3}\right)}{c^4} + \frac{b \left(\operatorname{arcsech}(cx)\, c x e^2 - \frac{2 \operatorname{arcsech}(cx) c d e}{x} - \frac{\operatorname{arcsech}(cx) d^2 c}{3x^3} + \frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{-c^2x^2+1}\, c^6 x^2 d^2 + \right.}{}\right.}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x)
```

```
[Out] c^3*(a/c^4*(c*x*e^2-2*c*d*e/x-1/3*d^2*c/x^3)+b/c^4*(arcsech(c*x)*c*x*e^2-2*
arcsech(c*x)*c*d*e/x-1/3*arcsech(c*x)*d^2*c/x^3+1/9*(-(c*x-1)/c/x)^(1/2)/c^
2/x^2*((c*x+1)/c/x)^(1/2)*(2*(-c^2*x^2+1)^(1/2)*c^6*x^2*d^2+18*c^4*d*e*x^2*
(-c^2*x^2+1)^(1/2)+(-c^2*x^2+1)^(1/2)*c^4*d^2+9*arcsin(c*x)*c^3*x^3*e^2)/(-
c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.31, size = 134, normalized size = 0.76

$$2\left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\operatorname{arsech}(cx)}{x}\right)bde + ae^2x + \frac{1}{9}\, bd^2 \left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c} - \frac{3\, \operatorname{arsech}(cx)}{x^3}\right) + \frac{\left(cx \operatorname{arsech}(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*((
c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*
x)/x^3) + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*e^2/c - 2*a*
d*e/x - 1/3*a*d^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asech}(cx)) \left(d + ex^2\right)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**4, x)
```

3.104 $\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^6} dx$

Optimal. Leaf size=213

$$-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}+\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{25x^5}+\frac{2bd\sqrt{\frac{1}{cx}}}{}$$

```
[Out] -1/5*d^2*(a+b*arcsech(c*x))/x^5-2/3*d*e*(a+b*arcsech(c*x))/x^3-e^2*(a+b*arc
sech(c*x))/x+1/25*b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/
x^5+2/225*b*d*(6*c^2*d+25*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(
1/2)/x^3+1/225*b*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.17, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 6301, 12, 1265, 453, 264}

$$-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(24c^4d^2+1\right.}{225x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (2*
b*d*(6*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(225*x^3) + (b*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[(1 + c*x)^(-1)]*Sq
rt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x) - (d^2*(a + b*ArcSech[c*x]))/(5*x^5)
 - (2*d*e*(a + b*ArcSech[c*x]))/(3*x^3) - (e^2*(a + b*ArcSech[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx=-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}+\Big($$

$$=-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}+\frac{}{1}$$

$$=\frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(c\right.}{3x^3}$$

$$=\frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}+\frac{2bd\left(6c^2d+25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3}$$

$$=\frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}+\frac{2bd\left(6c^2d+25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3}$$

Mathematica [A] time = 0.29, size = 134, normalized size = 0.63

$$\frac{-15a\left(3d^2+10dex^2+15e^2x^4\right)+b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(50dex^2\left(2c^2x^2+1\right)+3d^2\left(8c^4x^4+4c^2x^2+3\right)+225e^2x^4\right)-15b}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*
c^4*x^4)) - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcSech[c*x])/(225*x^5)
```

fricas [A] time = 1.07, size = 167, normalized size = 0.78

$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\left(15\,be^2x^4+10\,bdex^2+3\,bd^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(\left(24\,bc^5d^2+100\,bc^3de\right.\right.}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")

[Out] -1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*d*e*x^2 + 3*b*d^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - ((24*b*c^5*d^2 + 100*b*c^3*d*e + 225*b*c*e^2)*x^5 + 9*b*c*d^2*x + 2*(6*b*c^3*d^2 + 25*b*c*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^5

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2 (b \operatorname{arsech}(cx)+a)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^6, x)

maple [A] time = 0.08, size = 193, normalized size = 0.91

$$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{2de}{3cx^3}-\frac{d^2}{5cx^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e^2}{cx}-\frac{2\operatorname{arcsech}(cx)de}{3cx^3}-\frac{\operatorname{arcsech}(cx)d^2}{5cx^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^8d^2x^4+100c^6dex^4+12c^6d^2\right.}{225c^4x^4}\right.}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x)

[Out] c^5*(a/c^4*(-e^2/c/x-2/3/c*d*e/x^3-1/5*d^2/c/x^5)+b/c^4*(-arcsech(c*x)*e^2/c/x-2/3*arcsech(c*x)/c*d*e/x^3-1/5*arcsech(c*x)*d^2/c/x^5+1/225*(-(c*x-1)/c/x)^(1/2)/c^4/x^4*((c*x+1)/c/x)^(1/2)*(24*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2+9*c^4*d^2)))

maxima [A] time = 0.33, size = 175, normalized size = 0.82

$$\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\operatorname{arsech}(cx)}{x}\right)be^2+\frac{1}{75}\,bd^2\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\operatorname{arsech}(cx}{x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")

[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) + 2/9*b*d*e*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - a*e^2/x - 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^6,x)

[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^6, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{asech}\,(cx))\left(d + ex^2\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**6,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**6, x)
```

3.105 $\int \frac{(d+ex^2)^2 (a+b\,\mathrm{sech}^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=281

$$-\frac{d^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7x^7}-\frac{2de \left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5x^5}-\frac{e^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x^3}+\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{49x^7}+\frac{2bd\sqrt{\frac{1}{cx}}}{}$$

```
[Out] -1/7*d^2*(a+b*arcsech(c*x))/x^7-2/5*d*e*(a+b*arcsech(c*x))/x^5-1/3*e^2*(a+b
*arcsech(c*x))/x^3+1/49*b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^
(1/2)/x^7+2/1225*b*d*(15*c^2*d+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*
x^2+1)^(1/2)/x^5+1/11025*b*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/11025*b*c^2*(360*c^4*d^2+1176*
c^2*d*e+1225*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.20, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 6301, 12, 1265, 453, 271, 264}

$$-\frac{d^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7x^7}-\frac{2de \left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5x^5}-\frac{e^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x^3}+\frac{2bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(360c^4d^2\right.}{11025x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(49*x^7) + (2*
b*d*(15*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])
/(1225*x^5) + (b*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(11025*x^3) + (2*b*c^2*(360*c^4*d^2 + 1
176*c^2*d*e + 1225*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(11025*x) - (d^2*(a + b*ArcSech[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcSech[c
*x]))/(5*x^5) - (e^2*(a + b*ArcSech[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} + \Big($$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} + \frac{}{1}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(c\right.}{5x^5}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{2bd\left(15c^2d+49e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x}}{1225x^5}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{2bd\left(15c^2d+49e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x}}{1225x^5}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{2bd\left(15c^2d+49e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x}}{1225x^5}$$

Mathematica [A] time = 0.40, size = 160, normalized size = 0.57

$$\frac{-105a\left(15d^2+42dex^2+35e^2x^4\right)+b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(1225e^2x^4\left(2c^2x^2+1\right)+294dex^2\left(8c^4x^4+4c^2x^2+3\right)+45d^2\right.}{11025x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*
x^4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 4
2*d*e*x^2 + 35*e^2*x^4)*ArcSech[c*x])/(11025*x^7)
```

fricas [A] time = 0.55, size = 199, normalized size = 0.71

$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\left(35\,be^2x^4+42\,bdex^2+15\,bd^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(2\left(360\,bc^7d^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x)) - (2*(360*b*c^7*d^2 + 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 + (360*b*c^5
*d^2 + 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 + 225*b*c*d^2*x + 18*(15*b*c^3*d^
2 + 49*b*c*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2\left(b\,\mathrm{arsech}\,(cx)+a\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^8, x)
```

maple [A] time = 0.08, size = 225, normalized size = 0.80

$$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^7x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)e^2}{3c^3x^3}-\frac{\mathrm{arcsech}(cx)d^2}{7c^3x^7}-\frac{2\,\mathrm{arcsech}(cx)de}{5c^3x^5}+\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(720c^{10}d^2x^6+2352c^8de\,x^6\right.\right.}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x)
```

```
[Out] c^7*(a/c^4*(-1/3*e^2/c^3/x^3-1/7*d^2/c^3/x^7-2/5/c^3*d*e/x^5)+b/c^4*(-1/3*a
rcsech(c*x)*e^2/c^3/x^3-1/7*arcsech(c*x)*d^2/c^3/x^7-2/5*arcsech(c*x)/c^3*d
*e/x^5+1/11025*(-(c*x-1)/c/x)^(1/2)/c^6/x^6*((c*x+1)/c/x)^(1/2)*(720*c^10*d
^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+2
70*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*c^4*d^2)))
```

maxima [A] time = 0.32, size = 232, normalized size = 0.83

$$\frac{1}{245}\,bd^2\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}+21\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+35\,c^8\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{35\,\mathrm{arsech}\,(cx)}{x^7}\right)+\frac{2}{75}\,bde\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d^2*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/2
) + 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35*a
rcsech(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(
c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5
) + 1/9*b*e^2*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/
c - 3*arcsech(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{asech}\left(cx\right)\right)\left(d+ex^{2}\right)^{2}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**8,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**8, x)
```

3.106 $\int x^3 \left(d+ex^2\right)^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=278

$$\frac{1}{4}d^2x^4\left(a+b\mathrm{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\mathrm{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\mathrm{sech}^{-1}(cx)\right)-\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(1-c^2x^2\right)^{5/2}(}{120c^8}$$

```
[Out] 1/4*d^2*x^4*(a+b*arcsech(c*x))+1/3*d*e*x^6*(a+b*arcsech(c*x))+1/8*e^2*x^8*(
a+b*arcsech(c*x))+1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*(-c^2*x^2+1)^(3/2)*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/120*b*e*(8*c^2*d+9*e)*(-c^2*x^2+1)^(5/2
)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8+1/56*b*e^2*(-c^2*x^2+1)^(7/2)*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/24*b*(6*c^4*d^2+8*c^2*d*e+3*e^2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^8
```

Rubi [A] time = 0.24, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {266, 43, 6301, 12, 1251, 771}

$$\frac{1}{4}d^2x^4\left(a+b\mathrm{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\mathrm{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\mathrm{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}(}{72c^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt
[1 - c^2*x^2])/(24*c^8) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(72*c^8) - (b*e*(8*c^2*d + 9*e)*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(120*c^8) + (b*e^2*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(7/2))/(56*c^8) + (d^2*x^4
*(a + b*ArcSech[c*x]))/4 + (d*e*x^6*(a + b*ArcSech[c*x]))/3 + (e^2*x^8*(a +
 b*ArcSech[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)dx=\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}\right.$$

$$=-\frac{b\left(6c^4d^2+8c^2de+3e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8}+\frac{b\left(6c^4d^2+16c^2de\right.}{}$$

Mathematica [A] time = 0.31, size = 168, normalized size = 0.60

$$\frac{1}{24}\left(6ad^2x^4+8adex^6+3ae^2x^8-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(3c^6\left(70d^2x^2+56dex^4+15e^2x^6\right)+c^4\left(420d^2+224dex^2+54e^2\right.\right.}{105c^8}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (6*a*d^2*x^4 + 8*a*d*e*x^6 + 3*a*e^2*x^8 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 5
4*e^2*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/(105*c^8) + b*x
^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x])/24
```

fricas [A] time = 1.08, size = 227, normalized size = 0.82

$$\frac{315\,ac^7e^2x^8+840\,ac^7dex^6+630\,ac^7d^2x^4+105\left(3\,bc^7e^2x^8+8\,bc^7dex^6+6\,bc^7d^2x^4\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(45\,bc^6e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e^2*x^8 + 840*a*c^7*d*e*x^6 + 630*a*c^7*d^2*x^4 + 105*(3*
b*c^7*e^2*x^8 + 8*b*c^7*d*e*x^6 + 6*b*c^7*d^2*x^4)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) + 1)/(c*x)) - (45*b*c^6*e^2*x^7 + 6*(28*b*c^6*d*e + 9*b*c^4
*e^2)*x^5 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^3 + 4*(105*b
*c^4*d^2 + 112*b*c^2*d*e + 36*b*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)^2 (b \operatorname{arsech}(cx) + a)x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^3, x)
```

maple [A] time = 0.07, size = 212, normalized size = 0.76

$$\frac{\frac{a\left(\frac{1}{8}e^2c^8x^8+\frac{1}{3}c^8de\,x^6+\frac{1}{4}c^8d^2x^4\right)}{c^4} + \frac{b\left(\frac{\operatorname{arcsech}(cx)e^2c^8x^8}{8}+\frac{\operatorname{arcsech}(cx)c^8de\,x^6}{3}+\frac{\operatorname{arcsech}(cx)c^8x^4d^2}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\,\sqrt{\frac{cx+1}{cx}}\,\left(45c^6e^2x^6+168c^6de\,x^4+210c^6d^2x^2+54c^4e\right.}{2520}\right.}{c^4}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^4*(a/c^4*(1/8*e^2*c^8*x^8+1/3*c^8*d*e*x^6+1/4*c^8*d^2*x^4)+b/c^4*(1/8*a
rcsech(c*x)*e^2*c^8*x^8+1/3*arcsech(c*x)*c^8*d*e*x^6+1/4*arcsech(c*x)*c^8*x
^4*d^2-1/2520*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(45*c^6*e^2*x^6+
168*c^6*d*e*x^4+210*c^6*d^2*x^2+54*c^4*e^2*x^4+224*c^4*d*e*x^2+420*c^4*d^2+
72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)))
```

maxima [A] time = 0.33, size = 245, normalized size = 0.88

$$\frac{1}{8}\,ae^2x^8+\frac{1}{3}\,adex^6+\frac{1}{4}\,ad^2x^4+\frac{1}{12}\left(3\,x^4\operatorname{arsech}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bd^2+\frac{1}{45}\left(15\,x^6\operatorname{arsech}(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arcsech(c*x) +
(c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d^2 +
1/45*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3
*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*d*e + 1/280*(
35*x^8*arcsech(c*x) + (5*c^6*x^7*(1/(c^2*x^2) - 1)^(7/2) - 21*c^4*x^5*(1/(c
^2*x^2) - 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 35*x*sqrt(1/(c^2*
x^2) - 1))/c^7)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

[Out] int(x^3*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)

sympy [A] time = 16.46, size = 332, normalized size = 1.19

$$\begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{asech}(cx)}{4} + \frac{bdex^6\operatorname{asech}(cx)}{3} + \frac{be^2x^8\operatorname{asech}(cx)}{8} - \frac{bd^2x^2\sqrt{-c^2x^2+1}}{12c^2} - \frac{bdex^4\sqrt{-c^2x^2+1}}{15c^2} - \frac{be^2x^6\sqrt{-c^2x^2+}}{56c^2} \\ (a+\infty b)\left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d)**2*(a+b*asech(c*x)),x)

[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asech(c*x)/4 + b*d*e*x**6*asech(c*x)/3 + b*e**2*x**8*asech(c*x)/8 - b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(15*c**2) - b*e**2*x**6*sqrt(-c**2*x**2 + 1)/(56*c**2) - b*d**2*sqrt(-c**2*x**2 + 1)/(6*c**4) - 4*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e**2*x**4*sqrt(-c**2*x**2 + 1)/(140*c**4) - 8*b*d*e*sqrt(-c**2*x**2 + 1)/(45*c**6) - b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e**2*sqrt(-c**2*x**2 + 1)/(35*c**8), Ne(c, 0)), ((a + oo*b)*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), True))

3.107 $\int x\left(d+ex^2\right)^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=230

$$\frac{\left(d+ex^2\right)^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6e}-\frac{bd^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{6e}+\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}\left(3c^2d+\right.}{18c^6}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arcsech(c*x))/e+1/18*b*e*(3*c^2*d+2*e)*(-c^2*x^2+1)^(3
/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/30*b*e^2*(-c^2*x^2+1)^(5/2)*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/6*b*d^3*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*(3*c^4*d^2+3*c^2*d*e+e^2)*(1/(c*x+1))^(1/
2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6
```

Rubi [A] time = 0.25, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6299, 517, 446, 88, 63, 208}

$$\frac{\left(d+ex^2\right)^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(3c^4d^2+3c^2de+e^2\right)}{6c^6}-\frac{bd^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{\right.}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
 - c^2*x^2])/(6*c^6) + (b*e*(3*c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c
*x]*(1 - c^2*x^2)^(3/2))/(18*c^6) - (b*e^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*
x]*(1 - c^2*x^2)^(5/2))/(30*c^6) + ((d + e*x^2)^3*(a + b*ArcSech[c*x]))/(6*
e) - (b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/
(6*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^3}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{6e}$$

$$= \frac{\left(d+ex^2\right)^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^3}{x\sqrt{1-c^2x^2}}\,dx}{6e}$$

$$= \frac{\left(d+ex^2\right)^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int\frac{(d+ex)^3}{x\sqrt{1-c^2x}}\,dx, x \cdots\right.}{12e}$$

$$= \frac{\left(d+ex^2\right)^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int\left(\frac{e\left(3c^4d^2+3c^2de \cdots\right.}{c^4\sqrt{1-c^2x} \cdots}\right.\right.}{\cdots}$$

$$= -\frac{b\left(3c^4d^2+3c^2de+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6} + \frac{be\left(3c^2d+2e\right)\sqrt{\frac{1}{1+c \cdots}}}{1 \cdots}$$

$$= -\frac{b\left(3c^4d^2+3c^2de+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6} + \frac{be\left(3c^2d+2e\right)\sqrt{\frac{1}{1+c \cdots}}}{1 \cdots}$$

$$= -\frac{b\left(3c^4d^2+3c^2de+e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6} + \frac{be\left(3c^2d+2e\right)\sqrt{\frac{1}{1+c \cdots}}}{1 \cdots}$$

Mathematica [A] time = 0.32, size = 139, normalized size = 0.60

$$\frac{1}{6}ax^2\left(3d^2+3dex^2+e^2x^4\right)-\frac{b\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)\left(3c^4\left(15d^2+5dex^2+e^2x^4\right)+2c^2e\left(15d+2ex^2\right)+8e^2\right)}{90c^6}+\frac{1}{6}bx^2\mathrm{sech}^{-1} \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4))/6 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(8*e^2 + 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x
^4)))/(90*c^6) + (b*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcSech[c*x])/6
```

fricas [A] time = 1.00, size = 192, normalized size = 0.83

$$\frac{15\,ac^5e^2x^6+45\,ac^5dex^4+45\,ac^5d^2x^2+15\left(bc^5e^2x^6+3\,bc^5dex^4+3\,bc^5d^2x^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(3\,bc^4e^2x^5+\right.}{90\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^5*e^2*x^6 + 45*a*c^5*d*e*x^4 + 45*a*c^5*d^2*x^2 + 15*(b*c^5*e^
2*x^6 + 3*b*c^5*d*e*x^4 + 3*b*c^5*d^2*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + 1)/(c*x)) - (3*b*c^4*e^2*x^5 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^3 +
 (45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^2 (b\,\mathrm{arsech}\,(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x, x)
```

maple [A] time = 0.07, size = 180, normalized size = 0.78

$$\frac{\frac{a\left(\frac{1}{6}c^6e^2x^6+\frac{1}{2}c^6de\,x^4+\frac{1}{2}c^6d^2x^2\right)}{c^4}+\frac{b\left(\frac{\mathrm{arcsech}(cx)e^2c^6x^6}{6}+\frac{\mathrm{arcsech}(cx)c^6de\,x^4}{2}+\frac{\mathrm{arcsech}(cx)c^6x^2d^2}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\,\sqrt{\frac{cx+1}{cx}}\,\left(3c^4e^2x^4+15c^4de\,x^2+45d^2c^4+4c^2e^2x^2+3\right.}{90}\right.}{c^4}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

```
[Out] 1/c^2*(a/c^4*(1/6*c^6*e^2*x^6+1/2*c^6*d*e*x^4+1/2*c^6*d^2*x^2)+b/c^4*(1/6*a
rcsech(c*x)*e^2*c^6*x^6+1/2*arcsech(c*x)*c^6*d*e*x^4+1/2*arcsech(c*x)*c^6*x
^2*d^2-1/90*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(3*c^4*e^2*x^4+15*
c^4*d*e*x^2+45*c^4*d^2+4*c^2*e^2*x^2+30*c^2*d*e+8*e^2)))
```

maxima [A] time = 0.33, size = 185, normalized size = 0.80

$$\frac{1}{6}\,ae^2x^6+\frac{1}{2}\,adex^4+\frac{1}{2}\,ad^2x^2+\frac{1}{2}\left(x^2\,\mathrm{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd^2+\frac{1}{6}\left(3\,x^4\,\mathrm{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\right.}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arcsech(c*x) - x*s
qrt(1/(c^2*x^2) - 1)/c)*b*d^2 + 1/6*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*
x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d*e + 1/90*(15*x^6*arcs
ech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1
)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 6.16, size = 252, normalized size = 1.10

$$\begin{cases} \frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\,\mathrm{asech}\,(cx)}{2}+\frac{bdex^4\,\mathrm{asech}\,(cx)}{2}+\frac{be^2x^6\,\mathrm{asech}\,(cx)}{6}-\frac{bd^2\sqrt{-c^2x^2+1}}{2c^2}-\frac{bdex^2\sqrt{-c^2x^2+1}}{6c^2}-\frac{be^2x^4\sqrt{-c^2x^2+1}}{30c^2} \\ (a+\infty b)\left(\frac{d^2x^2}{2}+\frac{dex^4}{2}+\frac{e^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asech
(c*x)/2 + b*d*e*x**4*asech(c*x)/2 + b*e**2*x**6*asech(c*x)/6 - b*d**2*sqrt(
-c**2*x**2 + 1)/(2*c**2) - b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(6*c**2) - b*e**
2*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*d*e*sqrt(-c**2*x**2 + 1)/(3*c**4)
 - 2*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*e**2*sqrt(-c**2*x**2
+ 1)/(45*c**6), Ne(c, 0)), ((a + oo*b)*(d**2*x**2/2 + d*e*x**4/2 + e**2*x**
6/6), True))
```

3.108 $\int \frac{(d+ex^2)^2(a+b\text{sech}^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=370

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{sech}^{-1}(cx)\right)+dex^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\,\text{Li}_2\left(e^{2i\csc^{-1}(c}\right.}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

```
[Out] d*e*x^2*(a+b*arcsech(c*x))+1/4*e^2*x^4*(a+b*arcsech(c*x))-d^2*(a+b*arcsech(
c*x))*ln(1/x)+1/2*I*b*d^2*arccsc(c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2
)/(1+1/c/x)^(1/2)-b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-
1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+b*d^2*arccsc(c*x)*ln(1/x)
*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*I*b*d^2*polylog(2
,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-1/6*b*e*(6*c^2*d+e)*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c^3-1/12*b
*e^2*x^3*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c
```

Rubi [A] time = 1.10, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6303, 266, 43, 5790, 6742, 454, 95, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{sech}^{-1}(cx)\right)+dex^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{se}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] -(b*e*(6*c^2*d + e)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(6*c^3) - (b*e^
2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x^3)/(12*c) + ((I/2)*b*d^2*Sqrt[1 -
1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + d*e*x^
2*(a + b*ArcSech[c*x]) + (e^2*x^4*(a + b*ArcSech[c*x]))/4 - (b*d^2*Sqrt[1 -
 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x
)]*Sqrt[1 + 1/(c*x)]) + (b*d^2*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)
])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - d^2*(a + b*ArcSech[c*x])*Log[x^
(-1)] + ((I/2)*b*d^2*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])
])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x} dx = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2 \left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{x^5} dx, x, \frac{1}{x}\right)$$

$$= dex^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right)-d^2\left(a+b\text{sech}^{-1}(cx)\right)\text{l}$$

$$= dex^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right)-d^2\left(a+b\text{sech}^{-1}(cx)\right)\text{l}$$

$$= dex^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right)-d^2\left(a+b\text{sech}^{-1}(cx)\right)\text{l}$$

$$= -\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}+dex^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(\right.$$

$$= -\frac{be\left(6c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}+dex^2\left(a+\right.$$

$$= -\frac{be\left(6c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}+dex^2\left(a+\right.$$

$$= -\frac{be\left(6c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}+\frac{ibd^2\sqrt{1-}}{2\sqrt{-1}}$$

$$= -\frac{be\left(6c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}+\frac{ibd^2\sqrt{1-}}{2\sqrt{-1}}$$

$$= -\frac{be\left(6c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}+\frac{ibd^2\sqrt{1-}}{2\sqrt{-1}}$$

$$= -\frac{be\left(6c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}+\frac{ibd^2\sqrt{1-}}{2\sqrt{-1}}$$

Mathematica [A] time = 0.44, size = 176, normalized size = 0.48

$$ad^2\log(x)+adex^2+\frac{1}{4}ae^2x^4-\frac{bde\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^2}-\frac{be^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2x^2+2\right)}{12c^4}+\frac{1}{2}bd^2\text{Li}_2\left(-e^{-2\text{sech}^{-1}(cx)}\right)-\frac{1}{2}bd^2\text{sech}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 - (b*d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/c^2
 - (b*e^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2))/(12*c^4) + b*d
```

```
*e*x^2*ArcSech[c*x] + (b*e^2*x^4*ArcSech[c*x])/4 - (b*d^2*ArcSech[c*x]*(Arc
Sech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])]))/2 + a*d^2*Log[x] + (b*d^2*Poly
Log[2, -E^(-2*ArcSech[c*x])])/2
```

fricas [F] time = 1.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\mathrm{arsech}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsech(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^2 \left(b\,\mathrm{arsech}\left(cx\right) + a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x, x)
```

maple [A] time = 1.82, size = 286, normalized size = 0.77

$$\frac{a\,e^2x^4}{4}+ade\,x^2+a\,d^2\ln\left(cx\right)+\frac{b\mathrm{arcsech}\left(cx\right)^2d^2}{2}-\frac{b\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,x^3e^2}{12c}-\frac{b\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,xde}{c}+\frac{b\,\mathrm{arcsech}\left(cx\right)e^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x)
```

```
[Out] 1/4*a*e^2*x^4+a*d*e*x^2+a*d^2*ln(c*x)+1/2*b*arcsech(c*x)^2*d^2-1/12*b/c*(-(
c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3*e^2-b/c*(-(c*x-1)/c/x)^(1/2)*((c*
x+1)/c/x)^(1/2)*x*d*e+1/4*b*arcsech(c*x)*e^2*x^4+b*arcsech(c*x)*d*e*x^2-1/6
*b/c^3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x*e^2+b/c^2*d*e+1/6*b/c^4*e
^2-b*d^2*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*
b*d^2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,ae^2x^4+adex^2+ad^2\log(x)+\int be^2x^3\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)+2\,bdex\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)+\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate(b*e^2*x^3*log(sqrt(1/(
c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x)) + 2*b*d*e*x*log(sqrt(1/(c*x) + 1)*sq
rt(1/(c*x) - 1) + 1/(c*x)) + b*d^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1)
+ 1/(c*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asech}\,(cx))\left(d + ex^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x, x)
```

3.109 $\int \frac{\left(d+ex^2\right)^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=373

$$-\frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{4}bc^2d^2\,\text{sech}^{-1}(cx)+\frac{ibde\sqrt{1}}{\sqrt{}}$$

```
[Out] 1/4*b*c^2*d^2*arcsech(c*x)-1/2*d^2*(a+b*arcsech(c*x))/x^2+1/2*e^2*x^2*(a+b*
arcsech(c*x))-2*d*e*(a+b*arcsech(c*x))*ln(1/x)+I*b*d*e*arccsc(c*x)^2*(1-1/c
^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-2*b*d*e*arccsc(c*x)*ln(1-(I/
c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^
(1/2)+2*b*d*e*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1
/c/x)^(1/2)+I*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^
(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/4*b*c*d^2*(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2)/x-1/2*b*e^2*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c
```

Rubi [A] time = 1.05, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {6303, 266, 43, 5790, 12, 6742, 95, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{d^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\,\text{sec}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) - (b*e^2*Sqrt[-1 + 1/(
c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c) + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*
x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d^2*ArcSech[c*x])/4 -
 (d^2*(a + b*ArcSech[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcSech[c*x]))/2 - (2
*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (2*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCs
c[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - 2*d*e*(a + b*A
rcSech[c*x])*Log[x^(-1)] + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*
I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2 \left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - 2de\left(a+b\text{sech}^{-1}(cx)\right)$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - 2de\left(a+b\text{sech}^{-1}(cx)\right)$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - 2de\left(a+b\text{sech}^{-1}(cx)\right)$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - 2de\left(a+b\text{sech}^{-1}(cx)\right)$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} -$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{1}{4}bc^2d^2\text{sech}^{-1}(cx) - \frac{d}{}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{1}{4}bc^2d^2\text{sech}^{-1}(cx) - \frac{d}{}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(c}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(c}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(c}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(c}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

Mathematica [A] time = 0.84, size = 212, normalized size = 0.57

$$\frac{1}{4}\left(-\frac{2ad^2}{x^2} + 8ade\log(x) + 2ae^2x^2 - \frac{bd^2\sqrt{\frac{1-cx}{cx+1}}\left(-c^2x^2 + c^2x^2\sqrt{1-c^2x^2}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)+1\right)}{x^2(cx-1)} - \frac{2be^2\sqrt{\frac{1-cx}{cx+1}}\,(c}{c^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*e^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
))/c^2 - (2*b*d^2*ArcSech[c*x])/x^2 + 2*b*e^2*x^2*ArcSech[c*x] - (b*d^2*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 - c^2*x^2 + c^2*x^2*Sqrt[1 - c^2*x^2]*ArcTanh[Sqr
t[1 - c^2*x^2]]))/(x^2*(-1 + c*x)) - 4*b*d*e*ArcSech[c*x]*(ArcSech[c*x] + 2
*Log[1 + E^(-2*ArcSech[c*x])]) + 8*a*d*e*Log[x] + 4*b*d*e*PolyLog[2, -E^(-2
*ArcSech[c*x])])/4
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\text{arsech}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsech(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^2 (b\,\text{arsech}\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^3, x)
```

maple [A] time = 1.60, size = 252, normalized size = 0.68

$$\frac{a\,x^2e^2}{2} + 2ade\ln(cx) - \frac{a\,d^2}{2x^2} + bde\,\text{arcsech}\,(cx)^2 + \frac{cb\,d^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{4x} + \frac{b\,c^2d^2\text{arcsech}\,(cx)}{4} - \frac{b\,\text{arcsech}\,(cx)\,d^2}{2x^2} - \frac{b\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x)
```

```
[Out] 1/2*a*x^2*e^2+2*a*d*e*ln(c*x)-1/2*a*d^2/x^2+b*d*e*arcsech(c*x)^2+1/4*c*b*d^
2/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/4*b*c^2*d^2*arcsech(c*x)-1/2
*b*arcsech(c*x)*d^2/x^2-1/2/c*b*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x*
e^2+1/2*b*arcsech(c*x)*x^2*e^2+1/2*b/c^2*e^2-2*b*d*e*arcsech(c*x)*ln(1+(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-b*d*e*polylog(2,-(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ae^2x^2 - \frac{1}{8}\,bd^2\left(\frac{\frac{2\,c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)}{c} + \frac{4\,\text{arsech}\,(cx)}{x^2}\right) + 2\,ade\,\text{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^2*x^2 - 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*
x^2) - 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(
1/(c^2*x^2) - 1) - 1))/c + 4*arcsech(c*x)/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2
/x^2 + integrate(b*e^2*x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))
 + 2*b*d*e*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asech}\,(cx))\left(d+ex^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**3, x)
```

3.110 $\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=519

$$\frac{\sqrt{-d}\,\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,\left(a+bs\right.}{}$$

```
[Out] x*(a+b*arcsech(c*x))/e-b*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c/e+1/2*(
a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arcsech(c*x))*ln(1
+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(
1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3
/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*
(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylog(2,-c
*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*poly
log(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*
d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A] time = 1.27, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6303, 5792, 5662, 92, 205, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcSech[c*x]))/e - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]
])/(c*e) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x
])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x
])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(
3/2)) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2
)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c
*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*e^(3/2)) + (b*S
qrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(2*e^(3/2))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{ex^2} - \frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{ce} + \frac{d\,\text{Subst}\left(\int \left(\frac{a+b\cosh}{2\sqrt{e}\left(\sqrt{e}-\right.}\right.\right.}{}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(}{\sqrt{e}+\sqrt{-d}\,x}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\tan^{-1}\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{d\,\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\tan^{-1}\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{d\,\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-}}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\tan^{-1}\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\text{lo}}{2e^{3/2}}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\tan^{-1}\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\text{lo}}{2e^{3/2}}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\tan^{-1}\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\text{lo}}{2e^{3/2}}$$

Mathematica [C] time = 1.75, size = 921, normalized size = 1.77

$$4ac\sqrt{e}\,x - 4ac\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) + b\left(4\sqrt{e}\left(cx\operatorname{sech}^{-1}(cx) - 2\tan^{-1}\left(\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)\right)\right) - 2ic\sqrt{d}\left(-4i\sin^{-1}\left(\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] (4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[e]
*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]) - (2*I)*c*Sqrt[d]*((-4
*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d]
 + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E
^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Arc
Sech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1
 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2,
 (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2,
((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]) + (2*I)*c*S
qrt[d]*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((
(-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[
c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[
d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c
*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])])))/(4*c*e^(3/2))
```

fricas [F] time = 1.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d), x)
```

maple [C] time = 6.53, size = 411, normalized size = 0.79

$$\frac{ax}{e} - \frac{ad \arctan\left(\frac{xe}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b \operatorname{arcsech}(cx)\, x}{e} + \frac{cbd\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)} \frac{\left(\_R1^2c^2d+4\_R1^2e+c^2d\right)\left(\operatorname{arcsech}(cx)\ln\left(\frac{\_R1\cdots}{\cdots}\right.\right.}{\_R1(\cdots}\right.}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

```
[Out] a*x/e-a*d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+b*arcsech(c*x)/e*x+1/8*c*b/
e^2*d*sum((_R1^2*c^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsec
```

```
h(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/
x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*
e)*_Z^2+c^2*d))-2/c*b/e*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))-1/8*
c*b/e^2*d*sum((_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(
c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)
*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e}-\frac{x}{e}\right)+b\int\frac{x^2\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(d*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e) - x/e) + b*integrate(x^2*log(sqrt
(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+b\,\mathrm{asech}\,(cx)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))/(d + e*x**2), x)
```

3.111 $\int \frac{x\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=459

$$\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\right.}{2e}$$

```
[Out] -(a+b*arcsech(c*x))^2/b/e-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))^2)/e+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsech
(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsech(c*
x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))/e+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)^2)/e+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,-c*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

Rubi [A] time = 1.23, antiderivative size = 441, normalized size of antiderivative = 0.96, number of steps used = 26, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6303, 5792, 5660, 3718, 2190, 2279, 2391, 5800, 5562}

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}\,e}{\sqrt{c^2d}}\right.}{2e}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 - (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcS
ech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])
/(2*e) - ((a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/e + (b*PolyLog[
2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e) + (b*
PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e)
+ (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]
)/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d +
 e])])/(2*e) - (b*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*e)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{ex} - \frac{dx\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{e} + \frac{d\,\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \ldots\right.\right.}{e}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2be} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(cx)\right)}{e} - \frac{\sqrt{-d}\,\text{Subst}\left(\int \frac{a+b\cos\ldots}{\sqrt{e}-\ldots}\right.}{2e}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2be} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{e} + \frac{b\,\text{Subst}\left(\int \log\left(1+\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{e} + \frac{b\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{sech}^{-1}(cx)}\right)}{2e} - \ldots$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right.}{2e}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right.}{2e}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right.}{2e}$$

Mathematica [C] time = 0.41, size = 860, normalized size = 1.87

$$\frac{4ib\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\tanh^{-1}\left(\frac{\left(\sqrt{e}-ic\sqrt{d}\right)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{dc^2+e}}\right) + 4ib\sin^{-1}\left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}}+1}}{\sqrt{2}}\right)\tanh^{-1}\left(\frac{\left(i\sqrt{d}c+\sqrt{e}\right)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{dc^2+e}}\right) - \ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] ((4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c
*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcSin
[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e]
)*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSech[c*x]*Log[1 + E^(-2*A
rcSech[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
] + b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
```

```
cSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcS
ech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
] + (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log
[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*b*
ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + a*Log[d + e*x^2] + b*PolyLog[2
, -E^(-2*ArcSech[c*x])] - b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e
]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e
]))/(c*Sqrt[d]*E^ArcSech[c*x])])/(2*e)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx\,\text{arsech}\,(cx)+ax}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arcsech(c*x) + a*x)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsech}\,(cx)+a)x}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d), x)
```

maple [C] time = 0.89, size = 513, normalized size = 1.12

$$\frac{a\ln\left(c^2x^2e+c^2d\right)}{2e}+\frac{c^2b\left(\sum_{\_R1=\text{RootOf}\left(c^2d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2+1\right)\left(\text{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-\frac{1}{cx}-\cdots}{}\right)\cdots\right)}{\_R1^2c^2d+c^2d+2e}\cdots\right)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

```
[Out] 1/2*a/e*ln(c^2*e*x^2+c^2*d)+1/4*c^2*b*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)
*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((
_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*
c^2*d+4*e)*_Z^2+c^2*d))*d/e-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2)))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2)))-b/e*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog
(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*b*sum((_R1^2*c^2*d+c^2*d
+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R
1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{ex^2+d}\,dx+\frac{a\log\left(ex^2+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d
), x) + 1/2*a*log(e*x^2 + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{asech}\left(cx\right)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2), x)
```

3.112 $\int \frac{a+b\mathbf{sech}^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=469

$$\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\right.}{2\sqrt{-d}\,\sqrt{}}$$

```
[Out] 1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsech(c*x))
*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d
+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)
/e^(1/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog
(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b
*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/
2)
```

Rubi [A] time = 0.97, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6293, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\right.}{2\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-
d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a
 + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*
d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*
E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*Pol
yLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*Sqr
t[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (
c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e
])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6293

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(2*(p + 1
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(\right.}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}\,e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}\,e^x}\,dx, x, \text{sec}\right.}{2\sqrt{e}}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [C] time = 0.39, size = 849, normalized size = 1.81

$$2a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) - 4b\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\tanh^{-1}\left(\frac{\left(\sqrt{e}-ic\sqrt{d}\right)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{dc^2+e}}\right) + 4b\sin^{-1}\left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}}+1}}{\sqrt{2}}\right)\tanh^{-1}\left(\frac{\left(i\sqrt{d}\,c+\sqrt{e}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2),x]
```

```
[Out] (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqr
t[c^2*d + e]] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTa
nh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - I*b*Ar
cSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] - 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c*x]*Log[1 - (I*(Sqr
t[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSin[Sqrt[1 -
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(
```

```
c*Sqrt[d]*E^ArcSech[c*x])] - I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
cSech[c*x])] - I*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d
]*E^ArcSech[c*x])] + I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqr
t[d]*E^ArcSech[c*x])] + I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])] - I*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])])/(2*Sqrt[d]*Sqrt[e])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsech}\,(cx)+a}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d), x)
```

maple [C] time = 2.69, size = 302, normalized size = 0.64

$$\frac{a\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{cb\left(\sum_{\_R1=\text{RootOf}\left(c^2d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\text{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{c}}}{\_R1}\right.\right.}{\_R1^2c^2d+c^2d+2e}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/2*c*b*sum(_R1/(_R1^2*c^2*d+c^2*d+2*
e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog
((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*c*b*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsec
h(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/
x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*
e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{ex^2+d}\,dx+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d),
 x) + a*arctan(e*x/sqrt(d*e))/sqrt(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asech}{\left(cx \right)}}{d + ex^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x**2), x)
```

3.113 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=417

$$-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1\right.}{2d}$$

```
[Out] 1/2*(a+b*arcsech(c*x))^2/b/d-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*a
rcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcs
ech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2
*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

Rubi [A] time = 0.99, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6303, 5792, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}-\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}}{\sqrt{c^2}}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (a + b*ArcSech[c*x])^2/(2*b*d) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])
*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) -
((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c
^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^Arc
Sech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]
*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*S
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = -\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}\,e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{e^x(a}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d}}{c}}\right.}{}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log}{2d}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log}{2d}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log}{2d}$$

Mathematica [C] time = 0.93, size = 386, normalized size = 0.93

$$-2a\log\left(d+ex^2\right)+4a\log(x)+b\left(\text{Li}_2\left(-\frac{\left(dc^2+2e-2\sqrt{e\left(dc^2+e\right)}\right)e^{-2\text{sech}^{-1}(cx)}}{c^2d}\right)+\text{Li}_2\left(-\frac{\left(dc^2+2\left(e+\sqrt{e\left(dc^2+e\right)}\right)\right)e^{-2\text{sech}^{-1}(cx)}}{c^2d}\right)-2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + b*(-2*(ArcSech[c*x]^2 + I*ArcSin[Sqrt[1
+ e/(c^2*d)]]*(2*ArcTanh[(e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/Sqrt[e*(c^
2*d + e)]] - Log[(2*e - 2*Sqrt[e*(c^2*d + e)] + c^2*d*(1 + E^(2*ArcSech[c*x
])))/(c^2*d*E^(2*ArcSech[c*x]))] + Log[(2*(e + Sqrt[e*(c^2*d + e)]) + c^2*d
*(1 + E^(2*ArcSech[c*x])))/(c^2*d*E^(2*ArcSech[c*x]))]) + ArcSech[c*x]*(Log
[(2*e - 2*Sqrt[e*(c^2*d + e)] + c^2*d*(1 + E^(2*ArcSech[c*x])))/(c^2*d*E^(2
*ArcSech[c*x]))] + Log[(2*(e + Sqrt[e*(c^2*d + e)]) + c^2*d*(1 + E^(2*ArcSe
ch[c*x])))/(c^2*d*E^(2*ArcSech[c*x]))])) + PolyLog[2, -((c^2*d + 2*e - 2*Sq
rt[e*(c^2*d + e)])/(c^2*d*E^(2*ArcSech[c*x])))] + PolyLog[2, -((c^2*d + 2*(
e + Sqrt[e*(c^2*d + e)]))/(c^2*d*E^(2*ArcSech[c*x])))]))/(4*d)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsech}(cx)+a}{ex^3+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e*x^3 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx) + a}{\left(ex^2 + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x), x)
```

maple [C] time = 0.89, size = 3157, normalized size = 7.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d),x)
```

```
[Out] 1/4*b*c^2/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*b*c^2/(c^2*d+e)*arcsech(c*x)^2+
a/d*ln(c*x)-b/c^4/d^3*e^2/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*d+e))^(1/2)
-3*b/c^2/(c^2*d+e)/d^2*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^
2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)*e-2*
b/c^4/d^3*e^2/(c^2*d+e)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)-b/c
^2/d^2*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2
*(e*(c^2*d+e))^(1/2)-2*e))*e-b/c^4/d^3*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e^2-b/c^2/d^2*ar
csech(c*x)^2*(e*(c^2*d+e))^(1/2)+1/2*b/c^2/d^2*polylog(2,c^2*d*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*
d+e))^(1/2)+1/2*b*c^2/(c^2*d+e)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-5/2*b/(c^2*d+e)
/d*arcsech(c*x)^2*e-3/4*b/(c^2*d+e)/d*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*d+e))^(1/
2)+5/4*b/(c^2*d+e)/d*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e+1/4*b*(e*(c^2*d+e))^(1/2)/d/(c^2
*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*
(e*(c^2*d+e))^(1/2)-2*e))+b*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arcsech(c*x)^2+
2*b/c^2/d^2*arcsech(c*x)^2*e+2*b/c^4/d^3*arcsech(c*x)^2*e^2-1/2*a/d*ln(c^2*
e*x^2+c^2*d)-1/2*b*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(a
rcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1
-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2
*d+4*e)*_Z^2+c^2*d))/d+b*arcsech(c*x)^2/d-1/4*b/d*polylog(2,c^2*d*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*b
/d*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*
d+e))^(1/2)-2*e))*arcsech(c*x)-1/8*b*c^2/e/(c^2*d+e)*polylog(2,c^2*d*(1/c/x
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e
*(c^2*d+e))^(1/2)+1/2*b*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arcsech(c*x)*ln(1-c
^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/
2)-2*e))-3/2*b/(c^2*d+e)/d*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)+
5/2*b*e/(c^2*d+e)/d*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(
-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-2*b/c^2/d^2*e*ln(1-c^2*d*(1
```

```
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e)
)*arcsech(c*x)-2*b/c^4/d^3*e^2*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+b/c^2/d^2*ln(1-c
^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/
2)-2*e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)-4*b/c^2/(c^2*d+e)/d^2*arcsech(c*x
)^2*e^2+2*b/c^2/(c^2*d+e)/d^2*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e^2+1/8*b*c^2*(e*(c^2*d+e
))^(1/2)/e/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-2*b/c^4/d^3*arcsech(c*x)^2*e*(e*(c
^2*d+e))^(1/2)+b/c^4/d^3*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e*(e*(c^2*d+e))^(1/2)-2*b/c^4/
d^3*e^3/(c^2*d+e)*arcsech(c*x)^2+b/c^4/d^3*e^3/(c^2*d+e)*polylog(2,c^2*d*(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e)
)+1/4*b*c^2*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*arcsech(c*x)*ln(1-c^2*d*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/4
*b*c^2/e/(c^2*d+e)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-
c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)+2*b/c^4/
d^3*e^2/(c^2*d+e)*arcsech(c*x)^2*(e*(c^2*d+e))^(1/2)+3*b/c^2/(c^2*d+e)/d^2*
arcsech(c*x)^2*(e*(c^2*d+e))^(1/2)*e-3/2*b/c^2/(c^2*d+e)/d^2*polylog(2,c^2*
d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-
2*e))*(e*(c^2*d+e))^(1/2)*e+2*b/c^4/d^3*e*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*(e*(c
^2*d+e))^(1/2)+4*b/c^2/(c^2*d+e)/d^2*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*e^2+2*b/c^
4/d^3*e^3/(c^2*d+e)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(
-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{\log\left(ex^2+d\right)}{d}-\frac{2\log(x)}{d}\right)+b\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{ex^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c*x) + 1)*
sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^3 + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{asech}\,(cx)}{x\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x*(d + e*x**2)), x)
```

3.114 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=523

$$\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\text{sech}^{-}\right.}{}$$

```
[Out] -a/d/x-b*arcsech(c*x)/d/x+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3
/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*
(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arcsech(c
*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2
)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylog(2,c*(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2))
)*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylo
g(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e
)^(1/2)))*e^(1/2)/(-d)^(3/2)+b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/d
```

Rubi [A] time = 1.26, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6303, 5792, 5654, 74, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d - a/(d*x) - (b*ArcSech[c*x])/(
d*x) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(S
qrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(
3/2)) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])
*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^
(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqr
t[c^2*d + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSe
ch[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[
2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*(-d)^(3/
2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2
*d + e])])/(2*(-d)^(3/2))
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{d} - \frac{e\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)dx,x,\frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}dx,x,\frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\,\text{Subst}\left(\int \cosh^{-1}\left(\frac{x}{c}\right)dx,x,\frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right.\right.}{d}$$

$$= -\frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}dx,x,\frac{1}{x}\right)}{cd} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\right.}{2d}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx,x,\text{sech}^{-1}\right.}{2d}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}\,e^x}dx,x,\text{sec}\right.}{2d}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}}}{\sqrt{e}-\sqrt{c^2}}\right.}{2(-d)^{3/2}}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}}}{\sqrt{e}-\sqrt{c^2}}\right.}{2(-d)^{3/2}}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}}}{\sqrt{e}-\sqrt{c^2}}\right.}{2(-d)^{3/2}}$$

Mathematica [C] time = 1.86, size = 933, normalized size = 1.78

$$-4\sqrt{e}\,x\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)a - 4\sqrt{d}\,a + b\left(4\sqrt{d}\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 4\sqrt{d}\,\text{sech}^{-1}(cx) - 2i\sqrt{e}\,x\left(-4i\sin^{-1}\left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}}+1}}{\sqrt{2}}\right)\tanh\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (-4*a*Sqrt[d] - 4*a*Sqrt[e]*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[d]*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 4*Sqrt[d]*ArcSech[c*x] - (2*I)*Sqrt[e]*
x*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*S
qrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log
[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
```

```
ch[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]
*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + Poly
Log[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyL
og[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]) + (2*
I)*Sqrt[e]*x*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcT
anh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + Ar
cSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e]
 + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Arc
Sech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])])))/(4*d^(3/2)*x)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsech}\,(cx)+a}{ex^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e*x^4 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x^2), x)
```

maple [C] time = 6.50, size = 372, normalized size = 0.71

$$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{cb\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d}-\frac{b\,\text{arcsech}\,(cx)}{dx}+\frac{cbe\left(\sum_{\_R1=\text{RootOf}\left(c^2d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\text{arcsech}(cx)\ln\right.}{}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d),x)
```

```
[Out] -a/d/x-a*e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+c*b/d*(-(c*x-1)/c/x)^(1/2)
*((c*x+1)/c/x)^(1/2)-b*arcsech(c*x)/d/x+1/2*c*b*e/d*sum(_R1/(_R1^2*c^2*d+c^
2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)
+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*
_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*c*b*e/d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*
e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog
((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d}+\frac{1}{dx}\right)+b\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{ex^4+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(e*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d) + 1/(d*x)) + b*integrate(log(sqrt
(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^4 + d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{asech}\,(cx)}{x^2\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)), x)
```

3.115 $\int \frac{x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=631

$$-\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{e^3}-\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\mathrm{lo}\cdots}{e^3}$$

```
[Out] 1/2*d*(a+b*arcsech(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arcsech(c*x))/e^2+2*d*(
a+b*arcsech(c*x))^2/b/e^3+2*d*(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2)/e^3-d*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arc
sech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsec
h(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))^2)/e^3-b*d*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2
,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^
(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*d*arctanh((c^2*d+e)^(1/2)/c/x/e^(
1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(1/2)/e^(5/2)/(c^2*d+e)^(1/2)/(-1
+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/2*b*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c/e^
2
```

Rubi [A] time = 1.55, antiderivative size = 611, normalized size of antiderivative = 0.97, number of steps used = 32, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6303, 5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$-\frac{bd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}-\frac{bd\,\mathrm{PolyLog}\left(2,\cdots\right)}{e\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(b*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c*e^2) + (d*(a + b*ArcSech[c
*x]))/(2*e^2*(e + d/x^2)) + (x^2*(a + b*ArcSech[c*x]))/(2*e^2) - (b*d*Sqrt[
-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]
*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - (d
*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e])])/e^3 - (d*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcSech[c*x])*Log[1 - (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcS
ech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])
/e^3 + (2*d*(a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/e^3 - (b*d*Po
lyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^3 -
 (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/
e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d +
```

```
e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c
^2*d + e])])/e^3 + (b*d*PolyLog[2, -E^(2*ArcSech[c*x])])/e^3
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^5 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^2} dx = -\text{Subst}\left(\int \frac{a + b\cosh^{-1}\left(\frac{x}{c}\right)}{x^3 \left(e + dx^2\right)^2} dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a + b\cosh^{-1}\left(\frac{x}{c}\right)}{e^2x^3} - \frac{2d\left(a + b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e^3x} + \frac{d^2x\left(a + b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e + dx^2\right)^2}\right.\right.$$

$$= \frac{(2d)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2} dx, x, \frac{1}{x}\right)}{e^3} -$$

$$= \frac{d\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2\left(e + \frac{d}{x^2}\right)} + \frac{x^2\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2} + \frac{(2d)\,\text{Subst}\left(\int (a + bx)\tanh(x)\,dx\right.}{e^3}$$

$$= -\frac{b\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\,x}{2ce^2} + \frac{d\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2\left(e + \frac{d}{x^2}\right)} + \frac{x^2\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2} - \frac{d\left(a + b\right.}{}$$

$$= -\frac{b\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\,x}{2ce^2} + \frac{d\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2\left(e + \frac{d}{x^2}\right)} + \frac{x^2\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2} - \frac{d\left(a + b\right.}{}$$

$$= -\frac{b\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\,x}{2ce^2} + \frac{d\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2\left(e + \frac{d}{x^2}\right)} + \frac{x^2\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2} - \frac{bd\sqrt{-1}}{2e^{5/2}}$$

$$= -\frac{b\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\,x}{2ce^2} + \frac{d\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2\left(e + \frac{d}{x^2}\right)} + \frac{x^2\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2} - \frac{bd\sqrt{-1}}{2e^{5/2}}$$

$$= -\frac{b\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\,x}{2ce^2} + \frac{d\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2\left(e + \frac{d}{x^2}\right)} + \frac{x^2\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2} - \frac{bd\sqrt{-1}}{2e^{5/2}}$$

$$= -\frac{b\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\,x}{2ce^2} + \frac{d\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2\left(e + \frac{d}{x^2}\right)} + \frac{x^2\left(a + b\text{sech}^{-1}(cx)\right)}{2e^2} - \frac{bd\sqrt{-1}}{2e^{5/2}}$$

Mathematica [C] time = 5.62, size = 1278, normalized size = 2.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*((2*e*S
qrt[(1 - c*x)/(1 + c*x)])/c^2 + (2*e*x*Sqrt[(1 - c*x)/(1 + c*x)])/c - 2*e*x
^2*ArcSech[c*x] + (d^(3/2)*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)
*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c
*x]/2])/Sqrt[c^2*d + e]] + (16*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*
d + e]] - 8*d*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + 4*d*ArcSech[c*x]*
Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (8*I)
*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] -
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[1 + (I
*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (8*I)*d*ArcSin
[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[1 - (I*(Sqrt[e
] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (8*I)*d*ArcSin[Sqrt[1 -
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c
^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (8*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]
*E^ArcSech[c*x])] + 2*d*Log[x] - 2*d*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*
x*Sqrt[(1 - c*x)/(1 + c*x)]] + (d*Sqrt[e]*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[
(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d +
 e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (d*Sqrt[e]*Log[(2*Sqrt[e]
*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*
d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + 4*d*P
olyLog[2, -E^(-2*ArcSech[c*x])] - 4*d*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 4*d*PolyLog[2, (I*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 4*d*PolyLog[2, ((-I)*(Sqrt[e] +
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 4*d*PolyLog[2, (I*(Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]))/e^3
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^5\,\mathrm{arsech}\,(cx)+ax^5}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arcsech(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\mathrm{arsech}\,(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^2, x)
```

maple [C] time = 1.63, size = 870, normalized size = 1.38

$$\frac{a\,x^2}{2e^2}-\frac{c^2a\,d^2}{2e^3\left(c^2x^2e+c^2d\right)}-\frac{ad\ln\left(c^2x^2e+c^2d\right)}{e^3}-\frac{cb\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,x^3}{2\left(c^2x^2e+c^2d\right)e}-\frac{cb\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,xd}{2\left(c^2x^2e+c^2d\right)e^2}+\frac{c^2b\,\mathrm{arcsech}\,(cx)\,x^4}{2\left(c^2x^2e+c^2d\right)e}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*a*x^2/e^2-1/2*c^2*a/e^3*d^2/(c^2*e*x^2+c^2*d)-a/e^3*d*ln(c^2*e*x^2+c^2*
d)-1/2*c*b/(c^2*e*x^2+c^2*d)/e*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3
-1/2*c*b/(c^2*e*x^2+c^2*d)/e^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x*d
+1/2*c^2*b/(c^2*e*x^2+c^2*d)/e*arcsech(c*x)*x^4+c^2*b/(c^2*e*x^2+c^2*d)/e^2
*arcsech(c*x)*d*x^2+1/2*b/(c^2*e*x^2+c^2*d)/e*x^2+1/2*b/(c^2*e*x^2+c^2*d)/e
^2*d+1/2*b*(e*(c^2*d+e))^(1/2)/e^3/(c^2*d+e)*d*arctanh(1/4*(2*c^2*d*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/2*c
^2*b/e^3*d^2*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/
c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+2
*b/e^3*d*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+2*b/
e^3*d*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+2*b/e^3
*d*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+2*b/e^3*d*dilog(1-I*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/2*b/e^3*d*sum((_R1^2*c^2*d+c^2*
d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_
R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{d^2}{e^4x^2+de^3}-\frac{x^2}{e^2}+\frac{2\,d\log\left(ex^2+d\right)}{e^3}\right)+b\int\frac{x^5\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*d*
e*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^5\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.116 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=580

$$\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^2}+\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\right.}{2e^2}$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/e/(e+d/x^2)-(a+b*arcsech(c*x))^2/b/e^2-(a+b*arcsech
(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e^2+1/2*(a+b*arcs
ech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arc
sech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*b*polyl
og(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e^2+1/2*b*polylog(2,-c*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/e^2+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(
2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^
(1/2)))/e^2+1/2*b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))
*(-1+1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1
/2)
```

Rubi [A] time = 1.46, antiderivative size = 562, normalized size of antiderivative = 0.97, number of steps used = 30, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {6303, 5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}\,e}{\sqrt{c^2d}}\right.}{2e^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcSech[c*x])/(2*e*(e + d/x^2)) + (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh
[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2
*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + ((a + b*ArcSech[c*x])*Log[1
 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a
+ b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d
 + e])])/(2*e^2) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x]
)/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSech[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) - ((a + b*Ar
cSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/e^2 + (b*PolyLog[2, -((c*Sqrt[-d]*
E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e^2) + (b*PolyLog[2, (c*S
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + (b*PolyLog[
2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*e^2) + (
b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e
^2) - (b*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*e^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e^2x} - \frac{dx\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{dx\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\text{Subs}}{}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int(a+bx)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{d\,\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2be^2} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(cx)\right.}{e^2}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2be^2} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{2\operatorname{sech}^{-1}(cx}\right.}{e^2}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\mathrm{l}}{e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\mathrm{l}}{}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\mathrm{l}}{}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\mathrm{l}}{}$$

Mathematica [C] time = 1.29, size = 1208, normalized size = 2.08

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (
b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*b*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[A
rcSech[c*x]/2])/Sqrt[c^2*d + e]] + (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sq
rt[c^2*d + e]] - 4*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + 2*b*ArcSec
h[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
- (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (4*I)*b
*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 - (I*
(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[S
qrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d
+ e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] +
 Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*
Sqrt[d]*E^ArcSech[c*x])] + 2*b*Log[x] + 2*a*Log[d + e*x^2] - 2*b*Log[1 + Sq
rt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]] + (b*Sqrt[e]*Log[(
(2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[
e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e]
 + (b*Sqrt[e]*Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
 + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*
x)])/Sqrt[c^2*d + e] + 2*b*PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*b*PolyLog[2
, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*Pol
yLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*
PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] -
 2*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
)/(4*e^2)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^3\,\mathrm{arsech}\,(cx)+ax^3}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsech(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\mathrm{arsech}\,(cx)+a)x^3}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

maple [C] time = 1.11, size = 661, normalized size = 1.14

$$\frac{c^2ad}{2e^2\left(c^2x^2e+c^2d\right)}+\frac{a\ln\left(c^2x^2e+c^2d\right)}{2e^2}-\frac{c^2b\,x^2\mathrm{arcsech}\,(cx)}{2\left(c^2x^2e+c^2d\right)e}-\frac{b\sqrt{e\left(c^2d+e\right)}\;\mathrm{arctanh}\left(\frac{2c^2d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2+2c^2d+4}{4\sqrt{c^2de+e^2}}\right.}{2e^2\left(c^2d+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a/e^2*d/(c^2*e*x^2+c^2*d)+1/2*a/e^2*ln(c^2*e*x^2+c^2*d)-1/2*c^2*b*x
^2*arcsech(c*x)/(c^2*e*x^2+c^2*d)/e-1/2*b*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)
*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*
e)/(c^2*d*e+e^2)^(1/2))+1/4*c^2*b/e^2*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)
*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((
_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*
c^2*d+4*e)*_Z^2+c^2*d))*d-b/e^2*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2)))-b/e^2*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2)))-b/e^2*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2
*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*b/e^2*sum((_R1^2*c
^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{d}{e^{3}x^{2}+de^{2}}+\frac{\log\left(ex^{2}+d\right)}{e^{2}}\right)+b\int\frac{x^{3}\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{e^{2}x^{4}+2\,dex^{2}+d^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(sqrt
(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^{3}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^{2}+d\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}\left(a+b\,\mathrm{asech}\,(cx)\right)}{\left(d+ex^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.117 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2de}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/e/(e*x^2+d)+1/2*b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/d/e-1/2*b*arctanh(e^(1/2)*(-c^2*x^2+1)^(1/2)/(c^
2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/e^(1/2)/(c^2*d+e)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6299, 517, 446, 86, 63, 208}

$$-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2de}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcSech[c*x])/(2*e*(d + e*x^2)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(2*d*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])/Sqrt[c^2*d + e]])/(2*d*Sqrt[e]*Sq
rt[c^2*d + e])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{2e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}\left(d+ex^2\right)}\,dx}{2e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}(d+ex)}\,dx, x, x^2\right)}{4e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}(d+ex)}\,dx, x, x^2\right)}{4d} - \frac{\left(b\sqrt{\frac{1}{1+}}\right.}{}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{2c^2d} + \frac{(b}{}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2de} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan}{2d\sqrt{e}\sqrt{c}}$$

Mathematica [C] time = 1.05, size = 345, normalized size = 2.35

$$-\frac{\frac{2a}{d+ex^2} + \frac{b\sqrt{e}\log\left(\frac{4\left(\frac{c^2d^{3/2}\sqrt{e}x+ide}{\sqrt{c^2d+e}\left(\sqrt{d}+i\sqrt{e}x\right)}+\frac{de\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{ex-i\sqrt{d}\sqrt{e}}\right)}{b}\right)}{d\sqrt{c^2d+e}} + \frac{b\sqrt{e}\log\left(\frac{4\left(\frac{de+ic^2d^{3/2}\sqrt{e}x}{\sqrt{c^2d+e}\left(\sqrt{e}x+i\sqrt{d}\right)}+\frac{de\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{ex+i\sqrt{d}\sqrt{e}}\right)}{b}\right)}{d\sqrt{c^2d+e}} + \frac{2b\operatorname{sech}^{-1}(cx)}{d+ex^2} - \frac{2b\log\left(cx\right.}{}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*((2*a)/(d + e*x^2) + (2*b*ArcSech[c*x])/(d + e*x^2) + (2*b*Log[x])/d -
 (2*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/d
 + (b*Sqrt[e]*Log[(4*((I*d*e + c^2*d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(Sqr
```

```
t[d] + I*Sqrt[e]*x)) + (d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/((-I)*Sqrt
[d]*Sqrt[e] + e*x)))/b])/(d*Sqrt[c^2*d + e]) + (b*Sqrt[e]*Log[(4*((d*e + I*
c^2*d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x)) + (d*e*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(I*Sqrt[d]*Sqrt[e] + e*x)))/b])/(d*Sqrt[c
^2*d + e]))/e
```

fricas [B] time = 0.66, size = 602, normalized size = 4.10

$$\left[-\frac{2\,ac^2d^2 + 2\,ade - \sqrt{c^2de + e^2}\left(bex^2 + bd\right)\log\left(\frac{c^4d^2+4\,c^2de-\left(c^4de+2\,c^2e^2\right)x^2+4\left(c^3de+ce^2\right)x\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+4\,e^2+2\left(c^2ex^2-c^2d-\left(c^3d+2\,c\cdots\right.\right.}{ex^2+d}\right.}{4\left(c^2d^3e + d^2e^2\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*e - sqrt(c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c^4
*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d)) +
2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) - 1)/x) + 2*(b*c^2*d^2 + b*d*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2), -
1/2*(a*c^2*d^2 + a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*arctan((sqrt(
-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)
*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + (b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*
e^2)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^2 + b*
d*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e + d^2*
e^2 + (c^2*d^2*e^2 + d*e^3)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx) + a)x}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^2, x)
```

maple [B] time = 0.10, size = 840, normalized size = 5.71

$$-\frac{c^2a}{2e\left(c^2x^2e + c^2d\right)} - \frac{c^2b\,\mathrm{arcsech}\,(cx)}{2e\left(c^2x^2e + c^2d\right)} - \frac{c^3b\sqrt{-\frac{cx-1}{cx}}\;x\sqrt{\frac{cx+1}{cx}}\;\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2\sqrt{-c^2x^2+1}\left(\sqrt{-c^2de}+e\right)\left(\sqrt{-c^2de}-e\right)} + \frac{c^3b\sqrt{-\frac{cx-1}{cx}}\;x\sqrt{\frac{cx+1}{cx}}\;\ln\left(-\frac{2\left(\sqrt{-\cdots}\right.}{\cdots}\right.}{4\sqrt{-c^2x^2+1}\left(\sqrt{-c^2de}+e\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arcsech(c*x)-1
/2*c^3*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/((-c
^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)*arctanh(1/(-c^2*x^2+1)^(1/2))+1/4*c^3
```

```
*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/((-c^2*d*e
)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/
2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))
+1/4*c^3*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/((
-c^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(2*((-c^2*x^2
+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(
1/2)))-1/2*c*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2
)/d/((-c^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)*arctanh(1/(-c^2*x^2+1)^(1/2))
*e+1/4*c*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/d/
((-c^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(-2*((-c^2*
x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*
e)^(1/2)))*e+1/4*c*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1
)^(1/2)/d/((-c^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(
2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+
(-c^2*d*e)^(1/2)))*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(2\,c^2\int \frac{x^3}{2\left(c^2d^2x^2+\left(c^2dex^2-de\right)x^2+\left(c^2d^2x^2+\left(c^2dex^2-de\right)x^2-d^2\right)\sqrt{cx+1}\,\sqrt{-cx+1}\,-d^2\right)}\,dx+\frac{x^2\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*integrate(1/2*x^3/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 + (c^2*
d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 - d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - d^2
), x) + (x^2*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - x^2*log(c) - x^2*log(x
))/(d*e*x^2 + d^2) - 2*integrate(1/2*x/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x
^2 - d^2), x))*b - 1/2*a/(e^2*x^2 + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + b \operatorname{asech}{\left(c x \right)}\right)}{\left(d + e x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.118 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=542

$$-\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d^2}-\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1\right.}{2d^2}$$

```
[Out] -1/2*e*(a+b*arcsech(c*x))/d^2/(e+d/x^2)+1/2*(a+b*arcsech(c*x))^2/b/d^2-1/2*
(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2
*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/
2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,-c*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
^2-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+
1/c^2/x^2)^(1/2))*e^(1/2)*(-1+1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)^(1/2)/(-1+1/c/
x)^(1/2)/(1+1/c/x)^(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6303, 5792, 5788, 519, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}}{\sqrt{c^2}}\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -(e*(a + b*ArcSech[c*x]))/(2*d^2*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2*b
*d^2) + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e
]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqr
t[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])
/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c*S
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*Arc
Sech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^Ar
cSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt
[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2,
 (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2} + \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x \cdots\right.}{2(-d)^{3/2}}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{(a+b\cdots}{\frac{\sqrt{e}}{c}+\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\cdots}\right.}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\cdots}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\cdots}\right.}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\cdots}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\cdots}\right.}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\cdots}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\cdots}\right.}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\cdots}$$

Mathematica [F] time = 42.09, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2), x]
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsech}\,(cx)+a}{e^2x^5+2\,dex^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\mathrm{arsech}\,(cx)+a}{\left(ex^2+d\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x), x)
```

maple [C] time = 2.17, size = 3326, normalized size = 6.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x)
```

```
[Out] 1/8*b*c^2*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*b/d^2*s
um((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-
1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
-1/4*b/d^2*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2
*d-2*(e*(c^2*d+e))^(1/2)-2*e))-b/c^2/d^3*e*polylog(2,c^2*d*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-b/c^4/d^4*e^
2*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(
c^2*d+e))^(1/2)-2*e))+1/4*b*c^2/d/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*b*c^2/d
/(c^2*d+e)*arcsech(c*x)^2+1/2*b/c^2/d^3*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*d+e))^(
1/2)-b/c^2/d^3*arcsech(c*x)^2*(e*(c^2*d+e))^(1/2)+2*b/c^2/d^3*arcsech(c*x)^
2*e+2*b/c^4/d^4*e^2*arcsech(c*x)^2-1/2*b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*
arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e
)/(c^2*d*e+e^2)^(1/2))-5/2*b/d^2/(c^2*d+e)*arcsech(c*x)^2*e-3/4*b/d^2/(c^2*
d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(
e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*d+e))^(1/2)+5/4*b/d^2*e/(c^2*d+e)*polylog(
2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^
(1/2)-2*e))+b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arcsech(c*x)^2+1/4*b*(e*(c^
2*d+e))^(1/2)/d^2/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*a*c^2/d/(c^2*e*x^2+c^2*
d)+a/d^2*ln(c*x)-1/2*a/d^2*ln(c^2*e*x^2+c^2*d)-1/4*b*c^2/d/e/(c^2*d+e)*ln(1
-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(
1/2)-2*e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)-3*b/c^2/d^3/(c^2*d+e)*ln(1-c^2*
d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-
2*e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)*e+1/4*b*c^2*(e*(c^2*d+e))^(1/2)/d/e/
```

```
(c^2*d+e)*arcsech(c*x)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^
2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-2*b/c^4/d^4*e^2/(c^2*d+e)*ln(1-c^2*d*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*
e))*arcsech(c*x)*(e*(c^2*d+e))^(1/2)-1/2*b/d^2*ln(1-c^2*d*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-
1/2*b*c^2*x^2*arcsech(c*x)*e/(c^2*e*x^2+c^2*d)/d^2+b*arcsech(c*x)^2/d^2-2*b
/c^4/d^4*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e
*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*e^2+b/c^2/d^3*ln(1-c^2*d*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c
*x)*(e*(c^2*d+e))^(1/2)-3/2*b/d^2/(c^2*d+e)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*(e*
(c^2*d+e))^(1/2)+b/c^4/d^4*e^3/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-b/c^4/d^4*e^2/
(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*
d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*d+e))^(1/2)+3*b/c^2/d^3/(c^2*d+e)*arc
sech(c*x)^2*(e*(c^2*d+e))^(1/2)*e-3/2*b/c^2/d^3/(c^2*d+e)*polylog(2,c^2*d*(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e
))*(e*(c^2*d+e))^(1/2)*e+4*b/c^2/d^3/(c^2*d+e)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*
e^2-2*b/c^4/d^4*e*arcsech(c*x)^2*(e*(c^2*d+e))^(1/2)+5/2*b/d^2*e/(c^2*d+e)*
ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e
))^(1/2)-2*e))*arcsech(c*x)+1/2*b*c^2/d/(c^2*d+e)*ln(1-c^2*d*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*
x)+2*b/c^2/d^3/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e^2-2*b/c^4/d^4*e^3/(c^2*d+e)*
arcsech(c*x)^2-2*b/c^2/d^3*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*e+1/2*b*(e*(c^2*d+e)
)^(1/2)/d^2/(c^2*d+e)*arcsech(c*x)*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-4*b/c^2/d^3/(c^2*d+e)*arc
sech(c*x)^2*e^2+b/c^4/d^4*e*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*d+e))^(1/2)-1/8*b*c
^2/d/e/(c^2*d+e)*polylog(2,c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2
/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(e*(c^2*d+e))^(1/2)+2*b/c^4/d^4*e^2/(c
^2*d+e)*arcsech(c*x)^2*(e*(c^2*d+e))^(1/2)+2*b/c^4/d^4*e^3/(c^2*d+e)*ln(1-c
^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/
2)-2*e))*arcsech(c*x)+2*b/c^4/d^4*ln(1-c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)*e*(e*(c^2*d+e
))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.119 $\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=840

$$\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\text{sech}^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\text{se\ldots}\right.}{4e^{5/2}}$$

```
[Out] x*(a+b*arcsech(c*x))/e^2-b*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c/e^2+3
/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arcsech(c*x))*
ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/
e^(5/2)-3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*polylog(
2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)
^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*
polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(
c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2
)-1/4*d*(a+b*arcsech(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arcsech
(c*x))/e^2/(d/x+(-d)^(1/2)*e^(1/2))+1/2*b*d*arctan((1+1/c/x)^(1/2)*(c*d-(-d
)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/e^2
/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)+1/2*b*d*arct
an((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d
)^(1/2)*e^(1/2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*
e^(1/2))^(1/2)
```

Rubi [A] time = 3.09, antiderivative size = 840, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {6303, 5792, 5662, 92, 205, 5707, 5802, 93, 5800, 5562, 2190, 2279, 2391}

$$\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\text{sech}^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\text{se\ldots}\right.}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(d*(a + b*ArcSech[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*ArcS
ech[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSech[c*x]))/e^2
+ (b*d*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d +
Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqr
t[c*d + Sqrt[-d]*Sqrt[e]]*e^2) + (b*d*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*
Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*S
qrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*ArcTan[S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]])/(c*e^2) + (3*Sqrt[-d]*(a + b*ArcSech[
c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*
e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c
*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSech
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4
```

```
*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2,
-((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*e^(5/2)) +
(3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d
+ e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x]
)/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\,\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\,\mathrm{Subst}\left(\int\left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx, x$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\mathrm{Subst}\left(\int \frac{a}{\ }\right.}{\ }$$

$$= \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+\frac{x}{c}}\,\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\,\mathrm{Subst}\left(\int\left(\frac{a+b\cosh^{-1}(}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\right.}\right.\right.}{\ }$$

$$= \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} + \frac{d\,\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{d\,\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,d\right.}{2e^{5/2}}$$

$$= -\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} - \frac{b\tan^{-1}\left(\sqrt{-\right.}{\ }$$

$$= -\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} - \frac{b\tan^{-1}\left(\sqrt{-\right.}{\ }$$

$$= -\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} + \frac{bd\tan^{-1}\left(\right.}{2\sqrt{cd-\sqrt{-d}}}$$

$$= -\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} + \frac{bd\tan^{-1}\left(\right.}{2\sqrt{cd-\sqrt{-d}}}$$

$$= -\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} + \frac{bd\tan^{-1}\left(\right.}{2\sqrt{cd-\sqrt{-d}}}$$

$$= -\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} + \frac{bd\tan^{-1}\left(\right.}{2\sqrt{cd-\sqrt{-d}}}$$

$$= -\frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} + \frac{bd\tan^{-1}\left(\right.}{2\sqrt{cd-\sqrt{-d}}}$$

Mathematica [C] time = 1.77, size = 1270, normalized size = 1.51

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (4*a*Sqrt[e]*x + (2*a*d*Sqrt[e]*x)/(d + e*x^2) + 4*b*Sqrt[e]*x*ArcSech[c*x]
 + (b*d*ArcSech[c*x])/((-I)*Sqrt[d] + Sqrt[e]*x) + (b*d*ArcSech[c*x])/(I*Sq
rt[d] + Sqrt[e]*x) - 6*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (8*b*Sqrt[e]
*ArcTan[Tanh[ArcSech[c*x]/2]])/c + 12*b*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x
]/2])/Sqrt[c^2*d + e]] - 12*b*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^
2*d + e]] + (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[d]*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 - (I*(Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[d]*ArcSin[Sqrt[1
 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])
)/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(Sq
rt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[d]*ArcSin[
Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d
 + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (I*b*Sqrt[d]*Sqrt[e]*Log[((2*I)*Sqrt[
e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*
d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (I*b*Sqr
t[d]*Sqrt[e]*Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
+ (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x
)])/Sqrt[c^2*d + e] + (3*I)*b*Sqrt[d]*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*PolyLog[2, (I*(-Sqr
t[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*Poly
Log[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*
I)*b*Sqrt[d]*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])])/(4*e^(5/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arsech}(cx) + ax^4}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^4}{\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

maple [C] time = 18.23, size = 2016, normalized size = 2.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] a*x/e^2+1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d)-3/2*a/e^2*d/(d*e)^(1/2)*arctan(
x*e/(d*e)^(1/2))+c^2*b*x^3*arcsech(c*x)/e/(c^2*e*x^2+c^2*d)+3/2*c^2*b*arcse
ch(c*x)/e^2*d*x/(c^2*e*x^2+c^2*d)-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d
+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d/e^2-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/
((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^2/e^2*(e*(c^2*d+e))^(1/2)-1
/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))
/d^2/e+1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)
*d)^(1/2))/e^2/(c^2*d+e)/d*(e*(c^2*d+e))^(1/2)+1/c^2*b*(-(c^2*d-2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d+1/c^4*b*(-(c
^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e
)/d^2*(e*(c^2*d+e))^(1/2)+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2
*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^2-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d/e^2+1/c^4*b*((c^2*d+2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2/e^2*(e*(c^2*d+e))^(1/2)
-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/
d^2/e-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2))/e^2/(c^2*d+e)/d*(e*(c^2*d+e))^(1/2)+1/c^2*b*((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d-1/c^4*b*((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(
c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(
c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2))/(c^2*d+e)/d^2-2/c*b/e^2*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))-3/16*c*b/e^3*d*sum((_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*
d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+d
ilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z
^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/16*c*b/e^3*d*sum((_R1^2*c^2*d+4*_R1^2*e+c^2
*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/
_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

3.120 $\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=786

$$\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\right.}{4\sqrt{-d}\,e^{3/2}}$$

```
[Out] 1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arcsech(c*x))
*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d
+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-
d)^(1/2)-1/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*polylog
(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b
*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/
2)+1/4*(a+b*arcsech(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arcsech(c*x
))/e/(d/x+(-d)^(1/2)*e^(1/2))-1/2*b*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*
e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/e/(c*d-(-d)
^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)-1/2*b*arctan((1+1/c/x)
^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1
/2))^(1/2))/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] time = 1.57, antiderivative size = 786, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6303, 5707, 5802, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-}}{\sqrt{c^2}}\right.}{4\sqrt{-d}\,e^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcSech[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/
(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sq
rt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqr
t[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - (b*ArcTan[(Sqrt
[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*S
qrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sq
rt[e]]*e) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt
[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^
(3/2)) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
 + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1 +
(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/
2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e
]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqr
```

```
t[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d
]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*
PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqr
t[-d]*e^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right.\right.}{2e\left(-de-d^2x^2\right)}\right.\right.$$

$$= \frac{d\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\,\text{Subst}\left(\right.}{}$$

$$= \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}\,dx, x\right.}{4ce}$$

$$= \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}} - \frac{\text{Subs}}{}$$

$$= \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}\,e} - \frac{}{2}$$

$$= \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}\,e} - \frac{}{2}$$

$$= \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}\,e} - \frac{}{2}$$

$$= \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}\,e} - \frac{}{2}$$

$$= \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}\,e} - \frac{}{2}$$

Mathematica [C] time = 1.75, size = 1226, normalized size = 1.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*a*Sqrt[e]*x)/(d + e*x^2) + (b*ArcSech[c*x])/(I*Sqrt[d] - Sqrt[e]*x) -
(b*ArcSech[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
```

```
)/Sqrt[d] - (4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[
(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d]
 + (4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqr
t[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] - (I*b*ArcS
ech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
])/Sqrt[d] - (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 +
 (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d] + (I*
b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])])/Sqrt[d] + (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*
Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[
d] + (I*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])])/Sqrt[d] - (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])
/Sqrt[d] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqr
t[d]*E^ArcSech[c*x])])/Sqrt[d] + (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c
*x])])/Sqrt[d] + (I*b*Sqrt[e]*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqr
t[d] + Sqrt[e]*x)])/(Sqrt[d]*Sqrt[c^2*d + e]) - (I*b*Sqrt[e]*Log[(2*Sqrt[e]
*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*
d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/(Sqrt[d]*Sqrt[c^2*d + e
]) - (I*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])])/Sqrt[d] + (I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])])/Sqrt[d] + (I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d] - (I*b*PolyLog[2, (I*(Sqrt[e
] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d])/(4*e^(3/2))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

maple [C] time = 5.84, size = 1880, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)-1/2*c^2*b*arcsech(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/4*c*b/e*sum(_R1/(_R1^2*c^2
*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c
```

```
^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/
2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^
2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^2/e+1/c^4*b*(-(c^2*d-2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/d^3*(e*(c^2*d+e))^(1/2)+
1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2)
)/d^3-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*
d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-1/c^2*b*(-(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^2-1/c^4*b*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d
^3*(e*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+
e))^(1/2)-2*e)*d)^(1/2))*e/(c^2*d+e)/d^3+1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2/e-1/c^4*b*((c^2*d+2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/d^3*(e*(c^2*d+e))^(1/2)+1/
c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^3
+1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
)/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-1/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^2+1/c^4*b*((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3*(e*(c^2*d+e
))^(1/2)-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2))*e/(c^2*d+e)/d^3+1/4*c*b/e*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsec
h(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/
x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*
e)*_Z^2+c^2*d))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{asech}{\left(c x \right)}\right)}{\left(d + e x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.121 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=786

$$-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log}{4(-d)^{3/}}$$

```
[Out] -1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsech(c*x)
)*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2
)/e^(1/2)+1/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylo
g(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+
e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*
b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1
/2)+1/4*(-a-b*arcsech(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arcsech(c*
x))/d/(d/x+(-d)^(1/2)*e^(1/2))+1/2*b*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)
*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/d/(c*d-(-d
)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)+1/2*b*arctan((1+1/c/x
)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(
1/2))^(1/2))/d/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2
)
```

Rubi [A] time = 2.84, antiderivative size = 786, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {6293, 5792, 5707, 5802, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{c\sqrt{}}{}\right)}{4(-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcSech[c*x])/(4*d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSech[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*S
qrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d*
Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) + (b*ArcTan[(Sqr
t[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*
Sqrt[-1 + 1/(c*x)])])/(2*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]
*Sqrt[e]]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqr
t[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])*Lo
g[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/
2)*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sq
rt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])*L
og[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3
/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
```

```
c^2*d + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2,
 -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(3/2)
*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d
 + e])])/(4*(-d)^(3/2)*Sqrt[e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6293

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(2*(p + 1
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{e\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int\left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)}{d}+\frac{e\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\right.\right.}{}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right)-\frac{1}{4}\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)-\frac{1}{2}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)}-\frac{a+}{2d\sqrt{e}}\right.\right.$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\,\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+}}{\sqrt{-1-}}\right.}{2cd}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\,\text{ta}}{2d\sqrt{cd}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\,\text{ta}}{2d\sqrt{cd}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\,\text{ta}}{2d\sqrt{cd}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\,\text{ta}}{2d\sqrt{cd}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\,\text{ta}}{2d\sqrt{cd}}$$

Mathematica [C] time = 1.79, size = 1216, normalized size = 1.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*Sqrt[d]*x)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/((-I)*Sqrt[d]*Sqrt[
e] + e*x) + (b*Sqrt[d]*ArcSech[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + (2*a*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqr
t[c^2*d + e]])/Sqrt[e] + (4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt
[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]
])/Sqrt[e] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech
[c*x])])/Sqrt[e] + (I*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] + (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])])/Sqrt[e] + (I*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (2*b*ArcSin[Sqrt[1 - (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqr
t[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sq
rt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] + (2*b*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d
 + e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (I*b*Log[(2*Sqrt[e]*(I*
Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)
/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] - (I*b*Poly
Log[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqr
t[e] + (I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])])/Sqrt[e] + (I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d
 + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e])/(4*d^(3/2))
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arsech}(cx) + a}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^2, x)
```

maple [C] time = 6.06, size = 1870, normalized size = 2.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/2*a/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
+1/2*c^2*b*arcsech(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^3-1/c^4*b*(-(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4*(e*(c^2*d+e))^(
1/2)-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^
(1/2))/d^4*e+1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh
(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2
)-2*e)*d)^(1/2))/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+1/c^2*b*(-(c^2*d-2*(e*(c
^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))*e/(c^2*d+e)/d^3+1/c^4*
b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/
(c^2*d+e)*(e*(c^2*d+e))^(1/2)*e+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(
e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*e^2-1/2/c^2*b*((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^3+1/c^4*b*((c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4*(e*(c^2*d+e))^(
1/2)-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2))/d^4*e-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2))/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+1/c^2*b*((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(
(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/(c^2*d+e)/d^3-1/c^4*b*((c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e
*(c^2*d+e))^(1/2)*e+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arc
tan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*e^2-1/4*c*b/d*sum(_R1/(_R1^2*c^2*d+c^2*d+2
*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilo
g((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+
(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*c*b/d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arc
sech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1
/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d
+4*e)*_Z^2+c^2*d))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asech}(cx)}{(d + ex^2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x**2)**2, x)
```

3.122 $\int \frac{a+b\textbf{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=844

$$-\frac{a}{d^2x}-\frac{b\text{sech}^{-1}(cx)}{d^2x}+\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{be\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{be\tan^{-1}}{2d^2\sqrt{cd-}}$$

```
[Out] -a/d^2/x-b*arcsech(c*x)/d^2/x-3/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d
)^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*arcse
ch(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^
(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*polylog(2,
c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*po
lylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arcsech(c*x))/d^2/(-d/x+(-d)^(1
/2)*e^(1/2))-1/4*e*(a+b*arcsech(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+b*c*(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)/d^2-1/2*b*e*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(
1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/d^2/(c
*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)-1/2*b*e*arctan(
(1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(
1/2)*e^(1/2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(
1/2))^(1/2)
```

Rubi [A] time = 2.95, antiderivative size = 844, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {6303, 5792, 5654, 74, 5707, 5802, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$-\frac{a}{d^2x}-\frac{b\text{sech}^{-1}(cx)}{d^2x}+\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{be\tan^{-1}\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{be\tan^{-1}}{2d^2\sqrt{cd-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d^2 - a/(d^2*x) - (b*ArcSech[c*x
])/(d^2*x) + (e*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e
*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTan[(Sqrt
[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*S
qrt[-1 + 1/(c*x)])]])/(2*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d
]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/
(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])]])/(2*d^2*Sqrt[c*d - Sqrt[
-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcSech[c*x]
)*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)
^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x
])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcSech
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4
```

```
*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSec
h[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog
[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*(-d)^(5
/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcS
ech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*Poly
Log[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5
/2))
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{d^2} + \frac{e^2\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} - \frac{2e\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)dx, x\cdots$$

$$= -\frac{\text{Subst}\left(\int\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Su}\cdots}{\cdots}$$

$$= -\frac{a}{d^2x} - \frac{b\,\text{Subst}\left(\int \cosh^{-1}\left(\frac{x}{c}\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int\left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\cosh^{-1}\cdots}{2\sqrt{e}\left(\sqrt{e}+\sqrt{\cdots}\right.}\cdots\right.\right.}{d^2}$$

$$= -\frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{cd^2} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\cdots\right.}{d^2}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{sech}^{-1}(c\cdots\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\cdots\right.}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{sech}^{-1}(c\cdots\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\cdots\right.}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{sech}^{-1}(c\cdots\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\cdots\right.}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{sech}^{-1}(c\cdots\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\cdots\right.}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{sech}^{-1}(c\cdots\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\cdots\right.}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{sech}^{-1}(c\cdots\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\cdots\right.}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{sech}^{-1}(c\cdots\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\cdots\right.}$$

Mathematica [C] time = 1.39, size = 1305, normalized size = 1.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-4*a*Sqrt[d])/x + 4*b*c*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)] + (4*b*Sqrt[d]*
Sqrt[(1 - c*x)/(1 + c*x)])/x - (2*a*Sqrt[d]*e*x)/(d + e*x^2) - (4*b*Sqrt[d]
*ArcSech[c*x])/x - (b*Sqrt[d]*e*ArcSech[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x)
- (b*Sqrt[d]*e*ArcSech[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - 6*a*Sqrt[e]*ArcTan
[(Sqrt[e]*x)/Sqrt[d]] + 12*b*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[
c^2*d + e]] - 12*b*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]
]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] +
 (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*
x])] - (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])
)/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
cSech[c*x])] - (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d
]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[e]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*
Sqrt[d]*E^ArcSech[c*x])] + (I*b*e*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x
)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I
*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] - (I*b*e*Log[(2*Sqrt[e]*(I*Sqrt[d]*
Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^
2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (3*I)*b*Sqrt[e]*P
olyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] -
 (3*I)*b*Sqrt[e]*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^A
rcSech[c*x])] - (3*I)*b*Sqrt[e]*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[e]*PolyLog[2, (I*(Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/(4*d^(5/2))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsech}\,(cx) + a}{e^2x^6 + 2\,dex^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx) + a}{\left(ex^2 + d\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

maple [C] time = 12.21, size = 1952, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2*a/d^2*e*c^2*x/(c^2*e*x^2+c^2*d)-3/2*a/d^2*e/(d*e)^(1/2)*arctan
(x*e/(d*e)^(1/2))-b*arcsech(c*x)/d^2/x-1/2*b*arcsech(c*x)/d^2*e*c^2*x/(c^2*
e*x^2+c^2*d)+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctanh(c
*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-
2*e)*d)^(1/2))/d^5*(e*(c^2*d+e))^(1/2)-b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*
d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*e^2-b/c^4*(-(c^2*d-2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^3*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)-
b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctan(c*d*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d
^5*(e*(c^2*d+e))^(1/2)-b/c^2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*ar
ctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*e^2-b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*e^3*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*
d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)-1/2*b/c^2*(-(c^2*d-2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(
c^2*d+e))^(1/2)*e-b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*ar
ctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))
^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/2*b/c^2*((c^2*d+2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(
c^2*d+e))^(1/2)*e+b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arc
tan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/2*b/c^2*(-(c^2*d-2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4*e+b/c^4*(-(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5+1/2*
b/c^2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4
*e+b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2))/d^5+c*b/d^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-3/4*c*b/d^2*e*sum(
1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R
1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/4*c*b/d^2*e*sum(_R1/
(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R
1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Timed out
```

3.123 $\int \frac{x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=778

$$\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^3}+\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(\right.}{2e^3}$$

```
[Out] 1/4*(-a-b*arcsech(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arcsech(c*x))/e^2/(e+d/x^2)
-(a+b*arcsech(c*x))^2/b/e^3-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)/e^3+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*a
rcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*
arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))^2)/e^3+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1
/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/8*b*d*(c^2-1/x^2)/c/e
^2/(c^2*d+e)/(e+d/x^2)/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/8*b*(c^2*d+2*e)
*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(
1/2)/e^(5/2)/(c^2*d+e)^(3/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*b*arctanh
((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(1/2)/e^(
5/2)/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)
```

Rubi [A] time = 1.71, antiderivative size = 760, normalized size of antiderivative = 0.98, number of steps used = 35, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6303, 5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 382, 377, 208, 5800, 5562}

$$\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}+\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{}}{}\right.}{2e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*d*(c^2 - x^(-2)))/(8*c*e^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*Sq
rt[1 + 1/(c*x)]*x) - (a + b*ArcSech[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcS
ech[c*x])/(2*e^2*(e + d/x^2)) + (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*
d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]*Sq
rt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2
)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(8*e^(5/2
)*(c^2*d + e)^(3/2)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + ((a + b*ArcSech
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2
*e^3) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
- Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^
ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSech[c*x])
```

```
*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3)
- ((a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/e^3 + (b*PolyLog[2, -(
(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e^3) + (b*Pol
yLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) +
 (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])
/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(2*e^3) - (b*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*e^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^5 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e^3 x} - \frac{dx\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3} - \frac{dx\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} - \ldots\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{e^3}\ldots$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2be^3} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,d\ldots\right.}{e^3}$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \ldots$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \ldots$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \ldots$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \ldots$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \ldots$$

Mathematica [C] time = 7.74, size = 2000, normalized size = 2.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*(-1/16*(d*(((-I)*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*
x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e
]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*
x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)
*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d
+ e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*
x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/
e^(5/2) - (d*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2
*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[
e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqr
t[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt
[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1
+ c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*(I
*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) - (((7*I)/16)
*Sqrt[d]*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[x]/Sqrt[e] -
Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e]
+ Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d
]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*
d + e]))/Sqrt[d]))/e^(5/2) + (((7*I)/16)*Sqrt[d]*(-(ArcSech[c*x]/((-I)*Sqrt
[d]*Sqrt[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)
] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt
[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d
+ e]))/((-I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/e^(5/2) + (P
olyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/S
qrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*
Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)
*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sq
rt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt
[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])]))/(4*e^3) - (-PolyLog[2, -E^(-2*ArcSech[c*x])]
+ 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I
)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x
]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c
^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*
E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*
Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/S
qrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
 + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x]
)] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
))/(4*e^3))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^5 \operatorname{arsech}(cx) + ax^5}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arcsech(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^5}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

maple [C] time = 1.58, size = 1779, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/4*b/e^2/(c^2*d+e)*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(ar
csech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-
1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*
d+4*e)*_Z^2+c^2*d))-b/e^2/(c^2*d+e)*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2)))-b/e^2/(c^2*d+e)*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2)))-1/2*c^6*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arcsech(c*x)*d^2*x^2-1/2*
c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arcsech(c*x)*d*x^2-3/4*c^6*b/e/(c^2*e
*x^2+c^2*d)^2/(c^2*d+e)*arcsech(c*x)*d*x^4+1/2*a/e^3*ln(c^2*e*x^2+c^2*d)+c^
2*a/e^3*d/(c^2*e*x^2+c^2*d)-1/4*c^4*a*d^2/e^3/(c^2*e*x^2+c^2*d)^2-c^2*b/e^3
/(c^2*d+e)*d*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-
c^2*b/e^3/(c^2*d+e)*d*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2)))+1/4*c^4*b/e^3/(c^2*d+e)*d^2*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*
(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_
R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c
^2*d+4*e)*_Z^2+c^2*d))+1/4*c^2*b/e^3/(c^2*d+e)*d*sum((_R1^2*c^2*d+c^2*d+4*e
)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),
_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/8*c^4*b/e^2/(c^2*e*x^2+c
^2*d)^2/(c^2*d+e)*d^2-c^2*b/e^3/(c^2*d+e)*d*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2)))+1/4*c^2*b/e^2/(c^2*d+e)*sum((_R1^2+1)/(_R1^2*c^2*d+c^2
*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+
dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_
Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))*d-c^2*b/e^3/(c^2*d+e)*d*dilog(1+I*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-3/4*c^4*b/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arc
sech(c*x)*x^4-1/8*c^5*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(-(c*x-1)/c/x)^(1
/2)*((c*x+1)/c/x)^(1/2)*x*d^2-1/8*c^5*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(-(
c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3*d+1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2
/(c^2*d+e)*d*x^2-5/8*c^2*b*(e*(c^2*d+e))^(1/2)/e^3/(c^2*d+e)^2*d*arctanh(1/
4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e
+e^2)^(1/2))+1/8*c^4*b/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*x^4-3/4*b*(e*(c^2*d+e)
)^(1/2)/e^2/(c^2*d+e)^2*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-b/e^2/(c^2*d+e)*arcsech(c*x)
*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2/(c^2*d+e)*arcsech(c
*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a\left(\frac{4\,dex^2 + 3\,d^2}{e^5x^4 + 2\,de^4x^2 + d^2e^3} + \frac{2\,\log\left(ex^2 + d\right)}{e^3}\right) + b\int \frac{x^5 \log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*
x))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.124 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=173

$$\frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(c^2d+2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}$$

```
[Out] 1/4*x^4*(a+b*arcsech(c*x))/d/(e*x^2+d)^2-1/8*b*(c^2*d+2*e)*arctanh(e^(1/2)*
(-c^2*x^2+1)^(1/2)/(c^2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/e^(3/
2)/(c^2*d+e)^(3/2)+1/8*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
/e/(c^2*d+e)/(e*x^2+d)
```

Rubi [A] time = 0.19, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {264, 6301, 12, 446, 78, 63, 208}

$$\frac{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(c^2d+2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(8*e*(c^2*d + e)*(
d + e*x^2)) + (x^4*(a + b*ArcSech[c*x]))/(4*d*(d + e*x^2)^2) - (b*(c^2*d +
2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])
/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx &= \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{4d\sqrt{1-c^2x^2}\left(d+ex^2\right)^2}\,dx\\
&= \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{\sqrt{1-c^2x^2}\left(d+ex^2\right)^2}\,dx}{4d}\\
&= \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{x}{\sqrt{1-c^2x}\,(d+ex)^2}\,dx,x,x^2\right)}{8d}\\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{\left(b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+\cdots\right.}{16\cdots}\\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{\left(b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+\cdots\right.}{8\cdots}\\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+c\cdots}}{8de^{3/2}\left(c^2d\cdots\right.}
\end{aligned}$$

Mathematica [C] time = 1.59, size = 486, normalized size = 2.81

$$-\frac{\frac{8a}{d+ex^2}-\frac{4ad}{\left(d+ex^2\right)^2}+\frac{b\sqrt{e}\left(c^2d+2e\right)\log\left(\frac{16de^{3/2}\sqrt{c^2d+e}\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e}+\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e}-ic^2\sqrt{d}\,x+\sqrt{e}\right)}{b\left(c^2d+2e\right)\left(\sqrt{e}\,x-i\sqrt{d}\right)}\right)}{d\left(c^2d+e\right)^{3/2}}+\frac{b\sqrt{e}\left(c^2d+2e\right)\log\left(\frac{16de^{3/2}\sqrt{c^2d+e}\left(cx\cdots\right.}{\cdots}\right.}{d\left(\cdots\right.}}{16e^{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*e*Sqrt[(1 - c*x)/(1
+ c*x)]*(b + b*c*x))/((c^2*d + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcSech
[c*x])/(d + e*x^2)^2 + (4*b*Log[x])/d - (4*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*
x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*
d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt
[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*
(c^2*d + 2*e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2)) + (b*Sqrt
[e]*(c^2*d + 2*e)*Log[(16*d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d
]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[
(1 - c*x)/(1 + c*x)]))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*
d + e)^(3/2)))/e^2
```

fricas [B] time = 0.90, size = 1346, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 2*(4*a - b)*c^2*d^3*e + 2*(2*a - b)*d^2*e^2 - 2*(b*c^
2*d*e^3 + b*e^4)*x^4 + 4*(2*a*c^4*d^3*e + (4*a - b)*c^2*d^2*e^2 + (2*a - b)
*d*e^3)*x^2 - (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c
^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e - (
c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c
^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c
^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) - 1)/x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e
 + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
+ 1)/(c*x)) - 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e
^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e
^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3
*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + (4*a - b)*c^2*d^3*e + (2*a - b)*d
^2*e^2 - (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(2*a*c^4*d^3*e + (4*a - b)*c^2*d^2*e
^2 + (2*a - b)*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*
d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*arctan((sqrt(
-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)
*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2
*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c
^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x)
+ 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e
^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - ((
b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4
+ 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^
2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx) + a)x^3}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

maple [B] time = 0.11, size = 3331, normalized size = 19.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4
*b*arcsech(c*x)/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*b*arcsech(c*x)/e^2/(c^2*e
*x^2+c^2*d)-1/4*c^7*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^2/(-c*
x*e+(-c^2*d*e)^(1/2))*d/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-
c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))-1/4*c^
7*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e/(-c*x*e+(-c^2*d*e)^(1/2))*
d^2/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/
(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))+1/16*c^7*b*(-(c*x-1)/c/x)^
(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))*d/((c^2*d+e)/e)
^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)
^2/(-c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2
*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^7*b*(-(c*x-1)/c/x)^(1/
2)*x*((c*x+1)/c/x)^(1/2)*e/(-c*x*e+(-c^2*d*e)^(1/2))*d^2/((c^2*d+e)/e)^(1/2
)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-
c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)
^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x^
3*((c*x+1)/c/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))*d/((c^2*d+e)/e)^(1/2)/(
c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2
*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/
2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x
+1)/c/x)^(1/2)*e/(-c*x*e+(-c^2*d*e)^(1/2))*d^2/((c^2*d+e)/e)^(1/2)/(c*x*e+(
-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)
^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+
e)/(c*x*e+(-c^2*d*e)^(1/2)))-1/8*c^5*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)
^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))*d/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)
^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2-1/2*c^5*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x
+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^
2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2
*x^2+1)^(1/2))-1/2*c^5*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^2/(-c
*x*e+(-c^2*d*e)^(1/2))*d/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((
-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))+3/16*
c^5*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(
1/2))/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/(
(-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+
e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+3/16*c^5*b
*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))*d
/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2
*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)
^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+3/16*c^5*b*(-(c
*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/((c^
2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)
^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)
*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+3/16*c^5*b*(-(c*x-1)/c
/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))*d/((c^2*d+e)/
e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+
e)^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^
2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))-1/8*c^3*b*(-(c*x-1)/c/x)^(1/2
)*x*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/
2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2-1/4*c^3*b*(-(c*x-1)/c/x)^
(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^4/(-c*x*e+(-c^2*d*e)^(1/2))/d/(c*x*e+(-c^2*
d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2
```

```
)*arctanh(1/(-c^2*x^2+1)^(1/2))-1/4*c^3*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c
/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e
)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+
1)^(1/2))+1/8*c^3*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^4/(-c*x*
e+(-c^2*d*e)^(1/2))/d/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d
*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1
)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1
/2)))+1/8*c^3*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^
2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2
)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*
((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/
8*c^3*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^4/(-c*x*e+(-c^2*d*e)
^(1/2))/d/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)
^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2
*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+1/8*c^3*
b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/
((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*
d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(
1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.125 $\int \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=217

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4d^2e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\left(3c^2d+2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\left(c^2d+e\right)^{3/2}}-$$

```
[Out] 1/4*(-a-b*arcsech(c*x))/e/(e*x^2+d)^2+1/4*b*arctanh((-c^2*x^2+1)^(1/2))*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^2/e-1/8*b*(3*c^2*d+2*e)*arctanh(e^(1/2)*(-c^
2*x^2+1)^(1/2)/(c^2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*d+
e)^(3/2)/e^(1/2)-1/8*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d
/(c^2*d+e)/(e*x^2+d)
```

Rubi [A] time = 0.29, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6299, 517, 446, 103, 156, 63, 208}

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4d^2e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\left(3c^2d+2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\left(c^2d+e\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(8*d*(c^2*d + e)*
(d + e*x^2)) - (a + b*ArcSech[c*x])/(4*e*(d + e*x^2)^2) + (b*Sqrt[(1 + c*x)
^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*d^2*e) - (b*(3*c^2*d +
2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])
/Sqrt[c^2*d + e]])/(8*d^2*Sqrt[e]*(c^2*d + e)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\left(d+ex^2\right)^2}\,dx}{4e}$$

$$= -\frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}\left(d+ex^2\right)^2}\,dx}{4e}$$

$$= -\frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\,(d+ex)^2}\,dx, x, x^2\right)}{8e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\quad}{x} \right.}{8de\left(c^2d+\right.}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\quad}{x} \right.}{8d^2e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{c} \right.}{4c^2d^2e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{a+b\,\text{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-\right.}{4d^2e}$$

Mathematica [C] time = 1.10, size = 486, normalized size = 2.24

$$\frac{1}{16}\left(-\frac{4a}{e\left(d+ex^2\right)^2} - \frac{b\left(3c^2d+2e\right)\log\left(\frac{16d^2\sqrt{e}\sqrt{c^2d+e}\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e}+\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e}-ic^2\sqrt{d}\,x+\sqrt{e}\right)}{b\left(3c^2d+2e\right)\left(\sqrt{e}\,x-i\sqrt{d}\right)}\right)}{d^2\sqrt{e}\left(c^2d+e\right)^{3/2}} - \frac{b\left(3c^2d+2e\right)\log}{\quad}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) - (2*Sqrt[(1 - c*x)/(1 + c*x)]*(b + b*c*x))/(d*(c
^2*d + e)*(d + e*x^2)) - (4*b*ArcSech[c*x])/(e*(d + e*x^2)^2) - (4*b*Log[x]
)/(d^2*e) + (4*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1
+ c*x)]])/(d^2*e) - (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]*
(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*
Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*(3*c^2*d + 2*e)*((-I)*Sqrt
[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(c^2*d + e)^(3/2)) - (b*(3*c^2*d + 2*e)*Lo
g[(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d +
 e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x
)]))/(b*(3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(c^2*d + e)
^(3/2)))/16
```

fricas [B] time = 0.72, size = 1232, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 2*(4*a + b)*c^2*d^3*e + 2*(2*a + b)*d^2*e^2 + 2*(b*c^
2*d*e^3 + b*e^4)*x^4 + 4*(b*c^2*d^2*e^2 + b*d*e^3)*x^2 - (3*b*c^2*d^3 + (3*
b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)
*sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 +
 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^
2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^
2*d*e + e^2))/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*
c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2
 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 4*(b*c^4
*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
 1)/(c*x)) + 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^
2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3
+ (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*
e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + (4*a + b)*c^2*d^3*e + (2*a + b)*d^
2*e^2 + (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^2*d^2*e^2 + b*d*e^3)*x^2 + (3*b*
c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*
b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*arctan((sqrt(-c^2*d*e - e^2)*c*d*x*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)*(e*x^2 + d))/((c^2*d*e + e
^2)*x^2)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b
*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*
log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 2*(b*c^4*d^4 + 2*b*c^2*d^
3*e + b*d^2*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + ((b*
c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*
c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x}{(ex^2 + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^3, x)
```

maple [B] time = 0.10, size = 3289, normalized size = 15.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arcsech(c*
x)-1/4*c^7*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2
*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1
/2)+e)^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))-1/4*c^7*b*(-(c*x-
1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))*d/(c*x*e+
(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1
)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))+3/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x^3*(
(c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2)/(c*x*e
+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+
1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*
x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+3/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c
```

```
/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))*d/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*
d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2
)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c
*x*e+(-c^2*d*e)^(1/2)))+3/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(
1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1
/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(-2
*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+
(-c^2*d*e)^(1/2)))+3/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^
2/(-c*x*e+(-c^2*d*e)^(1/2))*d/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/
((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*ln(-2*((-c
^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2
*d*e)^(1/2)))+1/8*c^5*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^3/(-c*
x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^
2*d*e)^(1/2)+e)^2-1/2*c^5*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^
4/(-c*x*e+(-c^2*d*e)^(1/2))/d/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)
^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))-
1/2*c^5*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)
^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e
)^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))+5/16*c^5*b*(-(c*x-1)/c
/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^4/(-c*x*e+(-c^2*d*e)^(1/2))/d/((c^2*d+e
)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2
)+e)^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-
c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+5/16*c^5*b*(-(c*x-1)/c/x)^(
1/2)*x*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2
)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-
c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^
(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+5/16*c^5*b*(-(c*x-1)/c/x)^(1/2)*x^3*
((c*x+1)/c/x)^(1/2)*e^4/(-c*x*e+(-c^2*d*e)^(1/2))/d/((c^2*d+e)/e)^(1/2)/(c*
x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x
^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2
)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+5/16*c^5*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x
+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c
^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/(-c^2*x^2+1)^(
1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e
)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/8*c^3*b*e^4*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/
c/x)^(1/2)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(
1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/d-1/4*c^3*b*e^5*(-(c*x-1)/c/x)^(1/2)*x^3*(
(c*x+1)/c/x)^(1/2)/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^
2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2/d^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(
-c^2*x^2+1)^(1/2))-1/4*c^3*b*e^4*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)
/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/
((-c^2*d*e)^(1/2)+e)^2/d/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))+1
/8*c^3*b*e^5*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)/(-c*x*e+(-c^2*d*e
)^(1/2))/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^
2/((-c^2*d*e)^(1/2)+e)^2/d^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((
c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+1/8*c
^3*b*e^4*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c*x*e+(-c^2*d*e)^(1/2
))/((c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c
^2*d*e)^(1/2)+e)^2/d/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)
/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+1/8*c^3*b*e^5
*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)/(-c*x*e+(-c^2*d*e)^(1/2))/((c
^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e
)^(1/2)+e)^2/d^2/(-c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)
^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/8*c^3*b*e^4*(
-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c*x*e+(-c^2*d*e)^(1/2))/((c^2*d
+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1
/2)+e)^2/d/(-c^2*x^2+1)^(1/2)*ln(-2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)
*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-c^2*d*e)^(1/2)))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.126 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=741

$$-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d^3}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log}{2d^3}$$

```
[Out] 1/4*e^2*(a+b*arcsech(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arcsech(c*x))/d^3/(e+d/x^
2)+1/2*(a+b*arcsech(c*x))^2/b/d^3-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2
*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/
2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*b*polylo
g(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/8*b*e*
(c^2-1/x^2)/c/d^2/(c^2*d+e)/(e+d/x^2)/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/
8*b*(c^2*d+2*e)*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*e
^(1/2)*(-1+1/c^2/x^2)^(1/2)/d^3/(c^2*d+e)^(3/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^
(1/2)+b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*e^(1/2)*(
-1+1/c^2/x^2)^(1/2)/d^3/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)
```

Rubi [A] time = 1.54, antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6303, 5792, 5788, 519, 382, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}\,e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}-\frac{b\,\mathrm{PolyLog}\left(2,\frac{c}{}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -(b*e*(c^2 - x^(-2)))/(8*c*d^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*S
qrt[1 + 1/(c*x)]*x) + (e^2*(a + b*ArcSech[c*x]))/(4*d^3*(e + d/x^2)^2) - (e
*(a + b*ArcSech[c*x]))/(d^3*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2*b*d^3)
 + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqr
t[-1 + 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1
/(c*x)]) - (b*Sqrt[e]*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2
*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d + e)^(3/2)*Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[
-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSech
[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2
*d^3) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
+ Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^
ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLog[2, -((c*Sq
```

```
rt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d^3) - (b*PolyLog[
2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - (b*P
olyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d
^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])
])/(2*d^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\,\text{Subst}\left(\int \frac{x^5\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\,\text{Subst}\left(\int\left(\frac{e^2x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2ex\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right) d$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Subst}\left(\right.}{}$$

$$= \frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{\sqrt{-d}\left(a+b\,\mathrm{c}\right.}{2d\left(\sqrt{e}+\right.}\right.\right.}{d^2}$$

$$= \frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{sech}^{-1}(c\right.}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{sech}^{-1}(c\right.}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{sech}^{-1}(c\right.}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{sech}^{-1}(c\right.}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{e^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\,\text{sech}^{-1}(c\right.}{d^3\left(e+\frac{d}{x^2}\right)}$$

Mathematica [F] time = 63.22, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3), x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsech}\,(cx)+a}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^3*x), x)
```

maple [C] time = 2.78, size = 5713, normalized size = 7.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}a\left(\frac{2\,ex^2+3\,d}{d^2e^2x^4+2\,d^3ex^2+d^4}-\frac{2\,\log\left(ex^2+d\right)}{d^3}+\frac{4\,\log(x)}{d^3}\right)+b\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1)
+ 1/(c*x))/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.127 $\int \frac{x^4\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1272

$$\frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\,\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\,\text{sech}^{-1}(cx)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\cdots$$

```
[Out] 3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arcsech(c*x
))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)
/(-d)^(1/2)-3/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*b*po
lylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)
-3/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/
(-d)^(1/2)-1/8*b*d*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-
1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/e/(c*d-(-d)^(1/2)*e^(1/2))^(
3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)-1/8*b*d*arctan((1+1/c/x)^(1/2)*(c*d+(-d
)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/e/(
c*d-(-d)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/16*(a+b*arcs
ech(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2+3/16*(a+b*arcsech(
c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsech(c*x))*(-d)^(1/2)/e^(3
/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arcsech(c*x))/e^2/(d/x+(-d)^(1/2)*
e^(1/2))+1/16*b*c*(-d)^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/e^(3/2)/(c^2*
d+e)/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*(-d)^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2)/e^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))-3/8*b*arctan((1+1/c/x)^
(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/
2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2
)-3/8*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1
/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d
+(-d)^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] time = 2.26, antiderivative size = 1272, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6303, 5707, 5802, 96, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\,\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\,\text{sech}^{-1}(cx)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*e^(3/2)*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c
*x)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*
```

```
ArcSech[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcSech[
c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcSech[c*x]))/
(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcSech[c*x]))/(16*e^2*
(Sqrt[-d]*Sqrt[e] + d/x)) - (3*b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[
1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*Sqrt[c
*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*d*ArcTan[(Sqr
t[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*
Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqr
t[e])^(3/2)*e) - (3*b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)
])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*Sqrt[c*d - Sqrt[-
d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*d*ArcTan[(Sqrt[c*d + Sqr
t[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1
/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)
*e) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
- Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSech[c*x])*Log[1
+ (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^
(5/2)) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[
e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSech[c*x])*Log
[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]
*e^(5/2)) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e]))])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(16*Sqrt[-d]
*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2
*d + e])])/(16*Sqrt[-d]*e^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
```

```
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\operatorname{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int\left(-\frac{d^3\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+\cdots\right.}{8(-d)^{3/2}e^{3/2}\cdots}\right.\right.$$

$$= \frac{(3d)\operatorname{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\operatorname{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\,\mathrm{S}\cdots}{\cdots}$$

$$= \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3\left(a+\cdots\right.}{16e^2\left(\cdots\right.}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}\cdots\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\cdots\right.}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}\cdots\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\cdots\right.}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}\cdots\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\cdots\right.}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}\cdots\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\cdots\right.}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}\cdots\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\cdots\right.}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}\cdots\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\cdots\right.}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\operatorname{sech}\cdots\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\cdots\right.}$$

Mathematica [C] time = 6.22, size = 2022, normalized size = 1.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*(((-I)*Sqrt[e
]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] +
 Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]
/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 +
c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt
[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[
c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqr
t[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - ((I/16)*Sqrt[d]*((I*Sqrt[e]*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)
) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) -
 Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt
[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sq
rt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*S
qrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^
2*d + e)^(3/2))))/e^2 + (5*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*
(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1
 + c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqr
t[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*e^2) + (5*(-(ArcSech[c*x]/((-I)*Sq
rt[d]*Sqrt[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*
x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*
d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*e^2) -
 (((3*I)/32)*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[Arc
Sech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])]
- ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech
[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*L
og[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*
ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt
[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])))/(Sqrt[d]*e^(5/2)) - (((3*I)/3
2)*(-PolyLog[2, -E^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c
*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcS
ech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x]
)] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1
 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])))/(Sqrt[d]*e^(5/2)))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4\,\text{arsech}\,(cx) + ax^4}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx) + a)x^4}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

maple [C] time = 9.41, size = 3455, normalized size = 2.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 5/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/
2))/e/(c^2*d+e)/d^2-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*ar
ctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))
^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3+5/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2-1/c^4*b*(-(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*
(e*(c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arc
tan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)-1/c^4*b*((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3+1/8
*c^5*b*x^4/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)
^(1/2)-3/8*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^
(1/2))/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^(1/2)+3/8*b*((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^(
1/2)+1/8*c^5*b*x^2/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(-(c*x-1)/c/x)^(1/2)*((c
*x+1)/c/x)^(1/2)*d+3/8*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arcta
nh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1
/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d-3/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e^2*d*x+3/
16*c^3*b/e^2/(c^2*d+e)*d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln
((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c
^2*d))-7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2))/(c^2*d+e)^2/d^2+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d
+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3-7/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^2+1/c^4*b*((c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3-3/
16*c^3*b/e^2/(c^2*d+e)*d*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((
_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2
*d))-5/8*c^4*b*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arcsech(c*x)-3/4*b*(-(c^2*
d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(
```

```
1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)^2
/d+3/8*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/
e^2/(c^2*d+e)/d-3/4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*
d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2))/e/(c^2*d+e)^2/d-5/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e
*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/2)-1/c^
4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^
2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+5/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/2)+3/4/c
^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e
^2/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*
d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)-
3/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))
/e^2/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-5/8*c^6*b*x^3/e/(c^2*e*x^2+c^2*d)^2/
(c^2*d+e)*arcsech(c*x)*d-3/8*c^6*b*x/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arcs
ech(c*x)*d^2-3/8*c^4*b*x/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d*arcsech(c*x)-5/8
*c^4*a/(c^2*e*x^2+c^2*d)^2*x^3/e-3/16*c*b/e/(c^2*d+e)*sum(_R1/(_R1^2*c^2*d+
c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R
1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*
d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/16*c*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2
*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c
^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/8*a/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.128 $\int \frac{x^2\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1276

$$\frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\,\mathrm{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\,\mathrm{sech}^{-1}(cx}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{\right.}}$$

```
[Out] -1/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsech(c*
x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(
3/2)/e^(3/2)+1/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*b*p
olylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c
^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2
)+1/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(
e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3
/2)/e^(3/2)+1/8*b*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1
+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/(c*d-(-d)^(1/2)*e^(1/2))^(3/2
)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/8*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/
2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/(c*d-(-d
)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/16*(a+b*arcsech(c*x
))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(a+b*arcsech(c*x))/d
/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arcsech(c*x))/(-d)^(1/2)/e^(1/2)/(d
/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arcsech(c*x))/d/e/(d/x+(-d)^(1/2)*e^(1/
2))+1/16*b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/(c^2*d+e)/(-d)^(1/2)/e^(1/2)/
(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/(c^2*d+
e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))-1/8*b*arctan((1+1/c/x)^(1/2)
*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(
1/2))/d/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)-1/8
*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(
c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)
^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] time = 4.04, antiderivative size = 1276, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6303, 5792, 5707, 5802, 96, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\,\mathrm{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\,\mathrm{sech}^{-1}(cx}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16
*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcSech[c*
```

```
x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcSech[c*x])
/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/(16*Sqrt[-d]*Sqrt
[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (a + b*ArcSech[c*x])/(16*d*e*(Sqrt[-d]*Sq
rt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/
(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt
[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]
*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x
)])])/(8*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) + (
b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[
-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d +
 Sqrt[-d]*Sqrt[e])^(3/2)) - (b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1
+ 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*Sqrt[c
*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - ((a + b*ArcSech[c*
x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(
-d)^(3/2)*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcSech
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(1
6*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(16*(-d)^(3/
2)*e^(3/2)) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2
*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*
e^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
```

```
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3} + \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Sul}}{}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e} - \frac{3\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{a+b\text{sech}^{-1}(cx)}{16\sqrt{-d}\,\sqrt{e}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\text{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{sech}^{-1}(cx)}{16\sqrt{-d}\,\sqrt{e}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{}{16}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{}{16}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{}{16}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{}{16}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{}{16}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{}{16}$$

Mathematica [C] time = 6.14, size = 2030, normalized size = 1.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(((-1/16*I)*(((-I)*Sqrt[e]*Sqrt[(1
- c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x
)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e
]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*
Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^
2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]
*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])
/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*((I*Sqrt[e]*Sqrt[(1 - c*x)/(
1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec
h[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + S
qrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2
*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x +
Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c
*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(
3/2))))/(Sqrt[d]*e) - (-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[
x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*
x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c
*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*
x)]/Sqrt[c^2*d + e]))/Sqrt[d])/(16*d*e) - (-(ArcSech[c*x]/((-I)*Sqrt[d]*Sqr
t[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x
*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c
*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/
((-I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d])/(16*d*e) - ((I/32)*(
PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/
Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]
*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I
)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqr
t[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1
+ (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])])))/(d^(3/2)*e^(3/2)) - ((I/32)*(-PolyLog[2, -E
^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d
 + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (
I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt
[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt
[d]*E^ArcSech[c*x])])))/(d^(3/2)*e^(3/2)))
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{\left(ex^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

maple [C] time = 9.28, size = 2537, normalized size = 1.99

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/8*c^6*b*x/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d*arcsech(c*x)+1/8*c^4*b*x^3/d
*e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)-1/4/c^2*b*((c^2*d+2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e)
)^(1/2)+1/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e
)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)-1/8*b*(-(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d
+e))^(1/2)+1/8*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/2)-1/4/c^2*b*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-1/4/c^2
*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^
2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)+1/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)-1/4/c
^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2
*d+e)^2*e/d^3-1/8*c^5*b*x^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(-(c*x-1)/c/x)^(1
/2)*((c*x+1)/c/x)^(1/2)+1/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2
*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3+1/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3-1/8*c^4*b*x/(c^2*d
+e)/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)+1/8*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+
2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2+1/8*b*((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2+1/8*c^6*b
*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arcsech(c*x)-1/8*c^5*b*x^4/d*e/(c^2*d+e)
/(c^2*e*x^2+c^2*d)^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/8*c^4*a/(c^
2*e*x^2+c^2*d)^2/d*x^3-1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e*x-1/4*b*((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^2-1/4*b
```

```
*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*
d+e)^2/d^2+1/8*a/d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/16*c*b/d/(c^2*d+
e)*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/16*c*b/d/(c^2*d
+e)*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/16*c^3*b/e/(
c^2*d+e)*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/16*c^3*b/
e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

3.129 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1272

$$\frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\,\text{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\,\text{sech}^{-1}(c\right.}{16d^2\left(\frac{d}{x}+\sqrt{-d}\right.}$$

```
[Out] 3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsech(c*x
))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5
/2)/e^(1/2)-3/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*po
lylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)
-3/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/
2)/e^(1/2)-1/8*b*e*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-
1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/d/(c*d-(-d)^(1/2)*e^(1/2))^(
3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)-1/8*b*e*arctan((1+1/c/x)^(1/2)*(c*d+(-d
)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d/(
c*d-(-d)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/16*(a+b*arcs
ech(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2-5/16*(a+b*arcsech(
c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsech(c*x))*e^(1/2)/(-d)^(3
/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arcsech(c*x))/d^2/(d/x+(-d)^(1/2)*
e^(1/2))+1/16*b*c*e^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/(-d)^(3/2)/(c^2*
d+e)/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*e^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2)/(-d)^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))+5/8*b*arctan((1+1/c/x)^
(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/
2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2
)+5/8*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1
/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d
+(-d)^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] time = 4.91, antiderivative size = 1272, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6293, 5792, 5707, 5802, 96, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\,\text{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\,\text{sech}^{-1}(c\right.}{16d^2\left(\frac{d}{x}+\sqrt{-d}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*(-d)^(3/2)*(c^2*d +
e)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(
c*x)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a +
 b*ArcSech[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*Ar
```

```
cSech[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSech[c*
x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcSech[c*x]))/
(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (5*b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e
]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(
8*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*Arc
Tan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*S
qrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sq
rt[-d]*Sqrt[e])^(3/2)) + (5*b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 +
 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d^2*Sqrt[
c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c
*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqr
t[-1 + 1/(c*x)])])/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt
[e])^(3/2)) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSech[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(
-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*Arc
Sech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] - Sqrt[c^2*d + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[
e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d +
 e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x
])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6293

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(2*(p + 1
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx = -\operatorname{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int\left(\frac{e^2\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2e\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right.$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\operatorname{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\operatorname{Subst}\left(\int \frac{a+b}{}\right.}{}$$

$$= -\frac{\operatorname{Subst}\left(\int\left(\frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\operatorname{Subst}\left(\int\left(-\frac{d\left(a+b\cosh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\right.\right.}{}$$

$$= \frac{3\operatorname{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\operatorname{Subst}\left(\int \frac{a+b\cosh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\operatorname{Subst}\left(\int \frac{a+b\,\mathrm{co}}{-de}\right.}{8d}$$

$$= \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5\left(a+b\right.}{16d^2\left(\sqrt{}\right.}$$

$$= \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\mathrm{se}\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\mathrm{se}\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\mathrm{se}\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\mathrm{se}\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\mathrm{se}\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\mathrm{se}\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

Mathematica [C] time = 6.07, size = 2015, normalized size = 1.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*(((I/16)*(((-I)*Sqrt[e]*Sqrt[(1 -
c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
 - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e])
 - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sq
rt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*
Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x
*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(
d*(c^2*d + e)^(3/2))))/d^(3/2) - ((I/16)*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*
x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]
/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1
 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d
 + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c
^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1
 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2)))
)/d^(3/2) - (3*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[x]/Sqrt
[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sq
rt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (
Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)]/Sqr
t[c^2*d + e]))/Sqrt[d]))/(16*d^2) - (3*(-(ArcSech[c*x]/((-I)*Sqrt[d]*Sqrt[e
] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sq
rt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)
/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-
I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*d^2) - (((3*I)/32)
*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2]
)/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*
x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2
*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] -
 Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])
)/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])])))/(d^(5/2)*Sqrt[e]) - (((3*I)/32)*(-PolyLog
[2, -E^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt
[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[
e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 -
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])])))/(d^(5/2)*Sqrt[e]))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arsech}(cx) + a}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^3, x)
```

maple [C] time = 10.82, size = 3446, normalized size = 2.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 3/8*a/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4*c^4*a*x/d/(c^2*e*x^2+c^2*
d)^2+3/8*c^2*a/d^2*x/(c^2*e*x^2+c^2*d)-3/16*c^3*b/d/(c^2*d+e)*sum(_R1/(_R1^
2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/16*c^3*b/d/(c^2*d+e)*sum(1/_R1/
(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R
1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-5/8*b*(-(c^2*d-2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3-5/8*b*((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3+5/8
*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^
2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)-5/8*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)+5/4*b*((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*
e/d^3+5/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2))/(c^2*d+e)/d^4*(e*(c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*e^3*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(
(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^5-5/4/c^2*b*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d
^4*(e*(c^2*d+e))^(1/2)-7/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^
2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^4-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*e^2*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^5+9/4/c^2*b*(-
(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctanh(c*d*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2
*d+e)^2/d^4+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^3*arctan
h(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/
2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^5-7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*e*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+
```

```
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^4-1/c^4*b*((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^5+9/4/c^2
*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c
^2*d+e)^2/d^4+5/8*c^6*b*x/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)-3/16*c
*b/(c^2*d+e)/d^2*e*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/
c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3
/16*c*b/(c^2*d+e)/d^2*e*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln(
(_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^
2*d))+5/4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(1/c/x
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/(c^2*d+e)^2*e/d^3+7/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^4*(e*(c^2*d+e))^(1/2)+1/c^4*b*(
-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctanh(c*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^
2*d+e)^2/d^5*(e*(c^2*d+e))^(1/2)+3/8*c^4*b*x^3/d^2/(c^2*d+e)/(c^2*e*x^2+c^2
*d)^2*arcsech(c*x)*e^2+5/8*c^4*b*x/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*e*arcsec
h(c*x)+3/8*c^6*b*x^3/d*e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)+1/c^4*b
*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctan(c*d*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d
+e)/d^5*(e*(c^2*d+e))^(1/2)-7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2)*e*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^4*(e*(c^2*d+e))^(1/2)-1/c^4*b*
((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*
d+e)^2/d^5*(e*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*
(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^5*(e*(c^2*d+e))^(1/2)+1/8*c^
5*b*x^2/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^
(1/2)*e+1/8*c^5*b*x^4/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(-(c*x-1)/c/x)^(1/2
)*((c*x+1)/c/x)^(1/2)*e^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^3,x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.130 $\int x^5\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx$

Optimal. Leaf size=447

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7e^3}-\frac{8bd^{7/2}\sqrt{}}{}$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*a
rcsech(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arcsech(c*x))/e^3-1/1680*b*(105*c
^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/
c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7/e^(5/2)-8/105*b*d^(7
/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(
c*x+1)^(1/2)/e^3+1/840*b*(29*c^2*d-25*e)*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*
(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2-1/42*b*(e*x^2+d)^(5/2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2+1/1680*b*(23*c^4*d^2+12*c^2
*d*e-75*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(
1/2)/c^6/e^2
```

Rubi [A] time = 1.40, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6301, 12, 1615, 154, 157, 63, 217, 203, 93, 207}

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7e^3}+\frac{b\sqrt{\frac{1}{cx+1}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(1680*c^6*e^2) + (b*(29*c^2*d - 25*e)*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^
4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2
)^(5/2))/(42*c^2*e^2) + (d^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3
) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) + ((d + e*x^2)^(7/
2)*(a + b*ArcSech[c*x]))/(7*e^3) - (b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*
d*e^2 + 75*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 -
 c^2*x^2])/(c*Sqrt[d + e*x^2])]])/(1680*c^7*e^(5/2)) - (8*b*d^(7/2)*Sqrt[(1
+ c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^
2])]])/(105*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
```

```
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5\sqrt{d+ex^2}\,\left(a+b\operatorname{sech}^{-1}(cx)\right)dx = \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3}$$

$$= \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3}$$

$$= \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42c^2e^2} + \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}\right.}{3e^3}$$

$$= \frac{b\left(29c^2d-25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+}}{}$$

$$= \frac{b\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b\left(2\right.}{}$$

$$= \frac{b\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b\left(2\right.}{}$$

$$= \frac{b\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b\left(2\right.}{}$$

$$= \frac{b\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b\left(2\right.}{}$$

$$= \frac{b\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b\left(2\right.}{}$$

Mathematica [A] time = 3.10, size = 409, normalized size = 0.91

$$\frac{\sqrt{d+ex^2}\left(16ac^6\left(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6\right)+16bc^6\operatorname{sech}^{-1}(cx)\left(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6\right)-be\sqrt{\frac{1-}{cx}}\right.}{1680c^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*a*c^6*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)
 - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x
^2) + c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^6*(8*d^3 - 4*d^2*e*
x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*ArcSech[c*x]))/(1680*c^6*e^3) - (b*Sqrt[(1
- c*x)/(1 + c*x)]*Sqrt[-1 + c^2*x^2]*(Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*(10
5*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[(c^2*(d + e*x^2))/(c
^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e
])] + 128*c^9*d^(7/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/S
qrt[d + e*x^2]]))/(1680*c^9*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [A] time = 3.53, size = 1995, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 - 35*b*c^4*d^2*e
+ 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d
*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e
*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 64*(15*b*c^7*e^3
*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6
 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*
x^5 + 2*(11*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*
e^2 - 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^
7*e^3), 1/3360*(64*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 - 35*b*c^4*
d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d
 - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^
4 + (c^2*d*e - e^2)*x^2 - d*e)) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4
- 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 6
4*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 +
 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), -1/6720*(256*b*c^
7*sqrt(-d)*d^3*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sq
rt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 -
d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(-
e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*
(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + e^2) - 64*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*
d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)) + 1)/(c*x)) - 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^
2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 + 25*b*c^4*
e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt(-(c^2*x
```

```
^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), -1/3360*(128*b*c^7*sqrt(-d)
*d^3*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (1
05*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*arctan(1
/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 32*(15*b*c^7*e^3*
x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*
log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(240*a*c^7*e^3*x^6
+ 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*x
^5 + 2*(11*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*e
^2 - 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7
*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

maple [F] time = 3.72, size = 0, normalized size = 0.00

$$\int x^5\left(a+b\,\mathrm{arcsech}\,(cx)\right)\sqrt{e\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{105}\left(\frac{15\left(ex^2+d\right)^{\frac{3}{2}}x^4}{e}-\frac{12\left(ex^2+d\right)^{\frac{3}{2}}dx^2}{e^2}+\frac{8\left(ex^2+d\right)^{\frac{3}{2}}d^2}{e^3}\right)a+\frac{1}{105}\,b\left(\frac{\left(15\,e^3x^6+3\,de^2x^4-4\,d^2ex^2+8\,d^3\right)\sqrt{ex}}{e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(e*x^2 + d)^(3/2)*x^4/e - 12*(e*x^2 + d)^(3/2)*d*x^2/e^2 + 8*(e*x
^2 + d)^(3/2)*d^2/e^3)*a + 1/105*b*((15*e^3*x^6 + 3*d*e^2*x^4 - 4*d^2*e*x^2
 + 8*d^3)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e^3 - 105*i
ntegrate(1/105*(210*(c^2*e^3*x^6 - e^3*x^4)*x^5*log(sqrt(x)) + 105*(c^2*e^3
*x^6*log(c) - e^3*x^4*log(c))*x^5 + (210*(c^2*e^3*x^6 - e^3*x^4)*x^5*log(sq
rt(x)) + (15*(7*e^3*log(c) + e^3)*c^2*x^6 - 4*c^2*d^2*e*x^2 + 8*c^2*d^3 + 3
*(c^2*d*e^2 - 35*e^3*log(c))*x^4)*x^5)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x +
 1)))*sqrt(e*x^2 + d)/(c^2*e^3*x^6 - e^3*x^4 + (c^2*e^3*x^6 - e^3*x^4)*e^(1
/2*log(c*x + 1) + 1/2*log(-c*x + 1))), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + b\,\mathrm{asech}\,(cx))\,\sqrt{d + ex^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.131 $\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)dx$

Optimal. Leaf size=329

$$-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{2bd^{5/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^2}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcs
ech(c*x))/e^2+1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*arctan(e^(1/2)*(-c^2*x^
2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(3/2)+2
/15*b*d^(5/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)/e^2-1/20*b*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e-1/120*b*(c^2*d+9*e)*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^4/e
```

Rubi [A] time = 0.43, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6301, 12, 573, 154, 157, 63, 217, 203, 93, 207}

$$-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(15c^4d^2-10c^2de-9e^2\right)\text{t}}{120c^5e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2])/(120*c^4*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c
^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e) - (d*(d + e*x^2)^(3/2)*(a + b*ArcSech
[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + (b*(15
*c^4*d^2 - 10*c^2*d*e - 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(S
qrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(3/2)) + (2*b*d^
(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*S
qrt[1 - c^2*x^2])])/(15*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
```

```
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)dx = -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\Big($$

$$=-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+$$

$$=-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+$$

$$=-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c^2e}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}\right.}{3e^2}$$

$$=-\frac{b\left(c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{}}{2}$$

$$=-\frac{b\left(c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{}}{2}$$

$$=-\frac{b\left(c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{}}{2}$$

$$=-\frac{b\left(c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{}}{2}$$

$$=-\frac{b\left(c^2d+9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{}}{2}$$

Mathematica [A] time = 1.49, size = 365, normalized size = 1.11

$$-\frac{\sqrt{d+ex^2}\left(8ac^4\left(2d^2-dex^2-3e^2x^4\right)+8bc^4\text{sech}^{-1}(cx)\left(2d^2-dex^2-3e^2x^4\right)+be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2\left(7d+6e\right.\right.\right.}{120c^4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/120*(Sqrt[d + e*x^2]*(8*a*c^4*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^4*(2*d^2
- d*e*x^2 - 3*e^2*x^4)*ArcSech[c*x]))/(c^4*e^2) - (b*Sqrt[(1 - c*x)/(1 + c*
x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^2 -
10*c^2*d*e - 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*S
qrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 16*c^7*d^(5/2)*Sqrt[-d
 - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(120*c^7*e
^2*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [A] time = 1.99, size = 1669, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b
*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e
^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2
*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c
*x)) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*
x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e
*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^
2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2
 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2
- 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*
sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*
d*e - e^2)*x^2 - d*e)) + 16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)
*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2
4*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7*b*
c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/
(c^5*e^2), 1/480*(32*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sq
rt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2
- 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d
^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4
*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7
*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d
))/(c^5*e^2), 1/240*(16*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 -
 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*
x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)
*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) +
16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5
*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d}\,(b\,\mathrm{arsech}\,(cx) + a)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)
```

maple [F] time = 3.39, size = 0, normalized size = 0.00

$$\int x^3\,(a + b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\left(\frac{3\left(ex^2+d\right)^{\frac{3}{2}}x^2}{e}-\frac{2\left(ex^2+d\right)^{\frac{3}{2}}d}{e^2}\right)a+\frac{1}{15}b\left(\frac{\left(3e^2x^4+dex^2-2d^2\right)\sqrt{ex^2+d}\log\left(\sqrt{cx+1}\sqrt{-cx+1}+1\right)}{e^2}-15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(e*x^2 + d)^(3/2)*x^2/e - 2*(e*x^2 + d)^(3/2)*d/e^2)*a + 1/15*b*((3
*e^2*x^4 + d*e*x^2 - 2*d^2)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1
) + 1)/e^2 - 15*integrate(1/15*(30*(c^2*e^2*x^4 - e^2*x^2)*x^3*log(sqrt(x))
 + 15*(c^2*e^2*x^4*log(c) - e^2*x^2*log(c))*x^3 + (30*(c^2*e^2*x^4 - e^2*x^
2)*x^3*log(sqrt(x)) + (3*(5*e^2*log(c) + e^2)*c^2*x^4 - 2*c^2*d^2 + (c^2*d*
e - 15*e^2*log(c))*x^2)*x^3)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt
(e*x^2 + d)/(c^2*e^2*x^4 - e^2*x^2 + (c^2*e^2*x^4 - e^2*x^2)*e^(1/2*log(c*x
 + 1) + 1/2*log(-c*x + 1))), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\,\mathrm{asech}\,(cx)\right)\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.132 $\int x\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx$

Optimal. Leaf size=221

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e}-\frac{bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}-$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e-1/3*b*d^(3/2)*arctanh((e*x^2+d)^(1
/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*(3*
c^2*d+e)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(
1/2)*(c*x+1)^(1/2)/c^3/e^(1/2)-1/6*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*
x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^2
```

Rubi [A] time = 0.36, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6299, 517, 446, 102, 157, 63, 217, 203, 93, 207}

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e}-\frac{bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(
6*c^2) + ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) - (b*(3*c^2*d + e)*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sq
rt[d + e*x^2])])/(6*c^3*Sqrt[e]) - (b*d^(3/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{1-cx}\sqrt{1+cx}}dx}{3e}$$

$$= \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{3e}$$

$$= \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x}}dx,\right.}{6e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} -$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} +$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} +$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} -$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} -$$

Mathematica [A] time = 1.28, size = 307, normalized size = 1.39

$$\frac{\sqrt{d+ex^2}\left(2ac^2\left(d+ex^2\right)+2bc^2\operatorname{sech}^{-1}(cx)\left(d+ex^2\right)-be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\right)}{6c^2e}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{e}\sqrt{c^2(-d)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*a*c^2*(d +
 e*x^2) + 2*b*c^2*(d + e*x^2)*ArcSech[c*x]))/(6*c^2*e) + (b*Sqrt[(1 - c*x)/
(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(3*c^2*
d + e)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x
^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 2*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcT
an[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e*(-1 + c*x)*Sqrt
[d + e*x^2])
```

fricas [B] time = 2.02, size = 1382, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c
```

```
^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(-e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*d^(3/2
)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d
 - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*
d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x
^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 -
b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3
*e), -1/24*(4*b*c^3*sqrt(-d)*d*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sq
rt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d
^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*
d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2
*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 8*(
b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2)) + 1)/(c*x)) - 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), -1/12*(2*b*c^3*sqrt(-d)*d*arctan(
-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2
- 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d + b
*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e))
 - 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)) - 2*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d}\,(b\,\mathrm{arsech}\,(cx) + a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

maple [F] time = 2.55, size = 0, normalized size = 0.00

$$\int x\,(a + b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{\left(ex^2 + d\right)^{\frac{3}{2}}\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)}{e} - 3\int \frac{\sqrt{ex^2+d}\left(6\left(c^2ex^2 - e\right)x\log\left(\sqrt{x}\right) + 3\left(c^2ex^2\log(c) - e\log\right.\right.}{3\left(c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((e*x^2 + d)^(3/2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e - 3*integrat
e(1/3*sqrt(e*x^2 + d)*(6*(c^2*e*x^2 - e)*x*log(sqrt(x)) + 3*(c^2*e*x^2*log(
c) - e*log(c))*x + (6*(c^2*e*x^2 - e)*x*log(sqrt(x)) + ((3*e*log(c) + e)*c^
2*x^2 + c^2*d - 3*e*log(c))*x)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/(c
^2*e*x^2 + (c^2*e*x^2 - e)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) - e), x
))*b + 1/3*(e*x^2 + d)^(3/2)*a/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{asech}\,(cx)\right)\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.133 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [A] time = 5.52, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

maple [A] time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2 + d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d}\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right) - \sqrt{ex^2 + d}\right)a + b\int \frac{\sqrt{ex^2 + d}\,\log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(d/(sqrt(d*e)*abs(x))) - sqrt(e*x^2 + d))*a + b*integrate(
sqrt(e*x^2 + d)*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2 + d}\,\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asech}\,(cx))\,\sqrt{d + ex^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x, x)
```

3.134 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [A] time = 6.34, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

maple [A] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{\sqrt{d}} - \frac{\sqrt{ex^2+d}\,e}{d} + \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{dx^2}\right)a + b\int \frac{\sqrt{ex^2+d}\,\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
[Out] -1/2*(e*arcsinh(d/(sqrt(d*e)*abs(x)))/sqrt(d) - sqrt(e*x^2 + d)*e/d + (e*x^
2 + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(e*x^2 + d)*log(sqrt(1/(c*x) + 1)
*sqrt(1/(c*x) - 1) + 1/(c*x))/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asech}\,(cx))\,\sqrt{d+ex^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**3,x)
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**3, x)
```

3.135 $\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

Mathematica [A] time = 13.90, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2\,\text{arsech}\,(cx)+ax^2\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

maple [A] time = 2.84, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\,\text{arcsech}\,(cx)\right)\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(\frac{2\left(ex^2+d\right)^{\frac{3}{2}}x}{e}-\frac{\sqrt{ex^2+d}\,dx}{e}-\frac{d^2\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right)a+b\int\sqrt{ex^2+d}\,x^2\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(e*x^2 + d)^(3/2)*x/e - sqrt(e*x^2 + d)*d*x/e - d^2*arcsinh(e*x/sqrt
(d*e))/e^(3/2))*a + b*integrate(sqrt(e*x^2 + d)*x^2*log(sqrt(1/(c*x) + 1)*s
qrt(1/(c*x) - 1) + 1/(c*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\,\mathrm{asech}\,(cx)\right)\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.136 $\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx=\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx$$

Mathematica [A] time = 3.17, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{ex^2+d}\,(b\operatorname{arsech}(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\operatorname{arsech}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

maple [A] time = 1.62, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{arcsech}(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{ex^2+d}\,x+\frac{d\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}\right)a+b\int\sqrt{ex^2+d}\,\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(e*x^2 + d)*x + d*arcsinh(e*x/sqrt(d*e))/sqrt(e))*a + b*integrate(
sqrt(e*x^2 + d)*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\,\mathrm{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.137 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

maple [A] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2 + d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\left(\sqrt{e}\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right) - \frac{\sqrt{ex^2 + d}}{x}\right)a + b\int \frac{\sqrt{ex^2 + d}\,\log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] (sqrt(e)*arcsinh(e*x/sqrt(d*e)) - sqrt(e*x^2 + d)/x)*a + b*integrate(sqrt(e
*x^2 + d)*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2 + d}\,\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asech}\,(cx))\,\sqrt{d + ex^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**2, x)
```

3.138 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=312

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}}{9dx}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(c^2d+e\right.}{}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/d/x^3+1/9*b*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/x^3+2/9*b*(c^2*d+2*e)*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x+2/9*b*c*(c^2*
d+2*e)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x
^2+d)^(1/2)/d/(1+e*x^2/d)^(1/2)-1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*EllipticF(c*x
,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/(e
*x^2+d)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {264, 6301, 12, 474, 583, 524, 426, 424, 421, 419}

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}}{9dx}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*x^3) + (2*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*
x^2]*Sqrt[d + e*x^2])/(9*d*x) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3
*d*x^3) + (2*b*c*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d +
e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[1 + (e*x^2)/d]) - (b
*(c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e
*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{\left(d+ex^2\right)^{3}}{3dx^4\sqrt{1-c}}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\sqrt{1-c^2x^2}}\,dx}{3d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} -$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1}}{9dx}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1}}{9dx}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1}}{9dx}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1}}{9dx}$$

Mathematica [C] time = 4.21, size = 576, normalized size = 1.85

$$-\frac{3a\left(d+ex^2\right)^2}{dx^3} + \frac{2b\sqrt{\frac{1-cx}{cx+1}}\left(c^2d+2e\right)\left(d+ex^2\right)}{dx} - \frac{2ib\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)^2\sqrt{\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}}\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(\sqrt{e}\left(-3\sqrt{e}+2ic\sqrt{d}\right)F\left(i\sinh\right.\right.}{cd\sqrt{-\frac{(cx-1}{(cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*Sqrt[(1 - c*x)/(1 + c
*x)]*(d + e*x^2))/x^2 + (2*b*(c^2*d + 2*e)*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e
*x^2))/(d*x) - (3*a*(d + e*x^2)^2)/(d*x^3) - (3*b*(d + e*x^2)^2*ArcSech[c*x
])/(d*x^3) - ((2*I)*b*(c*Sqrt[d] - I*Sqrt[e])^2*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x
))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(
(c^2*d + 2*e)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d]
+ I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[
e])^2] + ((2*I)*c*Sqrt[d] - 3*Sqrt[e])*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c
^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] +
I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(c*d*Sqrt[-(((c*Sqrt[d] - I*Sqrt[
e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]))/(9*Sqrt[d + e*x^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsech}\,(cx))\sqrt{e\,x^2+d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}b\left(\frac{\left(ex^3+dx\right)\sqrt{ex^2+d}\,\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)}{dx^4}+3\int\frac{\left(3\,c^2dx^2\log(c)-\left(c^2ex^4-\left(3\,d\log(c)-d\right)c^2x^2+3\right.\right.}{3\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/3*b*((e*x^3 + d*x)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)
/(d*x^4) + 3*integrate(1/3*(3*c^2*d*x^2*log(c) - (c^2*e*x^4 - (3*d*log(c) -
 d)*c^2*x^2 + 3*d*log(c) - 6*(c^2*d*x^2 - d)*log(sqrt(x)))*e^(1/2*log(c*x +
 1) + 1/2*log(-c*x + 1)) - 3*d*log(c) + 6*(c^2*d*x^2 - d)*log(sqrt(x)))*sqr
t(e*x^2 + d)/((c^2*d*x^2 - d)*x^4 + (c^2*d*x^2 - d)*e^(1/2*log(c*x + 1) + 1
/2*log(-c*x + 1) + 4*log(x))), x)) - 1/3*(e*x^2 + d)^(3/2)*a/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asech}\,(cx))\sqrt{d+ex^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**4,x)
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**4, x)
```

3.139 $\int \frac{\sqrt{d+ex^2}\left(a+b\mathbf{sech}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=446

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^5}+\cdots$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*a
rcsech(c*x))/d^2/x^3+1/25*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/x^5+1/45*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)*(e*x^2+d)^(1/2)/d/x^3+1/75*b*(4*c^2*d+e)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x^3-2/15*b*e^2*(1/(c*x+1))^(1/2
)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/45*b*e*(2*c^2*d+
e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x
+1/75*b*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x
^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x-2/15*b*c*e^2*EllipticE(c*x,(-e/c^2/d)^(1/
2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)+1
/45*b*c*e*(2*c^2*d+e)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)+1/75*b*c*(8*c^4*d^2+3*c^2*
d*e-2*e^2)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)-1/75*b*c*(8*c^2*d-e)*(c^2*d+e)*Ellipt
icF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)
/d/(e*x^2+d)^(1/2)-2/45*b*c*e*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(e*x^2+d)^(1/2)+2/15*b*e^2
*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {271, 264, 6301, 12, 580, 583, 524, 426, 424, 421, 419}

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(24c^4d^2+19c^2\cdots\right.}{225d^2x}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (b*(12*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2])/(225*d*x^3) + (b*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*x) + (
b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(
25*d*x^5) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (2*e*(d +
e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(15*d^2*x^3) + (b*c*(24*c^4*d^2 + 19*c^2
*d*e - 31*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE
[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)
*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[
1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(225*c*d^2*Sqrt[d + e*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
```

```
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}+\Big(b \ldots$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3}+\Big(\ldots$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^5}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}\ldots$$

$$= \frac{b\left(12c^2d-e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-\ldots}}{25d\ldots}\ldots$$

$$= \frac{b\left(12c^2d-e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3}+\frac{b\left(24c^4d^2+19c^2de-\ldots\right)}{\ldots}\ldots$$

$$= \frac{b\left(12c^2d-e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3}+\frac{b\left(24c^4d^2+19c^2de-\ldots\right)}{\ldots}\ldots$$

$$= \frac{b\left(12c^2d-e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3}+\frac{b\left(24c^4d^2+19c^2de-\ldots\right)}{\ldots}\ldots$$

$$= \frac{b\left(12c^2d-e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3}+\frac{b\left(24c^4d^2+19c^2de-\ldots\right)}{\ldots}\ldots$$

Mathematica [C] time = 6.38, size = 641, normalized size = 1.44

$$\frac{\frac{15a\left(2ex^2-3d\right)\left(d+ex^2\right)^2}{x^5}+\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)\left(dex^2\left(19c^2x^2+8\right)+3d^2\left(8c^4x^4+4c^2x^2+3\right)-31e^2x^4\right)}{x^5}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\Big(-\big(c^2\left(24c^4d^2+19c^2de-31e^2\right)\left(d+e\ldots\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] ((15*a*(d + e*x^2)^2*(-3*d + 2*e*x^2))/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)*(d + e*x^2)*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4
*c^2*x^2 + 8*c^4*x^4)))/x^5 + (15*b*(d + e*x^2)^2*(-3*d + 2*e*x^2)*ArcSech[
c*x])/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(24*c^4*d^2 + 19*c^2*d*e -
31*e^2)*(d + e*x^2)) - (I*(c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*(Sqrt
[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] +
 I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((24*c^4*d^2 + 19*c^2*d
*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] +
 I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e
])^2] + 2*Sqrt[e]*((24*I)*c^3*d^(3/2) - 36*c^2*d*Sqrt[e] - (29*I)*c*Sqrt[d]
*e + 30*e^(3/2))*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[
d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sq
rt[e])^2]))/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqr
t[e])*(1 + c*x)))]))/c)/(225*d^2*Sqrt[d + e*x^2])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex^2+d}\,(b\operatorname{arsech}(cx)+a)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\operatorname{arsech}(cx)+a)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsech}(cx))\sqrt{e\,x^2+d}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

```
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}a\left(\frac{2\left(ex^2+d\right)^{\frac{3}{2}}e}{d^2x^3}-\frac{3\left(ex^2+d\right)^{\frac{3}{2}}}{dx^5}\right)+\frac{1}{15}b\left(\frac{\left(2\,e^2x^5-dex^3-3\,d^2x\right)\sqrt{ex^2+d}\,\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)}{d^2x^6}-15\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*a*(2*(e*x^2 + d)^(3/2)*e/(d^2*x^3) - 3*(e*x^2 + d)^(3/2)/(d*x^5)) + 1/
15*b*((2*e^2*x^5 - d*e*x^3 - 3*d^2*x)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqr
```

```
t(-c*x + 1) + 1)/(d^2*x^6) - 15*integrate(1/15*(15*c^2*d^2*x^2*log(c) - 15*
d^2*log(c) + (2*c^2*e^2*x^6 - c^2*d*e*x^4 + 3*(5*d^2*log(c) - d^2)*c^2*x^2
- 15*d^2*log(c) + 30*(c^2*d^2*x^2 - d^2)*log(sqrt(x)))*e^(1/2*log(c*x + 1)
+ 1/2*log(-c*x + 1)) + 30*(c^2*d^2*x^2 - d^2)*log(sqrt(x)))*sqrt(e*x^2 + d)
/((c^2*d^2*x^2 - d^2)*x^6 + (c^2*d^2*x^2 - d^2)*e^(1/2*log(c*x + 1) + 1/2*l
og(-c*x + 1) + 6*log(x))), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asech}\,(cx))\,\sqrt{d+ex^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**6, x)
```

3.140 $\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\,\mathbf{sech}^{-1}(cx)\right)\,dx$

<u>**Optimal**</u>. Leaf size=418

$$-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{7e^2}+\frac{2bd^{7/2}\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{1-c^2x^2}}\right)}{35e^2}-\ b$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arcs
ech(c*x))/e^2+1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*e^3)*arctan(
e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)/c^7/e^(3/2)+2/35*b*d^(7/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^
(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2-1/840*b*(13*c^2*d+25*e)*(e*x^2+d
)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e-1/42*b*(e*
x^2+d)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e+1/560
*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)*(e*x^2+d)^(1/2)/c^6/e
```

<u>**Rubi**</u> **[A]** time = 0.53, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6301, 12, 573, 154, 157, 63, 217, 203, 93, 207}

$$-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{7e^2}+\frac{b\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}\,\left(3c^4d^2-38c^2de-\right.}{560c^6e}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqr
t[1 - c^2*x^2]*Sqrt[d + e*x^2])/(560*c^6*e) - (b*(13*c^2*d + 25*e)*Sqrt[(1
+ c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^4*e)
 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2
))/(42*c^2*e) - (d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + ((d +
e*x^2)^(7/2)*(a + b*ArcSech[c*x]))/(7*e^2) + (b*(35*c^6*d^3 - 35*c^4*d^2*e
- 63*c^2*d*e^2 - 25*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]
*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^7*e^(3/2)) + (2*b*d^(7/2)*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1
- c^2*x^2])])/(35*e^2)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 43</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

<u>Rule 63</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) dx = -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}+\left(\right.$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}+$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}+$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42c^2e}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}\right.}{5e^2}$$

$$= -\frac{b\left(13c^2d+25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^4e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1-}}{}$$

$$= \frac{b\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e}-\frac{b\left(13c^2\right.}{}$$

$$= \frac{b\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e}-\frac{b\left(13c^2\right.}{}$$

$$= \frac{b\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e}-\frac{b\left(13c^2\right.}{}$$

$$= \frac{b\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e}-\frac{b\left(13c^2\right.}{}$$

$$= \frac{b\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e}-\frac{b\left(13c^2\right.}{}$$

Mathematica [A] time = 2.99, size = 382, normalized size = 0.91

$$\frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2x^2-1}\left(32c^9d^{7/2}\sqrt{d+ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)+\sqrt{c^2}\sqrt{e}\sqrt{c^2d+e}\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right.\right.}{560c^9e^2(cx-1)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/1680*(Sqrt[d + e*x^2]*(48*a*c^6*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*Sqrt
[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(
57*d^2 + 106*d*e*x^2 + 40*e^2*x^4)) + 48*b*c^6*(2*d - 5*e*x^2)*(d + e*x^2)^
2*ArcSech[c*x]))/(c^6*e^2) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[-1 + c^2*x^2
]*(Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*
e^2 - 25*e^3)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-
1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])] + 32*c^9*d^(7/2)*Sqrt[d + e*x^2]
*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]]))/(560*c^9*e^2*(-1 +
c*x)*Sqrt[d + e*x^2])
```

fricas [A] time = 3.98, size = 1989, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e
- 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d
*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e
*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 192*(5*b*c^7*e^3
*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 +
 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^
5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e
^2 + 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7
*e^2), 1/3360*(48*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*
d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d
 - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^
4 + (c^2*d*e - e^2)*x^2 - d*e)) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 +
 b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*
a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25
*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2), 1/6720*(192*b*c^7*s
qrt(-d)*d^3*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(
-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2
)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(-e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2
*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + e^2) + 192*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*
e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e
*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)
*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2), 1/3360*(96*b*c^7*sqrt(-d)*d^3*
arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(35*b
*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*arctan(1/2*(
2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 96*(5*b*c^7*e^3*x^6 +
 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*
x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a
*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*
(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 7
5*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arsech}\,(cx)+a)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^3, x)
```

maple [F] time = 3.37, size = 0, normalized size = 0.00

$$\int x^3\left(e\,x^2+d\right)^{\frac{3}{2}}(a+b\,\mathrm{arcsech}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{35}\left(\frac{5\left(ex^2+d\right)^{\frac{5}{2}}x^2}{e}-\frac{2\left(ex^2+d\right)^{\frac{5}{2}}d}{e^2}\right)a+\frac{1}{105}b\left(\frac{\left(\left(15\,e^3x^6+3\,de^2x^4-4\,d^2ex^2+8\,d^3\right)x^5+7\left(3\,de^2x^6+d^2ex^4-2\,d^3\right.\right.}{e^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(5*(e*x^2 + d)^(5/2)*x^2/e - 2*(e*x^2 + d)^(5/2)*d/e^2)*a + 1/105*b*((
(15*e^3*x^6 + 3*d*e^2*x^4 - 4*d^2*e*x^2 + 8*d^3)*x^5 + 7*(3*d*e^2*x^6 + d^2
*e*x^4 - 2*d^3*x^2)*x^3)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) +
 1)/(e^2*x^5) - 105*integrate(1/105*(105*(c^2*e^3*x^6*log(c) - e^3*x^4*log(
c))*x^5 + 105*(c^2*d*e^2*x^6*log(c) - d*e^2*x^4*log(c))*x^3 + ((15*(7*e^3*l
og(c) + e^3)*c^2*x^6 - 4*c^2*d^2*e*x^2 + 8*c^2*d^3 + 3*(c^2*d*e^2 - 35*e^3*
log(c))*x^4)*x^5 + 7*(3*(5*d*e^2*log(c) + d*e^2)*c^2*x^6 - 2*c^2*d^3*x^2 +
(c^2*d^2*e - 15*d*e^2*log(c))*x^4)*x^3 + 210*((c^2*e^3*x^6 - e^3*x^4)*x^5 +
 (c^2*d*e^2*x^6 - d*e^2*x^4)*x^3)*log(sqrt(x)))*e^(1/2*log(c*x + 1) + 1/2*l
og(-c*x + 1)) + 210*((c^2*e^3*x^6 - e^3*x^4)*x^5 + (c^2*d*e^2*x^6 - d*e^2*x
^4)*x^3)*log(sqrt(x)))*sqrt(e*x^2 + d)/(c^2*e^2*x^6 - e^2*x^4 + (c^2*e^2*x^
6 - e^2*x^4)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

```
[Out] Timed out
```

3.141 $\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=297

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}-\frac{bd^{5/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{5e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(d+ex^2\right.}{20c^2}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/e-1/5*b*d^(5/2)*arctanh((e*x^2+d)^(1
/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/40*b*(1
5*c^4*d^2+10*c^2*d*e+3*e^2)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(
1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(1/2)-1/20*b*(e*x^2+d)^(3/2)*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2-1/40*b*(7*c^2*d+3*e)*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^4
```

Rubi [A] time = 0.43, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6299, 517, 446, 102, 154, 157, 63, 217, 203, 93, 207}

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(15c^4d^2+10c^2de+3e^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}-\frac{bd^{5/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(7*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sq
rt[d + e*x^2])/(40*c^4) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^
2*x^2]*(d + e*x^2)^(3/2))/(20*c^2) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]
))/(5*e) - (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(40*c^5*Sq
rt[e]) - (b*d^(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x
^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(5*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^{5/2}}{x\sqrt{1-cx}\sqrt{1+cx}}\,d}{5e}$$

$$= \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^{5/2}}{x\sqrt{1-c^2x^2}}\,dx}{5e}$$

$$= \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/}}{x\sqrt{1-c^2}}\right.}{10e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c^2} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}\right.}{5e}$$

$$= -\frac{b\left(7c^2d+3e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+c}}{}$$

$$= -\frac{b\left(7c^2d+3e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+c}}{}$$

$$= -\frac{b\left(7c^2d+3e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+c}}{}$$

$$= -\frac{b\left(7c^2d+3e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+c}}{}$$

$$= -\frac{b\left(7c^2d+3e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+c}}{}$$

Mathematica [A] time = 1.54, size = 342, normalized size = 1.15

$$\frac{\sqrt{d+ex^2}\left(8ac^4\left(d+ex^2\right)^2+8bc^4\operatorname{sech}^{-1}(cx)\left(d+ex^2\right)^2-be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2\left(9d+2ex^2\right)+3e\right)\right)}{40c^4e}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^4*(d + e*x^2)^2 - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(3*e + c^2*(9*d + 2*e*x^2)) + 8*b*c^4*(d + e*x^2)^2*ArcSech[c*x]))/(
40*c^4*e) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt
[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[(c^2*(d + e*x
^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c
^2*d) - e])] + 8*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*
x^2])/Sqrt[-d - e*x^2]]))/(40*c^7*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [B] time = 1.57, size = 1667, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
 d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*
e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^
2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*
c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)
) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3
+ 3*(3*b*c^4*d*e + b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2
 + d))/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4
- 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*s
qrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*
c^2*d*e + 3*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*
x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e
^2)*x^2 - d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^5*e^
2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e
+ b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), -
1/160*(16*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt
(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2
 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*log(
8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*
e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + e^2) - 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^
2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(8*a*c^5*e^2
*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e +
 b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), -1
/80*(8*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*
x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*arctan(1
/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 16*(b*c^5*e^2*x^4
 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)) - 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*
d^2 - (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e + b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)^{\frac{3}{2}} (b \operatorname{arsech}(cx) + a) x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x, x)
```

maple [F] time = 2.43, size = 0, normalized size = 0.00

$$\int x \left(e x^2 + d\right)^{\frac{3}{2}} (a + b \operatorname{arcsech}(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(ex^2+d\right)^{\frac{5}{2}}a}{5e}+\frac{1}{15}b\left(\frac{\left(\left(3e^2x^4+dex^2-2d^2\right)x^3+5\left(dex^4+d^2x^2\right)x\right)\sqrt{ex^2+d}\log\left(\sqrt{cx+1}\sqrt{-cx+1}+1\right)}{ex^3}-15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(e*x^2 + d)^(5/2)*a/e + 1/15*b*(((3*e^2*x^4 + d*e*x^2 - 2*d^2)*x^3 + 5*
(d*e*x^4 + d^2*x^2)*x)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1
)/(e*x^3) - 15*integrate(1/15*(15*(c^2*e^2*x^4*log(c) - e^2*x^2*log(c))*x^3
 + 15*(c^2*d*e*x^4*log(c) - d*e*x^2*log(c))*x + ((3*(5*e^2*log(c) + e^2)*c^
2*x^4 - 2*c^2*d^2 + (c^2*d*e - 15*e^2*log(c))*x^2)*x^3 + 5*((3*d*e*log(c) +
 d*e)*c^2*x^4 + (c^2*d^2 - 3*d*e*log(c))*x^2)*x + 30*((c^2*e^2*x^4 - e^2*x^
2)*x^3 + (c^2*d*e*x^4 - d*e*x^2)*x)*log(sqrt(x)))*e^(1/2*log(c*x + 1) + 1/2
*log(-c*x + 1)) + 30*((c^2*e^2*x^4 - e^2*x^2)*x^3 + (c^2*d*e*x^4 - d*e*x^2)
*x)*log(sqrt(x)))*sqrt(e*x^2 + d)/(c^2*e*x^4 - e*x^2 + (c^2*e*x^4 - e*x^2)*
e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{asech}\,(cx)\right)\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

3.142 $\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x} dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x},x\right)$$

[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x,x]

[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x} dx = \int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Mathematica [A] time = 6.38, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x,x]

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\operatorname{arsech}(cx)\right)\sqrt{ex^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x, x)
```

maple [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsech}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(3\,d^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)-\left(ex^{2}+d\right)^{\frac{3}{2}}-3\,\sqrt{ex^{2}+d}\,d\right)a+b\int \frac{\left(ex^{2}+d\right)^{\frac{3}{2}}\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
[Out] -1/3*(3*d^(3/2)*arcsinh(d/(sqrt(d*e)*abs(x))) - (e*x^2 + d)^(3/2) - 3*sqrt(
e*x^2 + d)*d)*a + b*integrate((e*x^2 + d)^(3/2)*log(sqrt(1/(c*x) + 1)*sqrt(
1/(c*x) - 1) + 1/(c*x))/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asech}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x, x)
```

3.143 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3} \, dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3} \, dx=\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3} \, dx$$

Mathematica [A] time = 6.22, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]
```

fricas [A] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\text{arsech}\left(cx\right)\right)\sqrt{ex^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsech}\left(cx\right)+a\right)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^3, x)
```

maple [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsech}\left(c x \right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(3\,\sqrt{d}\,e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)-3\,\sqrt{ex^2+d}\,e-\frac{\left(ex^2+d\right)^{\frac{3}{2}}e}{d}+\frac{\left(ex^2+d\right)^{\frac{5}{2}}}{dx^2}\right)a+b\int\frac{\left(ex^2+d\right)^{\frac{3}{2}}\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}\cdots}\right.}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
[Out] -1/2*(3*sqrt(d)*e*arcsinh(d/(sqrt(d*e)*abs(x))) - 3*sqrt(e*x^2 + d)*e - (e*
x^2 + d)^(3/2)*e/d + (e*x^2 + d)^(5/2)/(d*x^2))*a + b*integrate((e*x^2 + d)
^(3/2)*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asech}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**3,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

3.144 $\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=26

$$\text{Int}\left(x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

Mathematica [A] time = 13.65, size = 0, normalized size = 0.00

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^4+adx^2+\left(bex^4+bdx^2\right)\text{arsech}\left(cx\right)\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arcsech(c*x))*sqrt(e*x^2
+ d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arsech}\left(cx\right)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^2, x)
```

maple [A] time = 2.73, size = 0, normalized size = 0.00

$$\int x^2 \left(e x^2 + d\right)^{\frac{3}{2}} (a + b \operatorname{arcsech}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}\left(\frac{8\left(ex^2+d\right)^{\frac{5}{2}}x}{e}-\frac{2\left(ex^2+d\right)^{\frac{3}{2}}dx}{e}-\frac{3\sqrt{ex^2+d}\,d^2x}{e}-\frac{3d^3\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right)a+b\int\left(ex^2+d\right)^{\frac{3}{2}}x^2\log\left(\sqrt{\frac{1}{cx}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(e*x^2 + d)^(5/2)*x/e - 2*(e*x^2 + d)^(3/2)*d*x/e - 3*sqrt(e*x^2 +
d)*d^2*x/e - 3*d^3*arcsinh(e*x/sqrt(d*e))/e^(3/2))*a + b*integrate((e*x^2 +
 d)^(3/2)*x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \left(e x^2 + d\right)^{3/2} \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asech}(cx)) \left(d + ex^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

3.145 $\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

Mathematica [A] time = 4.48, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2+ad+\left(bex^2+bd\right)\text{arsech}\left(cx\right)\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arsech}\left(cx\right)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a), x)
```

maple [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arcsech}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(2\left(ex^2+d\right)^{\frac{3}{2}}x+3\sqrt{ex^2+d}\,dx+\frac{3\,d^2\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}\right)a+b\int\left(ex^2+d\right)^{\frac{3}{2}}\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(e*x^2 + d)^(3/2)*x + 3*sqrt(e*x^2 + d)*d*x + 3*d^2*arcsinh(e*x/sqrt
(d*e))/sqrt(e))*a + b*integrate((e*x^2 + d)^(3/2)*log(sqrt(1/(c*x) + 1)*sqr
t(1/(c*x) - 1) + 1/(c*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int\left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{asech}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

3.146 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 6.87, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\mathrm{arsech}\left(cx\right)\right)\sqrt{ex^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsech}\left(cx\right)+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^2, x)
```

maple [A] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsech}\left(cx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(3\sqrt{ex^{2}+d}\,ex+3\,d\sqrt{e}\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)-\frac{2\left(ex^{2}+d\right)^{\frac{3}{2}}}{x}\right)a+b\int \frac{\left(ex^{2}+d\right)^{\frac{3}{2}}\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*sqrt(e*x^2 + d)*e*x + 3*d*sqrt(e)*arcsinh(e*x/sqrt(d*e)) - 2*(e*x^2
+ d)^(3/2)/x)*a + b*integrate((e*x^2 + d)^(3/2)*log(sqrt(1/(c*x) + 1)*sqrt(
1/(c*x) - 1) + 1/(c*x))/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{asech}\left(cx\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

3.147 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx$$

Mathematica [A] time = 16.58, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\mathrm{arsech}\left(cx\right)\right)\sqrt{ex^2+d}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsech}\left(cx\right)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsech}\left(c x\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{3\,\sqrt{ex^{2}+d}\,e^{2}x}{d}+3\,e^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)-\frac{2\left(ex^{2}+d\right)^{\frac{3}{2}}e}{dx}-\frac{\left(ex^{2}+d\right)^{\frac{5}{2}}}{dx^{3}}\right)a+b\int \frac{\left(ex^{2}+d\right)^{\frac{3}{2}}\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-\right.}{x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
[Out] 1/3*(3*sqrt(e*x^2 + d)*e^2*x/d + 3*e^(3/2)*arcsinh(e*x/sqrt(d*e)) - 2*(e*x^
2 + d)^(3/2)*e/(d*x) - (e*x^2 + d)^(5/2)/(d*x^3))*a + b*integrate((e*x^2 +
d)^(3/2)*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{asech}\left(cx\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**4,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

3.148 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\mathbf{sech}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=409

$$-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^5}+\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(c^2d+2e\right.}{75x^3}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/d/x^5+1/25*b*(e*x^2+d)^(3/2)*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+4/75*b*(c^2*d+2*e)*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/x^3+1/75*b*(8*c
^4*d^2+23*c^2*d*e+23*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2
)*(e*x^2+d)^(1/2)/d/x+1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*EllipticE(c*x,
(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/(1+e*x^
2/d)^(1/2)-1/75*b*(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*EllipticF(c*x,(-e
/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/(e*x^2
+d)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {264, 6301, 12, 474, 580, 583, 524, 426, 424, 421, 419}

$$-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{75dx}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (4*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqr
t[d + e*x^2])/(75*x^3) + (b*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*x) + (b*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^5
) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (b*c*(8*c^4*d^2 +
23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*Ell
ipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d +
 e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(75*c*d*Sqrt[d + e*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{\left(d+ex^2\right)^{5/}}{5dx^6\sqrt{1-c^2}}$$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^{5/2}}{x^6\sqrt{1-c^2x^2}}\,dx}{5d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5}$$

$$= \frac{4b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-}}{25x^5}$$

$$= \frac{4b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3}+\frac{b\left(8c^4d^2+23c^2de+2\right.}{}$$

$$= \frac{4b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3}+\frac{b\left(8c^4d^2+23c^2de+2\right.}{}$$

$$= \frac{4b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3}+\frac{b\left(8c^4d^2+23c^2de+2\right.}{}$$

$$= \frac{4b\left(c^2d+2e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3}+\frac{b\left(8c^4d^2+23c^2de+2\right.}{}$$

Mathematica [C] time = 6.37, size = 620, normalized size = 1.52

$$-\frac{15a\left(d+ex^2\right)^3}{x^5}+\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)\left(dex^2\left(23c^2x^2+11\right)+d^2\left(8c^4x^4+4c^2x^2+3\right)+23e^2x^4\right)}{x^5}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\left(-\left(c^2\left(8c^4d^2+23c^2de+23e^2\right)\left(d+ex^2\right)\right)-\frac{i(cx+1)(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] ((-15*a*(d + e*x^2)^3)/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*
x^2)*(23*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x
^4)))/x^5 - (15*b*(d + e*x^2)^3*ArcSech[c*x])/x^5 + (b*Sqrt[(1 - c*x)/(1 +
c*x)]*(-(c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*(d + e*x^2)) - (I*(c*Sqrt[d]
 - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*
Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[
```

```
e])*(1 + c*x))]*((8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[I*ArcSinh[Sqrt
[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d
] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*Sqrt[e]*((8*I)*c^3*d^(3/2)
- 12*c^2*d*Sqrt[e] + (7*I)*c*Sqrt[d]*e - 15*e^(3/2))*EllipticF[I*ArcSinh[Sq
rt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt
[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/Sqrt[-(((c*Sqrt[d] - I*Sqrt
[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]]))/c)/(75*d*Sqrt[d +
e*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\operatorname{arsech}(cx)\right)\sqrt{ex^2+d}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^6
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arsech}(cx)+a\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^6, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\operatorname{arcsech}(cx)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}b\left(\frac{\left(2e^2x^5-dex^3-3d^2x-5\left(e^2x^3+dex\right)x^2\right)\sqrt{ex^2+d}\,\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}+1\right)}{dx^6}-15\int\frac{\left(15c^2d^2x^2\log(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*b*((2*e^2*x^5 - d*e*x^3 - 3*d^2*x - 5*(e^2*x^3 + d*e*x)*x^2)*sqrt(e*x^
2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(d*x^6) - 15*integrate(1/15*(1
5*c^2*d^2*x^2*log(c) + 15*(c^2*d*e*x^2*log(c) - d*e*log(c))*x^2 - 15*d^2*lo
g(c) + (2*c^2*e^2*x^6 - c^2*d*e*x^4 + 3*(5*d^2*log(c) - d^2)*c^2*x^2 - 5*(c
^2*e^2*x^4 - (3*d*e*log(c) - d*e)*c^2*x^2 + 3*d*e*log(c))*x^2 - 15*d^2*log(
c) + 30*(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 - d^2)*log(sqrt(x)))*e^(1/2*
```

```
log(c*x + 1) + 1/2*log(-c*x + 1)) + 30*(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x
^2 - d^2)*log(sqrt(x)))*sqrt(e*x^2 + d)/((c^2*d*x^2 - d)*x^6 + (c^2*d*x^2 -
 d)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1) + 6*log(x))), x)) - 1/5*(e*x^2
+ d)^(5/2)*a/(d*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asech}\,(cx))\left(d + ex^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**6,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**6, x)
```

3.149 $\int \frac{(d+ex^2)^{3/2}(a+b\,\mathbf{sech}^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=556

$$\frac{2e(d+ex^2)^{5/2}(a+b\,\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\,\text{sech}^{-1}(cx))}{7dx^7}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\,(d+ex^2)^{5/2}}{49dx^7}+\frac{b\ldots}{}$$

```
[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*a
rcsech(c*x))/d^2/x^5+1/1225*b*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(1/(c*x+1))^(
1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/x^5+1/49*b*(e*x^2+d)^(5/2)*(1/(c*x+
1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/x^7+1/3675*b*(120*c^4*d^2+159*
c^2*d*e-37*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d
)^(1/2)/d/x^3+1/3675*b*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/36
75*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*EllipticE(c*x,(-e/
c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/
d)^(1/2)-2/3675*b*(c^2*d+e)*(120*c^6*d^3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3
)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/
d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {271, 264, 6301, 12, 580, 583, 524, 426, 424, 421, 419}

$$\frac{2e(d+ex^2)^{5/2}(a+b\,\text{sech}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\,\text{sech}^{-1}(cx))}{7dx^7}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\,(528c^4d^2e+24\ldots}{3675d^2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (b*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^3) + (b*(240*c^6*d^3 + 528*c^4
*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d^2*x) + (b*(30*c^2*d + 11*e)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^5)
+ (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2)
)/(49*d*x^7) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(7*d*x^7) + (2*e*(d
 + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(35*d^2*x^5) + (b*c*(240*c^6*d^3 + 52
8*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*S
qrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[1 + (e*
x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e^2 - 1
05*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[Ar
cSin[c*x], -(e/(c^2*d))])/(3675*c*d^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^8}\,dx = -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{7dx^7} + \frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{35d^2x^5} + $$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{7dx^7} + \frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{35d^2x^5} + $$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^7} - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\,\text{sech}^{-1}(cx\right.}{7dx^7}$$

$$= \frac{b\left(30c^2d+11e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{1225dx^5} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+}}{}$$

$$= \frac{b\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b\left(\right.}{}$$

$$= \frac{b\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b\left(\right.}{}$$

$$= \frac{b\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b\left(\right.}{}$$

$$= \frac{b\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b\left(\right.}{}$$

$$= \frac{b\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b\left(\right.}{}$$

Mathematica [C] time = 7.81, size = 728, normalized size = 1.31

$$\frac{\frac{105a\left(2ex^2-5d\right)\left(d+ex^2\right)^3}{x^7} + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)\left(de^2x^4\left(193c^2x^2+71\right)+3d^2ex^2\left(176c^4x^4+83c^2x^2+61\right)+15d^3\left(16c^6x^6+8c^4x^4+6c^2x^2+5\right)-247e^3x^6\right)}{x^7}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] ((105*a*(d + e*x^2)^3*(-5*d + 2*e*x^2))/x^7 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*
(1 + c*x)*(d + e*x^2)*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*
e*x^2*(61 + 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 +
 16*c^6*x^6)))/x^7 + (105*b*(d + e*x^2)^3*(-5*d + 2*e*x^2)*ArcSech[c*x])/x^
7 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(240*c^6*d^3 + 528*c^4*d^2*e + 193*
c^2*d*e^2 - 247*e^3)*(d + e*x^2)) - (I*(c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x)*
Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[
(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((240*c^6*
d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[((c
^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] +
I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*Sqrt[e]*((240*I)*c^5*d^(5/2) -
360*c^4*d^2*Sqrt[e] + (48*I)*c^3*d^(3/2)*e - 207*c^2*d*e^(3/2) - (173*I)*c*
Sqrt[d]*e^2 + 210*e^(5/2))*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))
/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt
[d] - I*Sqrt[e])^2]))/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[
d] + I*Sqrt[e])*(1 + c*x)))]))/c)/(3675*d^2*Sqrt[d + e*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\text{arsech}\left(cx\right)\right)\sqrt{ex^2 + d}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^8
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^{\frac{3}{2}}\left(b\,\text{arsech}\left(cx\right) + a\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^8, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2 + d\right)^{\frac{3}{2}}\left(a + b\,\text{arcsech}\left(cx\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{35}\,a\left(\frac{2\left(ex^2 + d\right)^{\frac{5}{2}}e}{d^2x^5} - \frac{5\left(ex^2 + d\right)^{\frac{5}{2}}}{dx^7}\right) - \frac{1}{105}\,b\left(\frac{\left(8\,e^3x^7 - 4\,de^2x^5 + 3\,d^2ex^3 + 15\,d^3x - 7\left(2\,e^3x^5 - de^2x^3 - 3\,d^2ex\right)x^2\right)}{d^2x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/35*a*(2*(e*x^2 + d)^(5/2)*e/(d^2*x^5) - 5*(e*x^2 + d)^(5/2)/(d*x^7)) - 1/
105*b*((8*e^3*x^7 - 4*d*e^2*x^5 + 3*d^2*e*x^3 + 15*d^3*x - 7*(2*e^3*x^5 - d
*e^2*x^3 - 3*d^2*e*x)*x^2)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1)
 + 1)/(d^2*x^8) + 105*integrate(1/105*(105*c^2*d^3*x^2*log(c) - 105*d^3*log
(c) + 105*(c^2*d^2*e*x^2*log(c) - d^2*e*log(c))*x^2 - (8*c^2*e^3*x^8 - 4*c^
2*d*e^2*x^6 + 3*c^2*d^2*e*x^4 - 15*(7*d^3*log(c) - d^3)*c^2*x^2 + 105*d^3*l
og(c) - 7*(2*c^2*e^3*x^6 - c^2*d*e^2*x^4 + 3*(5*d^2*e*log(c) - d^2*e)*c^2*x
^2 - 15*d^2*e*log(c))*x^2 - 210*(c^2*d^3*x^2 - d^3 + (c^2*d^2*e*x^2 - d^2*e
)*x^2)*log(sqrt(x)))*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) + 210*(c^2*d^
3*x^2 - d^3 + (c^2*d^2*e*x^2 - d^2*e)*x^2)*log(sqrt(x)))*sqrt(e*x^2 + d)/((
c^2*d^2*x^2 - d^2)*x^8 + (c^2*d^2*x^2 - d^2)*e^(1/2*log(c*x + 1) + 1/2*log(
-c*x + 1) + 8*log(x))), x))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^8, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**8,x)
```

```
[Out] Timed out
```

3.150 $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=356

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{8bd^{5/2}\sqrt{\frac{1}{cx+1}}}{}$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcs
ech(c*x))/e^3-1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*arctan(e^(1/2)*(-c^2*x^
2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(5/2)-8
/15*b*d^(5/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)/e^3-1/20*b*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2+d^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^
3+1/120*b*(19*c^2*d-9*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
*(e*x^2+d)^(1/2)/c^4/e^2
```

Rubi [A] time = 1.16, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6301, 12, 1615, 154, 157, 63, 217, 203, 93, 207}

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*(19*c^2*d - 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sq
rt[d + e*x^2])/(120*c^4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e^2) + (d^2*Sqrt[d + e*x^2]*(a + b*A
rcSech[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3) +
((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) - (b*(45*c^4*d^2 - 10*c^2*
d*e + 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^
2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(5/2)) - (8*b*d^(5/2)*Sqrt[(1 + c*
x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])]
)/(15*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
```

```
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}(\cdots}{\cdots}$$

$$=\frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}(\cdots}{\cdots}$$

$$=\frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}(\cdots}{\cdots}$$

$$=-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c^2e^2}+\frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d\cdots\right.}{\cdots}$$

$$=\frac{b\left(19c^2d-9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d\cdots\right.}{20c^2e^2}$$

$$=\frac{b\left(19c^2d-9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d\cdots\right.}{20c^2e^2}$$

$$=\frac{b\left(19c^2d-9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d\cdots\right.}{20c^2e^2}$$

$$=\frac{b\left(19c^2d-9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d\cdots\right.}{20c^2e^2}$$

$$=\frac{b\left(19c^2d-9e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d\cdots\right.}{20c^2e^2}$$

Mathematica [A] time = 1.60, size = 366, normalized size = 1.03

$$\frac{\sqrt{d+ex^2}\left(8ac^4\left(8d^2-4dex^2+3e^2x^4\right)+8bc^4\operatorname{sech}^{-1}(cx)\left(8d^2-4dex^2+3e^2x^4\right)-be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(c^2\left(6ex^2-13\cdots\right.\right.\right.}{120c^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^4*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) - b*e*Sqrt[(1 - c
*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^4*(8*d^2 - 4
*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x]))/(120*c^4*e^3) + (b*Sqrt[(1 - c*x)/(1 +
 c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(45*c^4*d^2
 - 10*c^2*d*e + 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e
]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 64*c^7*d^(5/2)*Sqrt
[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(120*c^
7*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [A] time = 1.85, size = 1679, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b
*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e
^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 +
 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x)) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e
^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sq
rt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e
 + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(
e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4
*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e
)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 +
(c^2*d*e - e^2)*x^2 - d*e)) + 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c
^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))
 + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e^2*x^3
 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^5*e^3), -1/480*(128*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*
e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(
c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d^2 - 10*b*c^2*d*e +
9*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^
2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^
2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
1)/(c*x)) - 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^
4*e^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
*sqrt(e*x^2 + d))/(c^5*e^3), -1/240*(64*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^
3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d^2 - 10*b*c^
2*d*e + 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^
2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2
)*x^2 - d*e)) - 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(24*a*c^5
*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e^2*x^3 - (13*b*c^4*d
*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*
e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^5}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

maple [F] time = 4.74, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arcsech}(cx))}{\sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\left(\frac{3\sqrt{ex^2+d}\,x^4}{e} - \frac{4\sqrt{ex^2+d}\,dx^2}{e^2} + \frac{8\sqrt{ex^2+d}\,d^2}{e^3}\right)a + \frac{1}{15}b\left(\frac{\left(3e^3x^6 - de^2x^4 + 4d^2ex^2 + 8d^3\right)\log\left(\sqrt{cx+1}\sqrt{-c\right.}}{\sqrt{ex^2+d}\,e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/15*(3*sqrt(e*x^2 + d)*x^4/e - 4*sqrt(e*x^2 + d)*d*x^2/e^2 + 8*sqrt(e*x^2
+ d)*d^2/e^3)*a + 1/15*b*((3*e^3*x^6 - d*e^2*x^4 + 4*d^2*e*x^2 + 8*d^3)*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(e*x^2 + d)*e^3) - 15*integrate(1/1
5*(30*(c^2*e^3*x^6 - e^3*x^4)*x^5*log(sqrt(x)) + 15*(c^2*e^3*x^6*log(c) - e
^3*x^4*log(c))*x^5 + (30*(c^2*e^3*x^6 - e^3*x^4)*x^5*log(sqrt(x)) + (3*(5*e
^3*log(c) + e^3)*c^2*x^6 + 4*c^2*d^2*e*x^2 + 8*c^2*d^3 - (c^2*d*e^2 + 15*e^
3*log(c))*x^4)*x^5)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*e^3*x^6
 - e^3*x^4 + (c^2*e^3*x^6 - e^3*x^4)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1
)))*sqrt(e*x^2 + d)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{\sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{asech}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.151 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=251

$$-\frac{d\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^2}+\frac{2bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2}-\cdots$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^2+1/6*b*(3*c^2*d-e)*arctan(e^(1/2)
*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3/
e^(3/2)+2/3*b*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2-d*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^2
-1/6*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c
^2/e
```

Rubi [A] time = 0.33, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 6301, 12, 573, 154, 157, 63, 217, 203, 93, 207}

$$-\frac{d\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^2}+\frac{2bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(
6*c^2*e) - (d*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^2 + ((d + e*x^2)^(3/2
)*(a + b*ArcSech[c*x]))/(3*e^2) + (b*(3*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^3*
e^(3/2)) + (2*b*d^(3/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d +
 e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
```

```
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\left(b\sqrt{\frac{1}{1+cx}}\right.$$

$$= -\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1\right.}{}$$

$$= -\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1\right.}{}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{(d+e}{}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{(d+e}{}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{(d+e}{}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{(d+e}{}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e}-\frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{(d+e}{}$$

Mathematica [**A**] time = 1.32, size = 406, normalized size = 1.62

$$-\frac{\sqrt{d+ex^2}\left(2ac^2\left(2d-ex^2\right)+2bc^2\text{sech}^{-1}(cx)\left(2d-ex^2\right)+be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\right)}{6c^2e^2}-\frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}e^{3/2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/6*(Sqrt[d + e*x^2]*(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 2*a*c^2*(2
*d - e*x^2) + 2*b*c^2*(2*d - e*x^2)*ArcSech[c*x]))/(c^2*e^2) - (b*Sqrt[(1 -
 c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(-3*(-c^2)^(3/2)*d*Sqrt[-(c^2*d) - e]*Sq
rt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^
2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*e^(3/2
)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(Sqrt[-c^2]*Sqrt[e]*Sqrt[1 - c
^2*x^2])/(c*Sqrt[-(c^2*d) - e])] + 4*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(S
qrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e^2*(-1 + c*x)*Sqrt[d
+ e*x^2])
```

fricas [**B**] time = 1.01, size = 1389, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(-e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*
d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((
c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 +
 (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2
*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*
e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 +
d))/(c^3*e^2), 1/24*(8*b*c^3*sqrt(-d)*d*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(-e)*log(8*c^4*e^2*x
^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^
4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e
^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(4*b*c^3*sqrt(-
d)*d*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3
*b*c^2*d - b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2
 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)
*x^2 - d*e)) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsech}\,(cx) + a)x^3}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

maple [F] time = 4.61, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(a + b\,\text{arcsech}\,(cx))}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{\sqrt{ex^2 + d}\,x^2}{e} - \frac{2\,\sqrt{ex^2 + d}\,d}{e^2}\right)a + \frac{1}{3}\,b\left(\frac{\left(e^2x^4 - dex^2 - 2\,d^2\right)\log\left(\sqrt{cx + 1}\,\sqrt{-cx + 1}\, + 1\right)}{\sqrt{ex^2 + d}\,e^2} - 3\int \frac{6\left(c^2e^2x^4 - e^2x^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(e*x^2 + d)*x^2/e - 2*sqrt(e*x^2 + d)*d/e^2)*a + 1/3*b*((e^2*x^4 -
 d*e*x^2 - 2*d^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(e*x^2 + d)*e^
2) - 3*integrate(1/3*(6*(c^2*e^2*x^4 - e^2*x^2)*x^3*log(sqrt(x)) + 3*(c^2*e
^2*x^4*log(c) - e^2*x^2*log(c))*x^3 + (6*(c^2*e^2*x^4 - e^2*x^2)*x^3*log(sq
rt(x)) + ((3*e^2*log(c) + e^2)*c^2*x^4 - 2*c^2*d^2 - (c^2*d*e + 3*e^2*log(c
))*x^2)*x^3)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*e^2*x^4 - e^2*
x^2 + (c^2*e^2*x^4 - e^2*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqr
t(e*x^2 + d)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{\sqrt{e x^2 + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(a + b \operatorname{asech}\left(cx\right)\right)}{\sqrt{d + ex^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.152 $\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}$$

```
[Out] -b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*d^(1/2)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)/e-b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2)
)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/e^(1/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1
/2)/e
```

Rubi [A] time = 0.30, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6299, 517, 446, 105, 63, 217, 203, 93, 207}

$$\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*Sqrt[e])
- (b*Sqrt[d]*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sq
rt[d]*Sqrt[1 - c^2*x^2])])/e
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{e}$$

$$=\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{x\sqrt{1-c^2x^2}}\,dx}{e}$$

$$=\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{2e}$$

$$=\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}+\frac{1}{2}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\right.$$

$$=\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2}\right.}{c^2}$$

$$=\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}-\frac{b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\right.}{}$$

$$=\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1}}{}$$

Mathematica [A] time = 0.56, size = 239, normalized size = 1.56

$$\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{e}\sqrt{c^2(-d)-e}\sqrt{\frac{c^2\left(d+ex^2\right)}{c^2d+e}}\sin^{-1}\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{c^2(-d)-e}}\right)+c^3\sqrt{d}\right.}{c^3e(cx-1)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqr
t[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2)
)/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*
d) - e])] + c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])
/Sqrt[-d - e*x^2]]))/(c^3*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [B] time = 0.82, size = 1102, normalized size = 7.20

$$\left[\frac{4\sqrt{ex^2+d}\,bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+bc\sqrt{d}\log\left(\frac{\left(c^4d^2-6c^2de+e^2\right)x^4-8\left(c^2d^2-de\right)x^2+4\left(\left(c^3d-ce\right)x^3-2cdx\right)\sqrt{ex^2+d}\sqrt{d}\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+8d^2}{x^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c
*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
```

```
*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - b*sqrt(-e)*log(8
*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e
*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2)) + e^2))/(c*e), 1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^
4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)
*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a
*c - 2*b*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*s
qrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 -
d*e)))/(c*e), -1/4*(2*b*c*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x
)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c
^2*d^2 - d*e)*x^2 - d^2)) - 4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) - 4*sqrt(e*x^2 + d)*a*c + b*sqrt(-e)*log(8*c^4*e
^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 +
 (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
 + e^2))/(c*e), -1/2*(b*c*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x
)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c
^2*d^2 - d*e)*x^2 - d^2)) - 2*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) - 2*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*arctan(1/2*(
2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)))/(c*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

maple [F] time = 2.73, size = 0, normalized size = 0.00

$$\int \frac{x (a + b \operatorname{arcsech}(cx))}{\sqrt{e x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{\sqrt{ex^2+d}\,\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)}{e} - \int \frac{2\left(c^2ex^2-e\right)x\log\left(\sqrt{x}\right)+\left(c^2ex^2\log(c)-e\log(c)\right)x+\left(2\left(c^2ex^2\right.\right.}{\left(c^2ex^2+\left(c^2ex^2-e\right)e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e - integrate((2*(
c^2*e*x^2 - e)*x*log(sqrt(x)) + (c^2*e*x^2*log(c) - e*log(c))*x + (2*(c^2*e
*x^2 - e)*x*log(sqrt(x)) + ((e*log(c) + e)*c^2*x^2 + c^2*d - e*log(c))*x)*e
^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*e*x^2 + (c^2*e*x^2 - e)*e^(1
/2*log(c*x + 1) + 1/2*log(-c*x + 1)) - e)*sqrt(e*x^2 + d)), x)) + sqrt(e*x^
2 + d)*a/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+b\,\mathrm{asech}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.153 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{a+b\,\mathrm{sech}^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]
```

fricas [A] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^3 + d*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx)+a}{\sqrt{ex^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsech}\,(cx)}{x\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\sqrt{ex^2+d}\,x}\,dx-\frac{a\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(sqrt(e*x^2
+ d)*x), x) - a*arcsinh(d/(sqrt(d*e)*abs(x)))/sqrt(d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\,(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x*sqrt(d + e*x**2)), x)
```

3.154 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 23.96, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{ex^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^5 + d*x^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx)+a}{\sqrt{ex^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

maple [A] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arcsech}\,(cx)}{x^3\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{ex^2+d}}{dx^2}\right)+b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\sqrt{ex^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(e*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(3/2) - sqrt(e*x^2 + d)/(d*x^2)) +
 b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(sqrt(e*x^2
 + d)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asech}\,(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

3.155 $\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}} dx = \int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 7.18, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

maple [A] time = 3.52, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a +b \,\mathrm{arcsech}\left(c x \right)\right)}{\sqrt{e \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \, a {\left(\frac{\sqrt{e x^{2} + d} x}{e} - \frac{d \operatorname{arsinh}\left(\frac{e x}{\sqrt{d e}}\right)}{e^{\frac{3}{2}}}\right)} + b \int \frac{x^{2} \log\left(\sqrt{\frac{1}{c x} + 1} \sqrt{\frac{1}{c x} - 1} + \frac{1}{c x}\right)}{\sqrt{e x^{2} + d}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(sqrt(e*x^2 + d)*x/e - d*arcsinh(e*x/sqrt(d*e))/e^(3/2)) + b*integrat
e(x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/sqrt(e*x^2 + d), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{asech}{\left(c x \right)}\right)}{\sqrt{d + e x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.156 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{a+b\,\mathrm{sech}^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsech}\,(cx)+a}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)
```

maple [A] time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsech}\,(cx)}{\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\sqrt{ex^{2}+d}}\,dx+\frac{a\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/sqrt(e*x^2 +
 d), x) + a*arcsinh(e*x/sqrt(d*e))/sqrt(e)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\,(cx)}{\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.157 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=221

$$-\frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{dx}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{s}\right.}{cd\sqrt{d+ex^2}}$$

```
[Out] -(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/d/x+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x+b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/(1+e*x^2/d)^(1/2)-b*(c^2*d
+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^
2/d)^(1/2)/c/d/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {264, 6301, 12, 475, 21, 423, 426, 424, 421, 419}

$$-\frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{dx}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{s}\right.}{cd\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(d*x) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*
Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sqrt[d + e*x^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} - \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{dx^2\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{x^2\sqrt{1-c^2x^2}}\,dx}{d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+\right.}{}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} - \frac{\left(be\sqrt{\frac{1}{1+cx}}\sqrt{1\right.}{}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} + \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1\right.}{}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} + \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1\right.}{}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+}{}$$

Mathematica [C] time = 4.38, size = 501, normalized size = 2.27

$$-\frac{a\left(\frac{d}{x}+ex\right)+\frac{b\sqrt{\frac{1-cx}{cx+1}}\left(\sqrt{e}x+i\sqrt{d}\right)\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(2i\sqrt{e}F\left(i\sinh^{-1}\left(\sqrt{\frac{(dc^2+e)(1-cx)}{\left(\sqrt{d}c+i\sqrt{e}\right)^2(cx+1)}}\right)\middle|\frac{\left(\sqrt{d}c+i\sqrt{e}\right)^2}{\left(c\sqrt{d}-i\sqrt{e}\right)^2}\right)+\left(c\sqrt{d}-i\sqrt{e}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{(dc^2}{(\sqrt{d}c+}}\right.\right.\right.}{\sqrt{-\frac{(cx-1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\sqrt{\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}}}}{d\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((a*(d/x + e*x) + b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2) - (b*Sqrt[(1 -
 c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/x + (b*(d + e*x^2)*ArcSech[c*x])/x
+ (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d]
 + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt[e]*x)*((c*Sqrt[d] - I*Sqrt[e])*
EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2
*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + (2*I)
*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*S
qrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2
]))/(Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(
1 + c*x)))]*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 +
c*x))]))/(d*Sqrt[d + e*x^2]))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{ex^4+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^4 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx) + a}{\sqrt{ex^2 + d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arcsech}\,(cx)}{x^2\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\left(\frac{\left(ex^3 + dx\right)\log\left(\sqrt{cx+1}\,\sqrt{-cx+1} + 1\right)}{\sqrt{ex^2 + d}\,dx^2} + \int \frac{c^2dx^2\log(c) - \left(c^2ex^4 - \left(d\log(c) - d\right)c^2x^2 + d\log(c) - 2\left(c^2dx^2\right.\right.}{\left(\left(c^2dx^2 - d\right)x^2 + \left(c^2dx^2 - d\right)e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*((e*x^3 + d*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(e*x^2 + d)*d*
x^2) + integrate((c^2*d*x^2*log(c) - (c^2*e*x^4 - (d*log(c) - d)*c^2*x^2 +
d*log(c) - 2*(c^2*d*x^2 - d)*log(sqrt(x)))*e^(1/2*log(c*x + 1) + 1/2*log(-c
*x + 1)) - d*log(c) + 2*(c^2*d*x^2 - d)*log(sqrt(x)))/(((c^2*d*x^2 - d)*x^2
 + (c^2*d*x^2 - d)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1) + 2*log(x)))*sqr
t(e*x^2 + d)), x)) - sqrt(e*x^2 + d)*a/(d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asech}\,(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

3.158 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=346

$$\frac{2e\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3dx^3}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{9d^2x}$$

```
[Out] -1/3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arcsech(c*x))*(e*x
^2+d)^(1/2)/d^2/x+1/9*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*
(e*x^2+d)^(1/2)/d/x^3+1/9*b*(2*c^2*d-5*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/9*b*c*(2*c^2*d-5*e)*EllipticE(c*x
,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e
*x^2/d)^(1/2)-2/9*b*(c^2*d-3*e)*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {271, 264, 6301, 12, 580, 583, 524, 426, 424, 421, 419}

$$\frac{2e\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3dx^3}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{9d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*d*x^3) + (b*(2*c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^
2*x^2]*Sqrt[d + e*x^2])/(9*d^2*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/
(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(3*d^2*x) + (b*c*(2*
c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[A
rcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)
*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*Ellipti
cF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
```

```
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+c\right.$$

$$=-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right.}{3}$$

$$=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}-\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(\right.}{3}$$

$$=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}+\frac{b\left(2c^2d-5e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+}}{9d^2x}$$

$$=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}+\frac{b\left(2c^2d-5e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+}}{9d^2x}$$

$$=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}+\frac{b\left(2c^2d-5e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+}}{9d^2x}$$

$$=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}+\frac{b\left(2c^2d-5e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+}}{9d^2x}$$

Mathematica [C] time = 5.03, size = 612, normalized size = 1.77

$$\frac{-\frac{3a\left(d-2ex^2\right)\left(d+ex^2\right)}{x^3}+\frac{b\sqrt{\frac{1-cx}{cx+1}}\left(2c^2d-5e\right)\left(d+ex^2\right)}{x}-\frac{b\sqrt{\frac{1-cx}{cx+1}}\left(\sqrt{e}x+i\sqrt{d}\right)\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(2\sqrt{e}\left(2ic^2d-c\sqrt{d}\sqrt{e}-6ie\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{\left(dc^2+\right.}{\left(\sqrt{d}c+i\right.}}\right.\right.\right.}{\sqrt{-\frac{(cx-1)(}{(cx+1)(}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] ((b*d*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*d*Sqrt[(1 - c*x)/(1
 + c*x)]*(d + e*x^2))/x^2 + (b*(2*c^2*d - 5*e)*Sqrt[(1 - c*x)/(1 + c*x)]*(d
 + e*x^2))/x - (3*a*(d - 2*e*x^2)*(d + e*x^2))/x^3 - (3*b*(d - 2*e*x^2)*(d
+ e*x^2)*ArcSech[c*x])/x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt[d]
+ I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt[e]*x
)*((2*c^3*d^(3/2) - (2*I)*c^2*d*Sqrt[e] - 5*c*Sqrt[d]*e + (5*I)*e^(3/2))*El
lipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(
1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*((2*I
)*c^2*d - c*Sqrt[d]*Sqrt[e] - (6*I)*e)*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c
^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] +
I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*
(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]*Sqrt[(c*(Sqrt[d] - I*Sqrt
[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]))/(9*d^2*Sqrt[d + e*x^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{ex^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^6 + d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex^2 + d}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^4 \sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a\left(\frac{2\,\sqrt{ex^2 + d}\,e}{d^2 x} - \frac{\sqrt{ex^2 + d}}{dx^3}\right) + \frac{1}{3}\,b\left(\frac{\left(2\,e^2x^5 + dex^3 - d^2x\right)\log\left(\sqrt{cx + 1}\,\sqrt{-cx + 1} + 1\right)}{\sqrt{ex^2 + d}\,d^2x^4} - 3\int \frac{3\,c^2d^2x^2\log(c) - \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*sqrt(e*x^2 + d)*e/(d^2*x) - sqrt(e*x^2 + d)/(d*x^3)) + 1/3*b*((2*e
^2*x^5 + d*e*x^3 - d^2*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(e*x^2
 + d)*d^2*x^4) - 3*integrate(1/3*(3*c^2*d^2*x^2*log(c) - 3*d^2*log(c) + (2*
c^2*e^2*x^6 + c^2*d*e*x^4 + (3*d^2*log(c) - d^2)*c^2*x^2 - 3*d^2*log(c) + 6
*(c^2*d^2*x^2 - d^2)*log(sqrt(x)))*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))
 + 6*(c^2*d^2*x^2 - d^2)*log(sqrt(x)))/(((c^2*d^2*x^2 - d^2)*x^4 + (c^2*d^2
*x^2 - d^2)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1) + 4*log(x)))*sqrt(e*x^2
 + d)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asech}(cx)}{x^4 \sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

3.159 $\int \frac{x^5\left(a+b\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=278

$$-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{8bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{ta}}{3e^3}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3+1/6*b*(9*c^2*d-e)*arctan(e^(1/2)
*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3/
e^(5/2)+8/3*b*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^3-d^2*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(1/
2)-2*d*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^3-1/6*b*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^2/e^2
```

Rubi [A] time = 1.12, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 6301, 12, 1615, 157, 63, 217, 203, 93, 207}

$$-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3}+\frac{8bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{ta}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(
6*c^2*e^2) - (d^2*(a + b*ArcSech[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d
 + e*x^2]*(a + b*ArcSech[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x
]))/(3*e^3) + (b*(9*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(S
qrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^3*e^(5/2)) + (8*b*d^(3
/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqr
t[1 - c^2*x^2])])/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}\right.}{3e^3}$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}\right.}{3e^3}$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}\right.}{3e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+\right.}{e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+\right.}{e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+\right.}{e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+\right.}{e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+\right.}{e^3}$$

Mathematica [A] time = 1.50, size = 436, normalized size = 1.57

$$\frac{-2ac^2\left(8d^2+4dex^2-e^2x^4\right)-2bc^2\text{sech}^{-1}(cx)\left(8d^2+4dex^2-e^2x^4\right)-be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)}{6c^2e^3\sqrt{d+ex^2}} - \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (-(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)) - 2*a*c^2*(8*d^2 +
4*d*e*x^2 - e^2*x^4) - 2*b*c^2*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcSech[c*x])/
(6*c^2*e^3*Sqrt[d + e*x^2]) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2
]*(-9*(-c^2)^(3/2)*d*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2
*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e
])] + Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*e^(3/2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d +
 e)]*ArcSin[(Sqrt[-c^2]*Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[-(c^2*d) - e])]
+ 16*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-
d - e*x^2]]))/(6*c^5*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [B] time = 2.59, size = 1771, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^2 +
d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 16*(b*c^3*d*e*x^2
+ b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*
e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 -
 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9*b*c^2*d^2 - b*d*e +
 (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x
)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^
2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2
)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(
b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8
*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt
(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^3*e^2*x^4 - 8*a
*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/24*(32*(b*c^
3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*
d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*
sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^
2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*
d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) +
4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^
2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^
3*d*e^3), 1/12*(16*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(-1/2*((c^3*d
 - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*
b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sq
rt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*
e - e^2)*x^2 - d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sq
rt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*
c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^5}{\left(ex^2 + d\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 4.65, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(a + b \operatorname{arcsech}(cx)\right)}{\left(e\,x^2 + d\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{x^4}{\sqrt{ex^2+d}\,e}-\frac{4\,dx^2}{\sqrt{ex^2+d}\,e^2}-\frac{8\,d^2}{\sqrt{ex^2+d}\,e^3}\right)a+b\int\frac{x^5\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4/(sqrt(e*x^2 + d)*e) - 4*d*x^2/(sqrt(e*x^2 + d)*e^2) - 8*d^2/(sqrt(
e*x^2 + d)*e^3))*a + b*integrate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1
) + 1/(c*x))/(e*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^5\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

3.160 $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}}-\frac{2b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{e}$$

```
[Out] -b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(
c*x+1)^(1/2)/c/e^(3/2)-2*b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/
2))*d^(1/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2+d*(a+b*arcsech(c*x))/e^2/(e
*x^2+d)^(1/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] time = 0.27, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 6301, 12, 573, 157, 63, 217, 203, 93, 207}

$$\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}}-\frac{2b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cSech[c*x]))/e^2 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sq
rt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*e^(3/2)) - (2*b*Sqrt[d]*Sqrt[(1 +
 c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2
])])/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx = \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int}{e^2}$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}}{2}$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \left(bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subs}$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Sub}$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} - \frac{2b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan}{e^2}$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tan^{-1}\Big(}{ce^{3/2}}$$

Mathematica [A] time = 1.78, size = 249, normalized size = 1.41

$$\frac{\left(2d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{e}\sqrt{c^2(-d)-e}\sqrt{\frac{c^2\left(d+ex^2\right)}{c^2d+e}}\sin^{-1}\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{c^2(-d)-e}}\right)+2c\right.}{c^3e^2(cx-1)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*d + e*x^2)*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (b*Sqrt[(1 - c
*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqr
t[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt
[-c^2]*Sqrt[-(c^2*d) - e])] + 2*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d
]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(c^3*e^2*(-1 + c*x)*Sqrt[d + e*x^2
])
```

fricas [B] time = 1.80, size = 1311, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*e*x^2 + b*d)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8
*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 +
 d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(b*c*e*x^2 + 2*b*c*d
)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(
b*c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
```

```
2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x
^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*sqrt(e)*arctan(1/2*(2
*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 2*(b*c*e*x^2 + 2*b*c*d
)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*
c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2
 + d))/(c*e^3*x^2 + c*d*e^2), -1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(-
1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 + b*d)
*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x
^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(a*c*e*x^2 + 2*a*c*d)*s
qrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*(2*(b*c*e*x^2 + b*c*d)*sqrt(-d)
*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x
^2 + b*d)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*
sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 -
 d*e)) - 2*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/(c*e^3
*x^2 + c*d*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^3}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 4.33, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arcsech}(cx))}{(e x^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{x^2}{\sqrt{ex^2 + d}\, e} + \frac{2\, d}{\sqrt{ex^2 + d}\, e^2}\right) + b \int \frac{x^3 \log\left(\sqrt{\frac{1}{cx} + 1}\, \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(x^2/(sqrt(e*x^2 + d)*e) + 2*d/(sqrt(e*x^2 + d)*e^2)) + b*integrate(x^3*l
og(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a + b \operatorname{asech}{\left(c x \right)}\right)}{\left(d + e x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**3*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.161 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}\,e}-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

```
[Out] b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)/e/d^(1/2)+(-a-b*arcsech(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6299, 517, 446, 93, 207}

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}\,e}-\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSech[c*x])/(e*Sqrt[d + e*x^2])) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(Sqrt[d]*e)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
```

```
x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex^2}}\,dx}{e}\\ &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e}\\ &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e}\\ &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{e}\\ &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}\,e}\end{aligned}$$

Mathematica [A] time = 0.63, size = 135, normalized size = 1.55

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{\sqrt{d}\,e(cx-1)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSech[c*x])/(e*Sqrt[d + e*x^2])) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*
Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[
-d - e*x^2]])/(Sqrt[d]*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [B] time = 0.61, size = 379, normalized size = 4.36

$$\left[-\frac{4\sqrt{ex^2+d}\,bd\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+4\sqrt{ex^2+d}\,ad-\left(bex^2+bd\right)\sqrt{d}\,\log\left(\frac{\left(c^4d^2-6\,c^2de+e^2\right)x^4-8\left(c^2d^2-de\right)x^2-4\left(\left(c^3d-c\right.\right.}{x^4}\right.}{4\left(de^2x^2+d^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x)) + 4*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(d)*log(((c^4*d^2 - 6*c
^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)
*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4))/(d*e
^2*x^2 + d^2*e), -1/2*(2*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(
```

```
c^2*x^2)) + 1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(-d)*ar
ctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)))/(d*e^2*x^2
 + d^2*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 2.42, size = 0, normalized size = 0.00

$$\int \frac{x\,(a + b \operatorname{arcsech}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x \log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}}\,dx - \frac{a}{\sqrt{ex^2 + d}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d
)^(3/2), x) - a/(sqrt(e*x^2 + d)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a + b \operatorname{asech}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.162 $\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

[Out] Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)),x]

[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 32.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)),x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]

fricas [A] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)}{e^2x^5+2\,dex^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

maple [A] time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{a + b \,\mathrm{arcsech}\left(cx\right)}{x \left(e \,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\operatorname{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{3}{2}}} - \frac{1}{\sqrt{ex^{2}+d}\,d}\right) + b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\left(ex^{2}+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(d/(sqrt(d*e)*abs(x)))/d^(3/2) - 1/(sqrt(e*x^2 + d)*d)) + b*inte
grate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((e*x^2 + d)^(3/2)
*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asech}{\left(cx \right)}}{x \left(d + ex^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

3.163 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 37.54, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{e^2x^7+2\,dex^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

maple [A] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsech}\left(cx\right)}{x^{3}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{3\,e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{5}{2}}}-\frac{3\,e}{\sqrt{ex^{2}+d}\,d^{2}}-\frac{1}{\sqrt{ex^{2}+d}\,dx^{2}}\right)+b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\left(ex^{2}+d\right)^{\frac{3}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] 1/2*a*(3*e*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(5/2) - 3*e/(sqrt(e*x^2 + d)*d^2
) - 1/(sqrt(e*x^2 + d)*d*x^2)) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(
c*x) - 1) + 1/(c*x))/((e*x^2 + d)^(3/2)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^{3}\,{\left(e\,x^{2}+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(3/2),x)
[Out] Timed out
```

3.164 $\int \frac{x^4(a+b\,\mathbf{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 9.91, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^4\operatorname{arsech}(cx)+ax^4\right)\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsech}(cx)+a)x^4}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 3.92, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arcsech}(cx))}{\left(e x^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{x^3}{\sqrt{ex^2+d}\,e} + \frac{3\,dx}{\sqrt{ex^2+d}\,e^2} - \frac{3\,d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{5}{2}}}\right)a + b\int \frac{x^4 \log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1} + \frac{1}{cx}\right)}{\left(ex^2+d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^3/(sqrt(e*x^2 + d)*e) + 3*d*x/(sqrt(e*x^2 + d)*e^2) - 3*d*arcsinh(e*
x/sqrt(d*e))/e^(5/2))*a + b*integrate(x^4*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x
) - 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{asech}(cx))}{\left(d + ex^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.165 $\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 4.71, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2\operatorname{arsech}(cx)+ax^2\right)\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arsech}(cx)+a\right)x^2}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a + b \operatorname{arcsech}(cx)\right)}{\left(e\, x^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{x}{\sqrt{ex^2 + d}\, e} - \frac{\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right) + b\int \frac{x^2 \log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(x/(sqrt(e*x^2 + d)*e) - arcsinh(e*x/sqrt(d*e))/e^(3/2)) + b*integrate(x
^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c\, x}\right)\right)}{{\left(e\, x^2 + d\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a + b \operatorname{asech}(cx)\right)}{\left(d + ex^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.166 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

```
[Out] x*(a+b*arcsech(c*x))/d/(e*x^2+d)^(1/2)+b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {191, 6291, 12, 421, 419}

$$\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt
[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sq
rt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 6291

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx$$
$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d}$$
$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{d+ex^2}}$$
$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

Mathematica [C] time = 1.39, size = 334, normalized size = 3.63

$$\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} + \frac{2ib\sqrt{\frac{1-cx}{cx+1}}\left(\sqrt{e}\,x-i\sqrt{d}\right)\sqrt{\frac{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}{(cx-1)\left(c\sqrt{d}-i\sqrt{e}\right)}}\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}\,x}{\sqrt{d}}-1}{1-cx}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-xc+\frac{i\sqrt{d}\,c}{\sqrt{e}}+\frac{i\sqrt{e}\,x}{\sqrt{d}}+1}{2-2cx}}\right)\right.}{d\left(c\sqrt{d}+i\sqrt{e}\right)\sqrt{d+ex^2}\sqrt{\frac{\frac{ic\sqrt{d}}{\sqrt{e}}+c(-x)+\frac{i\sqrt{e}\,x}{\sqrt{d}}+1}{1-cx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + ((2*I)*b*Sqrt[(1 - c*x)/(1 +
 c*x)]*Sqrt[((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] - I*Sqrt[e])*(-
1 + c*x))]*((-I)*Sqrt[d] + Sqrt[e]*x)*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] +
c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticF[ArcSin[Sqrt[(1 + (I*c*Sq
rt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(2 - 2*c*x)]], ((-4*I)*c*Sqrt
[d]*Sqrt[e])/(c*Sqrt[d] - I*Sqrt[e])^2])/(d*(c*Sqrt[d] + I*Sqrt[e])*Sqrt[(1
 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(1 - c*x)]*Sqrt[d +
 e*x^2])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 2.17, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arcsech}\,(cx)}{\left(e\,x^{2} + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{\left(ex^{2} + d\right)^{\frac{3}{2}}}\,dx + \frac{ax}{\sqrt{ex^{2} + d}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^
(3/2), x) + a*x/(sqrt(e*x^2 + d)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{\left(e\,x^{2} + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asech}\,(cx)}{\left(d + ex^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.167 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(c^2d+2e\right)\sqrt{\frac{e}{\ }}}{cd^2\sqrt{d+}}$$

```
[Out] (-a-b*arcsech(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arcsech(c*x))/d^2/(e*x^2
+d)^(1/2)+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1
/2)/d^2/x+b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)-b*(c^2*d+2*e)*EllipticF(c*x,(-e/c^
2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+
d)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {271, 191, 6301, 12, 583, 524, 426, 424, 421, 419}

$$-\frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(c^2d+2e\right)\sqrt{\frac{e}{\ }}}{cd^2\sqrt{d+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
^2*x) - (a + b*ArcSech[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcSech[
c*x]))/(d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt
[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d^2*Sqrt[1 + (e*x^2)/d])
 - (b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*
EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-d-2ex^2}{d^2x^2\sqrt{1-c^2x^2}\sqrt{\cdots}}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-d-2ex^2}{x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{\left(b\cdots\right.}{\cdots}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(bc\cdots\right.}{\cdots}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(bc\cdots\right.}{\cdots}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 4.49, size = 501, normalized size = 2.01

$$\frac{-\frac{a\left(d+2ex^2\right)}{x} + \frac{b\sqrt{\frac{1-cx}{cx+1}}\left(-c^2\left(d+ex^2\right) + \frac{(cx+1)\sqrt{\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}}\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(2\sqrt{e}\left(c\sqrt{d}-2i\sqrt{e}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{\left(dc^2+e\right)(1-cx)}{\left(\sqrt{d}c+i\sqrt{e}\right)^2(cx+1)}}\right)\middle|\frac{\left(\sqrt{d}c+i\sqrt{e}\right)^2}{\left(c\sqrt{d}-i\sqrt{e}\right)^2}\right)-i\left(c\sqrt{d}\cdots\right.}{\sqrt{-\frac{(cx-1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}}\right)}{c}}{d^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/x - (a*(d + 2*e*x^2))/
x - (b*(d + 2*e*x^2)*ArcSech[c*x])/x + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*
(d + e*x^2)) + ((1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*
Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[
e])*(1 + c*x))]*((-I)*(c*Sqrt[d] - I*Sqrt[e])^2*EllipticE[I*ArcSinh[Sqrt[((
c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] +
 I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*(c*Sqrt[d] - (2*I)*Sqrt[e])*Sq
rt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt
[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))
/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 +
c*x)))]))/c)/(d^2*Sqrt[d + e*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{e^2x^6+2\,dex^4+d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

maple [F] time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^2 (e x^2 + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{2\,ex}{\sqrt{ex^2 + d}\,d^2} + \frac{1}{\sqrt{ex^2 + d}\,dx}\right) + b \int \frac{\log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a*(2*e*x/(sqrt(e*x^2 + d)*d^2) + 1/(sqrt(e*x^2 + d)*d*x)) + b*integrate(lo
g(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((e*x^2 + d)^(3/2)*x^2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)}{x^2 \left(e\,x^2 + d\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asech}(cx)}{x^2 (d + ex^2)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

3.168 $\int \frac{x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{d^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^3}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{5/2}}$$

```
[Out] -1/3*d^2*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(3/2)-b*arctan(e^(1/2)*(-c^2*x^2+
1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/e^(5/2)-8/3*b
*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*d^(1/2)*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)/e^3+2*d*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(1/2)-1/3*b*d*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/(e*x^2+d)^(1
/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^3
```

Rubi [A] time = 1.28, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 6301, 12, 1614, 157, 63, 217, 203, 93, 207}

$$-\frac{d^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^3}-\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -(b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*e^2*(c^2*d +
 e)*Sqrt[d + e*x^2]) - (d^2*(a + b*ArcSech[c*x]))/(3*e^3*(d + e*x^2)^(3/2))
 + (2*d*(a + b*ArcSech[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a +
 b*ArcSech[c*x]))/e^3 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[
e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*e^(5/2)) - (8*b*Sqrt[d]*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^
2*x^2])])/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} + \Big(b$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} + \frac{(b}{}$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} + \frac{(b}{}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{}}{}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{}}{}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{}}{}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{}}{}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{}}{}$$

Mathematica [A] time = 1.97, size = 348, normalized size = 1.28

$$\frac{a\left(c^2d+e\right)\left(8d^2+12dex^2+3e^2x^4\right)+b\left(c^2d+e\right)\text{sech}^{-1}(cx)\left(8d^2+12dex^2+3e^2x^4\right)-bde\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)}{3e^3\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(b*d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)) + a*(c^2*d + e)*(
8*d^2 + 12*d*e*x^2 + 3*e^2*x^4) + b*(c^2*d + e)*(8*d^2 + 12*d*e*x^2 + 3*e^2
*x^4)*ArcSech[c*x])/(3*e^3*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*Sqrt[(1 - c*
x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(3*Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sq
rt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqr
t[-c^2]*Sqrt[-(c^2*d) - e])] + 8*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[
d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(3*c^3*e^3*(-1 + c*x)*Sqrt[d + e*
x^2])
```

fricas [B] time = 1.19, size = 2415, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
 + b*d*e^2)*x^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*
d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sq
rt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(8*b*c^3*d^3 + 8*b*c*d^2*e
 + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 8*(b*c^3*d^
3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x
^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 +
4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) + 8*d^2)/x^4) - 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 +
 a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b*c^2*d*e^2*x^3 + b*c^2
*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c
*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(
3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e
^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*s
qrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 -
d*e)) - 2*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(
b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/
(c^2*x^2)) + 1)/(c*x)) - 4*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)
*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e +
 e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*
x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(8*a*c^3*
d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d
*e^2)*x^2 - (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*
(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/12*(16*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e
^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*(
(c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^
2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*lo
g(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^
4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + e^2) - 4*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)
*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3
*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b*c^2*d*e^2*x^3
 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3
*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2)
, -1/6*(8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d
^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*
d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d
- e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4
 + (c^2*d*e - e^2)*x^2 - d*e)) - 2*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*
e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(8*a*c^3*d^3 + 8*a*c*d^
2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b
*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2
+ d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 +
 c*d*e^5)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx) + a)x^5}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 4.58, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arcsech}(cx))}{(e x^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{3x^4}{(ex^2+d)^{\frac{3}{2}}e} + \frac{12dx^2}{(ex^2+d)^{\frac{3}{2}}e^2} + \frac{8d^2}{(ex^2+d)^{\frac{3}{2}}e^3}\right)a + b\int \frac{x^5 \log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1} + \frac{1}{cx}\right)}{(ex^2+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*(3*x^4/((e*x^2 + d)^(3/2)*e) + 12*d*x^2/((e*x^2 + d)^(3/2)*e^2) + 8*d^2
/((e*x^2 + d)^(3/2)*e^3))*a + b*integrate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/
(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

3.169 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{d}\,e^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[Out] 1/3*d*(a+b*arcsech(c*x))/e^2/(e*x^2+d)^(3/2)+2/3*b*arctanh((e*x^2+d)^(1/2)/
d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2/d^(1/2)+(-a
-b*arcsech(c*x))/e^2/(e*x^2+d)^(1/2)+1/3*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(-c^2*x^2+1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {266, 43, 6301, 12, 573, 152, 93, 207}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{d}\,e^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*e*(c^2*d + e)*S
qrt[d + e*x^2]) + (d*(a + b*ArcSech[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a +
 b*ArcSech[c*x])/(e^2*Sqrt[d + e*x^2]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*Sqrt[d]*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx = \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-2d\ }{3e^2x\sqrt{1-c^2x}}$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-2d-3ex^2}{x\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}}{3e^2}$$

$$= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{-2d-3}{x\sqrt{1-c^2x}\,(d}\right.}{6e^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+c}}\right.}{}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+c}}\right.}{}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{\left(2b\sqrt{\frac{}{1+}}\right.}{}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{2b\sqrt{\frac{1}{1+c}}}{}$$

Mathematica [A] time = 0.38, size = 218, normalized size = 1.22

$$\frac{-a\left(c^2d+e\right)\left(2d+3ex^2\right)-b\left(c^2d+e\right)\text{sech}^{-1}(cx)\left(2d+3ex^2\right)+be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)}{3e^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}} - \frac{2b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2}}{3\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2) - a*(c^2*d + e)*(2*d +
 3*e*x^2) - b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcSech[c*x])/(3*e^2*(c^2*d + e)*
(d + e*x^2)^(3/2)) - (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sqrt[
-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(3*Sqrt[d
]*e^2*(-1 + c*x)*Sqrt[d + e*x^2])
```

fricas [B] time = 0.78, size = 786, normalized size = 4.39

$$\left[-\frac{2\left(2bc^2d^3+2bd^2e+3\left(bc^2d^2e+bde^2\right)x^2\right)\sqrt{ex^2+d}\,\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(bc^2d^3+\left(bc^2de^2+be^3\right)x^4+bd^2e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 +
(b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(d
)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d
- c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^
2 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^
2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3
 + d^2*e^4)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2
*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c
*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4
+ (c^2*d^2 - d*e)*x^2 - d^2)) - (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)
/(c*x)) - (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 - (b*c*d
*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c
^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4
)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsech}(cx) + a)x^3}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 4.48, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(a + b\operatorname{arcsech}(cx))}{\left(e\,x^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a\left(\frac{3\,x^2}{\left(ex^2 + d\right)^{\frac{3}{2}}e} + \frac{2\,d}{\left(ex^2 + d\right)^{\frac{3}{2}}e^2}\right) + b\int \frac{x^3\log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*inte
grate(x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5
/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

3.170 $\int \frac{x\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[Out] 1/3*(-a-b*arcsech(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*arctanh((e*x^2+d)^(1/2)/d^(
1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^(3/2)/e-1/3*b*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2
)
```

Rubi [A] time = 0.29, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6299, 517, 446, 96, 93, 207}

$$-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d + e)*
Sqrt[d + e*x^2]) - (a + b*ArcSech[c*x])/(3*e*(d + e*x^2)^(3/2)) + (b*Sqrt[(
1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*
x^2])])/(3*d^(3/2)*e)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6299

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)),
 x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\left(d+ex^2\right)^{3/2}}\,dx}{3e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\ }{x}\right.}{6de}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \right.}{3de}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} - \frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{d}\sqrt{}}\right.}{3d^{3/2}e}$$

Mathematica [A] time = 0.32, size = 204, normalized size = 1.32

$$\frac{-ad\left(c^2d+e\right)-bd\left(c^2d+e\right)\text{sech}^{-1}(cx)-be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)}{3de\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}} - \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{}}\right.}{3d^{3/2}e(cx-1)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(a*d*(c^2*d + e)) - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2) -
 b*d*(c^2*d + e)*ArcSech[c*x])/(3*d*e*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*S
qrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]
*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(3*d^(3/2)*e*(-1 + c*x)*Sqrt[d + e*x
^2])
```

fricas [B] time = 1.41, size = 692, normalized size = 4.49

$$\left[-\frac{4\left(bc^2d^3+bd^2e\right)\sqrt{ex^2+d}\,\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(bc^2d^3+\left(bc^2de^2+be^3\right)x^4+bd^2e+2\left(bc^2d^2e+bde^2\right)x^2\right)\sqrt{d}\,\log}{12\left(c^2d^5e+d^4e^2+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(b*c^2*d^3 + b*d^2*e)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*
x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 +
d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*
d^2*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(
e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4
*e^2 + d^3*e^3)*x^2), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 -
 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*
x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*(b*c^2*d^3 + b*d^2*e)*sqrt(e*x^2 + d)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*c^2*d^3 + a*d^2
*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^
2 + d^3*e^3)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx)+a)x}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 2.58, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+b\,\mathrm{arcsech}\,(cx))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx-\frac{a}{3\left(ex^2+d\right)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d
)^(5/2), x) - 1/3*a/((e*x^2 + d)^(3/2)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.171 $\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

[Out] Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)),x]

[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 43.68, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)),x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

maple [A] time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsech}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\,a\left(\frac{3\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{5}{2}}}-\frac{3}{\sqrt{ex^{2}+d}\,d^{2}}-\frac{1}{\left(ex^{2}+d\right)^{\frac{3}{2}}d}\right)+b\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\,\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\left(ex^{2}+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*a*(3*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(5/2) - 3/(sqrt(e*x^2 + d)*d^2) -
 1/((e*x^2 + d)^(3/2)*d)) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x)
- 1) + 1/(c*x))/((e*x^2 + d)^(5/2)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

3.172 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 54.75, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{e^3x^9+3\,de^2x^7+3\,d^2ex^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

maple [A] time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{a + b \,\mathrm{arcsech}\left(c x\right)}{x^{3} \left(e \,x^{2}+d \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\, a \left(\frac{15 \, e \operatorname{arsinh}\left(\frac{d}{\sqrt{d e} {\left| x \right|}}\right)}{d^{\frac{7}{2}}} - \frac{15 \, e}{\sqrt{e x^{2} + d} d^{3}} - \frac{5 \, e}{{\left(e x^{2} + d\right)}^{\frac{3}{2}} d^{2}} - \frac{3}{{\left(e x^{2} + d\right)}^{\frac{3}{2}} d x^{2}}\right) + b \int \frac{\log\left(\sqrt{\frac{1}{c x} + 1} \sqrt{\frac{1}{c x} - 1} + \frac{1}{c x}\right)}{{\left(e x^{2} + d\right)}^{\frac{5}{2}} x^{3}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(15*e*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(7/2) - 15*e/(sqrt(e*x^2 + d)*d
^3) - 5*e/((e*x^2 + d)^(3/2)*d^2) - 3/((e*x^2 + d)^(3/2)*d*x^2)) + b*integr
ate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((e*x^2 + d)^(5/2)*x
^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.173 $\int \frac{x^6\left(a+b\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 14.89, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

fricas [A] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^6\,\text{arsech}\,(cx)+ax^6\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^6*arcsech(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsech}\,(cx)+a)x^6}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

maple [A] time = 4.40, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + b \operatorname{arcsech}(cx))}{(e x^2 + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(\frac{3x^5}{(ex^2+d)^{\frac{3}{2}}e} + \frac{5dx\left(\frac{3x^2}{(ex^2+d)^{\frac{3}{2}}e} + \frac{2d}{(ex^2+d)^{\frac{3}{2}}e^2}\right)}{e} + \frac{5dx}{\sqrt{ex^2+d}\,e^3} - \frac{15d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{7}{2}}}\right)a + b\int \frac{x^6 \log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}\cdots}\right.}{(ex^2+d)^{\frac{5}{2}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^5/((e*x^2 + d)^(3/2)*e) + 5*d*x*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d
/((e*x^2 + d)^(3/2)*e^2))/e + 5*d*x/(sqrt(e*x^2 + d)*e^3) - 15*d*arcsinh(e*
x/sqrt(d*e))/e^(7/2))*a + b*integrate(x^6*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x
) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^6 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.174 $\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 13.54, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4\,\text{arsech}\,(cx)+ax^4\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsech}\,(cx)+a\right)x^4}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

maple [A] time = 3.95, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b \,\mathrm{arcsech}\left(c x\right)\right)}{\left(e \,x^{2}+d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left(x \left(\frac{3 \, x^{2}}{\left(e x^{2} + d\right)^{\frac{3}{2}} e} + \frac{2 \, d}{\left(e x^{2} + d\right)^{\frac{3}{2}} e^{2}}\right) + \frac{x}{\sqrt{e x^{2} + d} e^{2}} - \frac{3 \, \operatorname{arsinh}\left(\frac{e x}{\sqrt{d e}}\right)}{e^{\frac{5}{2}}}\right) a + b \int \frac{x^{4} \log\left(\sqrt{\frac{1}{c x} + 1} \sqrt{\frac{1}{c x} - 1} + \frac{1}{c x}\right)}{\left(e x^{2} + d\right)^{\frac{5}{2}}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*(x*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + x/(sq
rt(e*x^2 + d)*e^2) - 3*arcsinh(e*x/sqrt(d*e))/e^(5/2))*a + b*integrate(x^4*
log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

3.175 $\int \frac{x^2\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}}-\frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx}}{3}$$

```
[Out] 1/3*x^3*(a+b*arcsech(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*x*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2)-1/3*b*c*EllipticE(
c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(
c^2*d+e)/(1+e*x^2/d)^(1/2)+1/3*b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/e/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {264, 6301, 12, 471, 423, 426, 424, 421, 419}

$$\frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}}-\frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -(b*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d + e
)*Sqrt[d + e*x^2]) + (x^3*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (
b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d))])/(3*d*e*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (b*Sqrt[(1 + c*
x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*
d))])/(3*c*d*e*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{3d\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{1-c\cdots}}{\sqrt{d+e\cdots}}}{3d\left(c^2d+e\right)}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\cdots}{\sqrt{1-c\cdots}}}{3de}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+e\cdots}\right.}{3de\left(c^2d+e\right)\sqrt{\cdots}}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2\cdots}}{3de\left(c^2d+e\right)\cdots}$$

Mathematica [C] time = 2.66, size = 488, normalized size = 1.98

$$\frac{ax^3 - \frac{b\sqrt{\frac{1-cx}{cx+1}}\left(d+ex^2\right)(ex-cd)}{e\left(c^2d+e\right)} + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\sqrt{\frac{c\left(\sqrt{e}x+i\sqrt{d}\right)}{(cx+1)\left(\sqrt{e}+ic\sqrt{d}\right)}}\left(\left(\sqrt{e}+ic\sqrt{d}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{\left(dc^2+e\right)(1-cx)}{\left(\sqrt{d}c+i\sqrt{e}\right)^2(cx+1)}}\right)\middle|\frac{\left(\sqrt{d}\cdots\right.}{\left(c\sqrt{d}\cdots\right.}\cdots\right)\cdots\right.}{ce\left(c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{(cx-1)\left(\sqrt{e}+ic\sqrt{d}\right)}{(cx+1)\left(\sqrt{e}-ic\sqrt{d}\right)}}}\cdots}{3d\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (a*x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(e*(c^2*d
 + e)) + b*x^3*ArcSech[c*x] + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[(
c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(I*
Sqrt[d] + Sqrt[e]*x))/((I*c*Sqrt[d] + Sqrt[e])*(1 + c*x))]*(d + e*x^2)*((I*
c*Sqrt[d] + Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*S
qrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] -
I*Sqrt[e])^2] - 2*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/
((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[
d] - I*Sqrt[e])^2]))/(c*(c*Sqrt[d] + I*Sqrt[e])*e*Sqrt[((I*c*Sqrt[d] + Sqrt
[e])*(-1 + c*x))/(((-I)*c*Sqrt[d] + Sqrt[e])*(1 + c*x))]))/(3*d*(d + e*x^2)
^(3/2))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2\operatorname{arsech}(cx) + ax^2\right)\sqrt{ex^2+d}}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 3.60, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arcsech}(cx))}{(e x^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a\left(\frac{x}{(ex^2 + d)^{\frac{3}{2}}e} - \frac{x}{\sqrt{ex^2 + d}\,de}\right) + b\int \frac{x^2 \log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^
2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.176 $\int \frac{a+b\,\mathbf{sech}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=266

$$\frac{2x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(\right.}{3cd^2\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*arcsech(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arcsech(c*x))/d^2/(e*
x^2+d)^(1/2)+1/3*b*e*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d
^2/(c^2*d+e)/(e*x^2+d)^(1/2)+1/3*b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(1+e*x^2/d)^(1/2)+2
/3*b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x
^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {192, 191, 6291, 12, 527, 524, 426, 424, 421, 419}

$$\frac{2x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(\right.}{3cd^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*e*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d^2*(c^2*d
+ e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) +
(2*x*(a + b*ArcSech[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*
d^2*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c
*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*c*d^2*Sqrt
[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 6291

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{3d+2e}{3d^2\sqrt{1-c^2x^2}\,(}$$

$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{3d+2ex^2}{\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}dx}{3d^2}$$

$$= \frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\right.}{}$$

$$= \frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{\left(2b\sqrt{\frac{1}{1+}}\right.}{}$$

$$= \frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{\left(bc^2\sqrt{\frac{}{1}}\right.}{}$$

$$= \frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{1+cx}}}{}$$

Mathematica [C] time = 5.50, size = 517, normalized size = 1.94

$$\frac{ax\left(3d+2ex^2\right) + \frac{b\sqrt{\frac{1-cx}{cx+1}}\left(d+ex^2\right)(ex-cd)}{c^2d+e} - \frac{ib\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(d+ex^2\right)\sqrt{\frac{c\left(\sqrt{d}-i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}}\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{(cx+1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(\left(c\sqrt{d}-i\sqrt{e}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{\left(dc^2+e\right.}{\left(\sqrt{d}c+i\sqrt{\right.}}}\right.\right.\right.}{c\left(c\sqrt{d}+i\sqrt{e}\right)\sqrt{-\frac{(cx-1}{(cx+1}}}}{3d^2\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(c^2*d + e) + a*x
*(3*d + 2*e*x^2) + b*x*(3*d + 2*e*x^2)*ArcSech[c*x] - (I*b*Sqrt[(1 - c*x)/(
1 + c*x)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e
])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1
 + c*x))]*(d + e*x^2)*((c*Sqrt[d] - I*Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c
^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] +
I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] - 2*(3*c*Sqrt[d] + (2*I)*Sqrt[e])*E
llipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*
(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(c*(c*
Sqrt[d] + I*Sqrt[e])*Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d
] + I*Sqrt[e])*(1 + c*x)))]))/(3*d^2*(d + e*x^2)^(3/2))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arsech}\,(cx)+a)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^
2*e*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsech}(cx)}{(e\,x^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{ex^2 + d}\,d^2} + \frac{x}{(ex^2 + d)^{\frac{3}{2}}\,d}\right) + b\int \frac{\log\left(\sqrt{\frac{1}{cx} + 1}\,\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.177 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\mathbf{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=596

$$\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\right.}{f^7(m+}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arcsech
(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arcsech(c*x))/f^5/(5+m)+e^3*(f*x)
^(7+m)*(a+b*arcsech(c*x))/f^7/(7+m)+b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1
+m)*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m
^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*(f*x)^(1+m)*hypergeom([1/2, 1
/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6/f/(1+m)/
(2+m)/(4+m)/(6+m)-b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3
*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)-
b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*(f*x)^(3+m)*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/f^3/(4+m)/(5+m)/(6+m)/(7+m)-b*e^3*(f*x)^(5+
m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/f^5/(6+m)/(7+m)
```

Rubi [A] time = 2.55, antiderivative size = 576, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {270, 6301, 1809, 1267, 459, 364}

$$\frac{3d^2e(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^3(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\right.}{f^7(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*
c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*(f*x)^(1 + m)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^6*f*(2 + m)*(3 + m)*(4 + m)*(5
+ m)*(6 + m)*(7 + m))) - (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*(
f*x)^(3 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^4*f^3
*(4 + m)*(5 + m)*(6 + m)*(7 + m)) - (b*e^3*(f*x)^(5 + m)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^2*f^5*(6 + m)*(7 + m)) + (d^3*(f*x)^
(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a + b*A
rcSech[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcSech[c*x]))/
(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcSech[c*x]))/(f^7*(7 + m)) + (b
*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13
*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/(c^6*(2 + m)
*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) dx = \frac{d^3(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{3d\cdots}{\cdots}$$

$$= -\frac{be^3(fx)^{5+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^5(6+m)(7+m)} + \frac{d^3(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx\cdots\right.}{f(1+m)}$$

$$= -\frac{be^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1\cdots}}{c^4f^3(4+m)(5+m)(6+m)(7+m)}$$

$$= -\frac{be\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(\cdots\right.\right.}{c^6f(2+m)(3+m)(4+\cdots}$$

$$= -\frac{be\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(\cdots\right.\right.}{c^6f(2+m)(3+m)(4+\cdots}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]), x]
```

fricas [F] time = 1.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\text{arsech}\left(cx\right)\right)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsech(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^3 (b\,\text{arsech}\left(cx\right)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \left(e\,x^2+d\right)^3 (a+b\,\text{arcsech}\left(cx\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^3f^mx^7x^m}{m+7}+\frac{3\,ade^2f^mx^5x^m}{m+5}+\frac{3\,ad^2ef^mx^3x^m}{m+3}+\frac{(fx)^{m+1}ad^3}{f(m+1)}+\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)be^3f^mx^7x^m+3\left(m^3+1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*x
^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((m^3 + 9*m^2 + 23*m +
15)*b*e^3*f^m*x^7*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5*x^m +
3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3*x^m + (m^3 + 15*m^2 + 71*m + 1
05)*b*d^3*f^m*x*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - ((m^3 + 9*m^2
+ 23*m + 15)*b*e^3*f^m*x^7*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x
^5*x^m + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3*x^m + (m^3 + 15*m^2 +
 71*m + 105)*b*d^3*f^m*x*x^m)*log(x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
 - integrate((b*c^2*e^3*f^m*(m + 7)*x^2*log(c) - (e^3*f^m*(m + 7)*log(c) -
e^3*f^m)*b)*x^6*x^m/(c^2*(m + 7)*x^2 - m - 7), x) - integrate(3*(b*c^2*d*e^
2*f^m*(m + 5)*x^2*log(c) - (d*e^2*f^m*(m + 5)*log(c) - d*e^2*f^m)*b)*x^4*x^
m/(c^2*(m + 5)*x^2 - m - 5), x) - integrate(3*(b*c^2*d^2*e*f^m*(m + 3)*x^2*
log(c) - (d^2*e*f^m*(m + 3)*log(c) - d^2*e*f^m)*b)*x^2*x^m/(c^2*(m + 3)*x^2
 - m - 3), x) - integrate((b*c^2*d^3*f^m*(m + 1)*x^2*log(c) - (d^3*f^m*(m +
 1)*log(c) - d^3*f^m)*b)*x^m/(c^2*(m + 1)*x^2 - m - 1), x) + integrate(((m^
3 + 9*m^2 + 23*m + 15)*b*c^2*e^3*f^m*x^8*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)
*b*c^2*d*e^2*f^m*x^6*x^m + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^2*e*f^m*x^4
*x^m + (m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2*x^m)/((m^4 + 16*m^3 +
86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 + ((m^4 + 16*m^3 + 86*m^2 + 17
6*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105)*sqrt(c*x + 1)*sqr
t(-c*x + 1) - 86*m^2 - 176*m - 105), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m\,(a+b\,\mathrm{asech}\,(cx))\left(d+ex^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*asech(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2)**3, x)
```

3.178 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=372

$$\frac{d^2(fx)^{m+1}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{f^5(m+5)}-\frac{be^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\sqrt{}}{c^2f^3(m+4}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arcsech(c
*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arcsech(c*x))/f^5/(5+m)+b*(c^4*d^2*(2+m
)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*(f*x)^(1+m)
*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^
(1/2)/c^4/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)-b*e*(e*(3+m)^2+2*c^2*d*(m^2+9*m
+20))*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/f/
(4+m)/(5+m)/(m^2+5*m+6)-b*e^2*(f*x)^(3+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)/c^2/f^3/(4+m)/(5+m)
```

Rubi [A] time = 0.43, antiderivative size = 352, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {270, 6301, 12, 1267, 459, 364}

$$\frac{d^2(fx)^{m+1}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{f^5(m+5)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,(f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*(f*x)^(1 + m)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^4*f*(2 + m)*(3 + m)*(4 + m)*(5
+ m))) - (b*e^2*(f*x)^(3 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c
^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)) + (d^2*(f*x)^(1 + m)*(a + b*ArcSech[c*x]
))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) +
 (e^2*(f*x)^(5 + m)*(a + b*ArcSech[c*x]))/(f^5*(5 + m)) + (b*(d^2/(1 + m)^2
 + (e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)))/(c^4*(2 + m)*(3 + m)*(4 + m
)*(5 + m)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric
2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right) dx = \frac{d^2(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)} + \cdots$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)} + \cdots$$

$$= -\frac{be^2(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^3(4+m)(5+m)} + \frac{d^2(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(\cdots\right.}{f(1+m)}$$

$$= -\frac{be\left(e(3+m)^2+2c^2d\left(20+9m+m^2\right)\right)(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1\cdots}}{c^4f(2+m)(4+m)\left(15+8m+m^2\right)}$$

$$= -\frac{be\left(e(3+m)^2+2c^2d\left(20+9m+m^2\right)\right)(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1\cdots}}{c^4f(2+m)(4+m)\left(15+8m+m^2\right)}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]), x]
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\operatorname{arsech}(cx)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsech(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^2 (b\operatorname{arsech}(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \left(e\,x^2+d\right)^2 (a+b\operatorname{arcsech}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^2f^mx^5x^m}{m+5}+\frac{2\,adef^mx^3x^m}{m+3}+\frac{\left(fx\right)^{m+1}ad^2}{f(m+1)}+\frac{\left(\left(m^2+4\,m+3\right)be^2f^mx^5x^m+2\left(m^2+6\,m+5\right)bdef^mx^3x^m+\left(m^2+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d
^2/(f*(m + 1)) + (((m^2 + 4*m + 3)*b*e^2*f^m*x^5*x^m + 2*(m^2 + 6*m + 5)*b*
d*e*f^m*x^3*x^m + (m^2 + 8*m + 15)*b*d^2*f^m*x*x^m)*log(sqrt(c*x + 1)*sqrt(
-c*x + 1) + 1) - ((m^2 + 4*m + 3)*b*e^2*f^m*x^5*x^m + 2*(m^2 + 6*m + 5)*b*d
*e*f^m*x^3*x^m + (m^2 + 8*m + 15)*b*d^2*f^m*x*x^m)*log(x))/(m^3 + 9*m^2 + 2
3*m + 15) - integrate((b*c^2*e^2*f^m*(m + 5)*x^2*log(c) - (e^2*f^m*(m + 5)*
log(c) - e^2*f^m)*b)*x^4*x^m/(c^2*(m + 5)*x^2 - m - 5), x) - integrate(2*(b
*c^2*d*e*f^m*(m + 3)*x^2*log(c) - (d*e*f^m*(m + 3)*log(c) - d*e*f^m)*b)*x^2
*x^m/(c^2*(m + 3)*x^2 - m - 3), x) - integrate((b*c^2*d^2*f^m*(m + 1)*x^2*l
og(c) - (d^2*f^m*(m + 1)*log(c) - d^2*f^m)*b)*x^m/(c^2*(m + 1)*x^2 - m - 1)
, x) + integrate(((m^2 + 4*m + 3)*b*c^2*e^2*f^m*x^6*x^m + 2*(m^2 + 6*m + 5)
*b*c^2*d*e*f^m*x^4*x^m + (m^2 + 8*m + 15)*b*c^2*d^2*f^m*x^2*x^m)/((m^3 + 9*
m^2 + 23*m + 15)*c^2*x^2 - m^3 + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 -
 9*m^2 - 23*m - 15)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 9*m^2 - 23*m - 15), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^m\,{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{asech}(cx)) \left(d + ex^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2)**2, x)
```

3.179 $\int (fx)^m \left(d+ex^2\right) \left(a+b\,\text{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=206

$$\frac{d(fx)^{m+1}\left(a+b\,\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\,\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,(fx)^{m+1}\left(c^2d(m+2)(m+3)+e(m+\right.}{c^2f(m+1)^2(m+2)(m+3}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arcsech(c*x))/f
^3/(3+m)+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^2/f/(1+m)^2/(2+
m)/(3+m)-b*e*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
/c^2/f/(2+m)/(3+m)
```

Rubi [A] time = 0.18, antiderivative size = 192, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {14, 6301, 12, 459, 364}

$$\frac{d(fx)^{m+1}\left(a+b\,\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\,\text{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,(fx)^{m+1}\left(\frac{e}{c^2(m+2)(m+3)}+\frac{d}{(m+1)^2}\right){}_2F_1\left(\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(c^2*f*(2 + m)*(3 + m))) + (d*(f*x)^(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m
)) + (e*(f*x)^(3 + m)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) + (b*(d/(1 + m)^2
 + e/(c^2*(2 + m)*(3 + m)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x
]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{d(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)}+\left(b\sqrt{\frac{}{1}}\right.$$

$$= \frac{d(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)}+\frac{\left(b\sqrt{\frac{}{1}}\right.}{}$$

$$= -\frac{be(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f(2+m)(3+m)}+\frac{d(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx\right.}{f(1+m)}$$

$$= -\frac{be(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f(2+m)(3+m)}+\frac{d(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx\right.}{f(1+m)}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]), x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2+ad+\left(bex^2+bd\right)\text{arsech}\,(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)(b\,\text{arsech}\,(cx)+a)\,(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right)(a+b\,\text{arcsech}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{aef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} ad}{f(m+1)} + \frac{\left(bef^m(m+1)x^3x^m + bdf^m(m+3)xx^m\right)\log\left(\sqrt{cx+1}\sqrt{-cx+1}+1\right) - \left(bef^m(m+1)x^3 x\right.}{m^2+4m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + ((b*e*f^m*(m + 1)
*x^3*x^m + b*d*f^m*(m + 3)*x*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - (
b*e*f^m*(m + 1)*x^3*x^m + b*d*f^m*(m + 3)*x*x^m)*log(x))/(m^2 + 4*m + 3) -
integrate((b*c^2*e*f^m*(m + 3)*x^2*log(c) - (e*f^m*(m + 3)*log(c) - e*f^m)*
b)*x^2*x^m/(c^2*(m + 3)*x^2 - m - 3), x) - integrate((b*c^2*d*f^m*(m + 1)*x
^2*log(c) - (d*f^m*(m + 1)*log(c) - d*f^m)*b)*x^m/(c^2*(m + 1)*x^2 - m - 1)
, x) + integrate((b*c^2*e*f^m*(m + 1)*x^4*x^m + b*c^2*d*f^m*(m + 3)*x^2*x^m
)/((m^2 + 4*m + 3)*c^2*x^2 + ((m^2 + 4*m + 3)*c^2*x^2 - m^2 - 4*m - 3)*sqrt
(c*x + 1)*sqrt(-c*x + 1) - m^2 - 4*m - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^m\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(a + b \operatorname{asech}{\left(cx \right)}\right) \left(d + ex^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2), x)
```

3.180 $\int \frac{(fx)^m \left(a+b\,\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m \left(a+b\,\text{sech}^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m \left(a+b\,\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\,\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m \left(a+b\,\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [A] time = 2.48, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\,\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arsech}\,(cx)+a)\left(fx\right)^m}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsech}\,(cx)+a)\left(fx\right)^m}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

maple [A] time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsech}(cx))}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a) (fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f x)^m \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{asech}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))/(d + e*x**2), x)
```

3.181 $\int \frac{(fx)^m\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 7.55, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]
```

fricas [A] time = 1.05, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b\operatorname{arsech}(cx)+a)\left(fx\right)^m}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsech}(cx)+a)\left(fx\right)^m}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsech}(cx))}{(e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f x)^m \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.182 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$$

Mathematica [A] time = 1.08, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(aex^2+ad+\left(bex^2+bd\right)\mathrm{arsech}\left(cx\right)\right)\sqrt{ex^2+d}\,\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)*(f*
x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsech}\left(cx\right)+a\right)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

maple [A] time = 1.79, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsech}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^{2}+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsech}\left(cx\right)+a\right)\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(f\,x\right)^{m}\,\left(e\,x^{2}+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
[Out] Timed out
```

3.183 $\int (fx)^m \sqrt{d+ex^2} \left(a+b\,\mathbf{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sqrt{d+ex^2}(fx)^m\left(a+b\,\mathrm{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx=\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

maple [A] time = 1.72, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m(a+b\,\mathrm{arcsech}\,(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)\,(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m\,(a+b\,\mathrm{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

3.184 $\int \frac{(fx)^m\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

fricas [A] time = 1.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b\operatorname{arsech}(cx)+a)\left(fx\right)^m}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsech}(cx)+a)\left(fx\right)^m}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

maple [A] time = 2.58, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arcsech}\left(cx\right)\right)}{\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)\left(fx\right)^{m}}{\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(f\,x\right)^{m}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{asech}\left(cx\right)\right)}{\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

3.185 $\int \frac{(fx)^m\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(fx)^m\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arsech}\,(cx)+a)\,(fx)^m}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsech}\,(cx)+a)\,(fx)^m}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 2.46, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsech}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsech}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{asech}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

3.186 $\int \frac{x^{11}\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=473

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}-\frac{b\sqrt{1-c^2x^2}}{90c^{13}x\sqrt{\frac{1}{cx}}}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arcsech(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b
*arcsech(c*x))/c^12+7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(-1+
1/c/x)^(1/2)/(1+1/c/x)^(1/2)-13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/
c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+
1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/90*b*(c^2*x^2+1)^(9/2)*(
-c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+4/15*b*arctanh((c
^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)
-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

Rubi [A] time = 1.58, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 43, 6309, 12, 6742, 848, 50, 63, 208, 783}

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}-\frac{b\sqrt{1-c^2x^2}}{90c^{13}x\sqrt{\frac{1}{cx}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt
[1 + 1/(c*x)]*x) + (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2
)^(5/2))/(150*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) + (3*b*Sqrt[1 -
c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]
*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqrt[-1 + 1/(c*x)]
*Sqrt[1 + 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/(2*c^12) +
 ((1 - c^4*x^4)^(3/2)*(a + b*ArcSech[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)
*(a + b*ArcSech[c*x]))/(10*c^12) + (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 +
c^2*x^2]])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6309

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcSech[c*x], v, x] + Dist[(b*Sqrt[1 - c^2*x^2])/(c*x*
Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]), Int[SimplifyIntegrand[v/(x*Sqrt[1 -
c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^{11}\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/}}{}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+\right.}{3c^{1}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{7b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{13b\sqrt{1-c^2x^2}\left(1+c\right.}{150c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{7b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{13b\sqrt{1-c^2x^2}\left(1+c\right.}{150c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1}}$$

Mathematica [A] time = 0.44, size = 213, normalized size = 0.45

$$\frac{-105a\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)-840b\log\left(-\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^4x^4}-cx+1\right)-105b\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x\right.}{3150c^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*Sqrt[(1 - c*x)/(
1 + c*x)]*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 + 35
*c^8*x^8))/(-1 + c*x) - 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8)
*ArcSech[c*x] + 840*b*Log[x*(1 - c*x)] - 840*b*Log[1 - c*x - Sqrt[(1 - c*x)
/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(3150*c^12)
```

fricas [A] time = 0.67, size = 393, normalized size = 0.83

$$-\frac{105\left(3\,bc^{10}x^{10}-3\,bc^{8}x^{8}+4\,bc^{6}x^{6}-4\,bc^{4}x^{4}+8\,bc^{2}x^{2}-8\,b\right)\sqrt{-c^{4}x^{4}+1}\,\log\left(\frac{cx\sqrt{-\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}+1}{cx}\right)-\left(35\,bc^{9}x^{9}+5\,bc^{7}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3150*(105*(3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6*x^6 - 4*b*c^4*x^4 + 8*b
*c^2*x^2 - 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
+ 1)/(c*x)) - (35*b*c^9*x^9 + 5*b*c^7*x^7 + 78*b*c^5*x^5 + 36*b*c^3*x^3 + 7
68*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 420*(b*c^2*x^
2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
 - 1)/(c^2*x^2 - 1)) - 420*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 +
1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + 105*(3*a*c^10*x
^10 - 3*a*c^8*x^8 + 4*a*c^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 - 8*a)*sqrt(-c^
4*x^4 + 1))/(c^14*x^2 - c^12)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 6.40, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(a+b\,\mathrm{arcsech}\left(cx\right)\right)}{\sqrt{-c^{4}x^{4}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{30}\,a\left(\frac{3\left(-c^{4}x^{4}+1\right)^{\frac{5}{2}}}{c^{12}}-\frac{10\left(-c^{4}x^{4}+1\right)^{\frac{3}{2}}}{c^{12}}+\frac{15\sqrt{-c^{4}x^{4}+1}}{c^{12}}\right)+\frac{1}{30}\,b\left(\frac{\left(3\,c^{12}x^{12}+c^{8}x^{8}+4\,c^{4}x^{4}-8\right)\log\left(\sqrt{cx+1}\,\sqrt{\right.}\right.}{\sqrt{c^{2}x^{2}+1}\,\sqrt{cx+1}\,\sqrt{-cx+1}\,c^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*lo
g(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-
c*x + 1)*c^12) - 30*integrate(1/30*(30*c^10*x^21*log(c) + 60*c^10*x^21*log(
sqrt(x)) + (60*c^10*x^21*log(sqrt(x)) + (3*c^10*x^10*(10*log(c) + 1) + 3*c^
8*x^8 + 4*c^6*x^6 + 4*c^4*x^4 + 8*c^2*x^2 + 8)*x^11)*e^(1/2*log(c*x + 1) +
```

```
1/2*log(-c*x + 1)))/((c^10*x^10*e^(log(c*x + 1) + log(-c*x + 1)) + c^10*x^1
0*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11} \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{\sqrt{1 - c^4 x^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^11*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Timed out
```

3.187 $\int \frac{x^7\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=316

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)^{5/2}}{30c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)}{18c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+}}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arcsech(c*x))/c^8+1/18*b*(c^2*x^2+1)^(3/2)*(-c^
2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/30*b*(c^2*x^2+1)^(5
/2)*(-c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/3*b*arctanh
((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/
2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^8
```

Rubi [A] time = 1.38, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 43, 6309, 12, 6742, 848, 50, 63, 208, 783}

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)^{5/2}}{30c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)}{18c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(3*c^9*Sqrt[-1 + 1/(c*x)]*Sqrt[1 +
 1/(c*x)]*x) + (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[-1 +
1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(
30*c^9*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*
ArcSech[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)*(a + b*ArcSech[c*x]))/(6*c^8)
 + (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(3*c^9*Sqrt[-1 + 1/(c*x
)]*Sqrt[1 + 1/(c*x)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6309

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcSech[c*x], v, x] + Dist[(b*Sqrt[1 - c^2*x^2])/(c*x*
Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]), Int[SimplifyIntegrand[v/(x*Sqrt[1 -
c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^7 \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\int}{c\sqrt{-1+}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\int}{6c^9\sqrt{-1}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)S}{12c}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)S}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)S}{6c^9\sqrt{-}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)S}{6c^9\sqrt{-}}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sec}\right.}{6c^8}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{18c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{30c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} -$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} + \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{18c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} - \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{30c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x} -$$

Mathematica [A] time = 0.38, size = 178, normalized size = 0.56

$$\frac{-15a\sqrt{1-c^4x^4}\left(c^4x^4+2\right) - 30b\log\left(-\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^4x^4}-cx+1\right) - 15b\sqrt{1-c^4x^4}\left(c^4x^4+2\right)\text{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1-c}{cx+}}}{}}{90c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[
1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c*x) - 15*b*Sqrt[1 - c^4*x^4
]*(2 + c^4*x^4)*ArcSech[c*x] + 30*b*Log[x*(1 - c*x)] - 30*b*Log[1 - c*x - S
qrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(90*c^8)
```

fricas [A] time = 0.58, size = 336, normalized size = 1.06

$$-\frac{15\left(bc^6x^6 - bc^4x^4 + 2\,bc^2x^2 - 2\,b\right)\sqrt{-c^4x^4+1}\,\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - \left(3\,bc^5x^5 + bc^3x^3 + 28\,bcx\right)\sqrt{-c^4x^4+1}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*(15*(b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b)*sqrt(-c^4*x^4 + 1)*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (3*b*c^5*x^5 + b*c^3*x^
3 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*(b*c^2
*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2)) - 1)/(c^2*x^2 - 1)) - 15*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*x^4
+ 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + 15*(a*c^6*x^6
 - a*c^4*x^4 + 2*a*c^2*x^2 - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 5.73, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(a + b\,\mathrm{arcsech}\left(cx\right)\right)}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,a\left(\frac{\left(-c^4x^4+1\right)^{\frac{3}{2}}}{c^8} - \frac{3\,\sqrt{-c^4x^4+1}}{c^8}\right) + \frac{1}{6}\,b\left(\frac{\left(c^8x^8 + c^4x^4 - 2\right)\log\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,+1\right)}{\sqrt{c^2x^2+1}\,\sqrt{cx+1}\,\sqrt{-cx+1}\,c^8} - 6\int \frac{6\,c^6x^{13}\log(c)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*x
^8 + c^4*x^4 - 2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*
sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8) - 6*integrate(1/6*(6*c^6*x^13*log(c) + 12
*c^6*x^13*log(sqrt(x)) + (12*c^6*x^13*log(sqrt(x)) + (c^6*x^6*(6*log(c) + 1
) + c^4*x^4 + 2*c^2*x^2 + 2)*x^7)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))
/((c^6*x^6*e^(log(c*x + 1) + log(-c*x + 1)) + c^6*x^6*e^(1/2*log(c*x + 1) +
 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7 \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^7*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\,(a+b\,\mathrm{asech}\,(cx))}{\sqrt{-\,(cx-1)\,(cx+1)\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Integral(x**7*(a + b*asech(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

3.188 $\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=159

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{b\sqrt{1-c^2x^2}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

```
[Out] 1/2*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^5/x/(-1+1/c/x)^(1/2)/
(1+1/c/x)^(1/2)-1/2*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^5/x/(-1+1/c/x)
^(1/2)/(1+1/c/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^4
```

Rubi [A] time = 0.16, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {261, 6309, 12, 1252, 848, 50, 63, 208}

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{b\sqrt{1-c^2x^2}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(2*c^5*Sqrt[-1 + 1/(c*x)]*Sqrt[1 +
 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/(2*c^4) + (b*Sqrt[1
 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(2*c^5*Sqrt[-1 + 1/(c*x)]*Sqrt[1 +
1/(c*x)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 6309

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcSech[c*x], v, x] + Dist[(b*Sqrt[1 - c^2*x^2])/(c*x*
Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]), Int[SimplifyIntegrand[v/(x*Sqrt[1 -
c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}+\frac{\left(b\sqrt{1-c^2x^2}\right)\int -\frac{\sqrt{1-c^4x^4}}{2c^4x\sqrt{1-c^2x^2}}\,dx}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}$$

$$=-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}$$

$$=-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{4c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}$$

$$=-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{4c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}$$

$$=-\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \right.}{4c^5\sqrt{-1+\frac{1}{cx}}}$$

$$=-\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \right.}{2c^7\sqrt{-1+}}$$

$$=-\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^4}+\frac{b\sqrt{1-c^2x^2}\tanh^{-1}\left(\sqrt{\right.}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+}}$$

Mathematica [A] time = 0.35, size = 140, normalized size = 0.88

$$-\frac{a\sqrt{1-c^4x^4}+\frac{b\sqrt{1-c^4x^4}}{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}+b\log\left(-\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^4x^4}-cx+1\right)+b\sqrt{1-c^4x^4}\operatorname{sech}^{-1}(cx)-b\log(x(1-cx))}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(a*Sqrt[1 - c^4*x^4] + (b*Sqrt[1 - c^4*x^4])/(Sqrt[(1 - c*x)/(1 + c*x)
]*(1 + c*x)) + b*Sqrt[1 - c^4*x^4]*ArcSech[c*x] - b*Log[x*(1 - c*x)] + b*Lo
g[1 - c*x - Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/c^4
```

fricas [B] time = 0.67, size = 279, normalized size = 1.75

$$\frac{2\sqrt{-c^4x^4+1}\,bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{-c^4x^4+1}\left(bc^2x^2-b\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(bc^2x^2-b\right)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}\,cx\sqrt{}}{c^2x^2-1}\right.}{4\left(c^6x^2-c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*sqrt(-c^
4*x^4 + 1)*(b*c^2*x^2 - b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x)) - (b*c^2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(
-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) - 2*sq
rt(-c^4*x^4 + 1)*(a*c^2*x^2 - a))/(c^6*x^2 - c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsech}(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

maple [F] time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+b\operatorname{arcsech}(cx)\right)}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}b\left(\frac{\left(c^4x^4-1\right)\log\left(\sqrt{cx+1}\sqrt{-cx+1}+1\right)}{\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{-cx+1}\,c^4}-2\int\frac{2c^2x^5\log(c)+4c^2x^5\log\left(\sqrt{x}\right)+\left(4c^2x^5\log\left(\sqrt{x}\right)+\left(c^2x^2\right.\right.}{2\left(c^2x^2e^{\left(\log(cx+1)+\log(-cx+1)\right)}+c^2x^2e^{\left(\frac{1}{2}\log(\right.}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*((c^4*x^4 - 1)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 +
1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate(1/2*(2*c^2*x^5*log(c) +
4*c^2*x^5*log(sqrt(x)) + (4*c^2*x^5*log(sqrt(x)) + (c^2*x^2*(2*log(c) + 1)
+ 1)*x^3)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*x^2*e^(log(c*x +
1) + log(-c*x + 1)) + c^2*x^2*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqr
t(c^2*x^2 + 1)), x)) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1 - c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(a + b \operatorname{asech}\left(cx\right)\right)}{\sqrt{-\left(cx - 1\right)\left(cx + 1\right)\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

3.189 $\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{a+b\,\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

fricas [A] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}\,(b\,\text{arsech}\,(cx)+a)}{c^4x^5-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^5 - x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsech}\,(cx)+a}{\sqrt{-c^4x^4+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

maple [A] time = 3.20, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsech}\left(cx\right)}{x\sqrt{-c^{4}x^{4}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\,a\left(\log\left(\sqrt{-c^{4}x^{4}+1}+1\right)-\log\left(\sqrt{-c^{4}x^{4}+1}-1\right)\right)+b\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\sqrt{-\left(c^{2}x^{2}+1\right)\left(cx+1\right)\left(cx-1\right)}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(sqrt(-(c^2*x^2 +
1)*(c*x + 1)*(c*x - 1))*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asech}\left(cx\right)}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

3.190 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Mathematica [A] time = 3.54, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

fricas [A] time = 1.24, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^4x^4+1}\,(b\,\mathrm{arsech}\,(cx)+a)}{c^4x^9-x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^9 - x^5), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsech}\,(cx)+a}{\sqrt{-c^4x^4+1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arcsech}\,(cx)}{x^5\sqrt{-c^4x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(c^4\log\left(\sqrt{-c^4x^4 + 1} + 1\right) - c^4\log\left(\sqrt{-c^4x^4 + 1} - 1\right) + \frac{2\sqrt{-c^4x^4 + 1}}{x^4}\right)a + b\int \frac{\log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{\sqrt{-(c^2x^2 + 1)(cx + 1)(cx - 1)}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x)
 - 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1 - c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asech}\,(cx)}{x^5\sqrt{-(cx - 1)(cx + 1)\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```